Experimental and Numerical Investigations of the Aeroelastic Stability of Bluff Structures



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A thesis submitted for the degree of *Doctor of Philosophy* February 2012

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Abstract

The study of the dynamic interactions between the wind and civil engineering structures has become increasingly important over the last few decades. Most of these structures are aerodynamically 'bluff' and are becoming more flexible. Bluff body aeroelasticity is a very challenging research area due to the unsteadiness and nonlinearity of the aerodynamic loading.

This thesis presents the investigation of three aeroelastic phenomena affecting bluff bodies: Vortex Induced Vibration (VIV), Galloping and Torsional Flutter. For each instability, extensive experimental studies are carried out in the wind tunnel. Innovative analysis, based on the Common-base Proper Orthogonal Decomposition (CPOD) method, is used to study the flow visualization data.

The VIV phenomenon is studied on a flexible tube with a circular cross-section, supported from its midpoint. A CPOD-based input-output model is developed to describe the system. The galloping instability is studied on a generic bridge section. A complete analysis of the aeroelastic behaviour of the structure is presented and a new polynomial empirical model is developed, which reflects accurately the nonlinear nature of the system. The torsional flutter phenomenon is extensively studied for two different structures: a generic bridge deck and a rectangular cylinder. The Motion Induced Vortex is identified as the fundamental cause of this aeroelastic phenomenon, on the basis of the analysis of the flow around the oscillating rectangle. In addition, it is demonstrated that the quasi-steady theory is not adapted to estimate the onset velocity of torsional flutter.

Finally, a 2D aeroelastic simulation code based on the Discrete Vortex Method (DVM) is developed. The nonlinear aerodynamics around the body are well reproduced, allowing the simulation of all the aeroelastic instabilities investigated experimentally.

Acknowledgements

I would like to acknowledge my supervisor Professor G. Dimitriadis for his constant guidance and the confidence he put in me during these four years. I enjoyed the interesting discussions we had, leading to good ideas in the manner to orient my research work... or bad ones to be discarded immediately.

I also want to thank all the members of the research group, especially Norizham Abdul Razak, who spent so much time with me in the wind tunnel laboratory, preparing our experiments. Thanks also to: Emile Kagambage, Jose Ignacio Rothkegel, Chandra Shekhar Prasad and Damien Watrin for their contributions to my work through the corridor's discussions, set-up installations, etc. Experimental work is impossible without the support of a technical team. Therefore, I wish to thank Antonio Martinez and Didier Lathouwers for supporting my suggestions about the design of experimental apparatus.

I also wish to thank my family and my friends for their support, received in all its forms. Finally and foremost, my thoughts go to Séverine, for all the rest...

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Nomenclature

Roman Symbols

\bar{u} Time averaged horizon	tal airspeed
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- \bar{v} Time averaged vertical airspeed
- **D** (C)POD co-variance matrix
- U CPOD global matrix (horizontal component)
- V CPOD global matrix (vertical component)
- Y CPOD global matrix (structural response)
- u Velocity vector
- A Amplitude of LCO (pitch dof)
- a_k Damping coefficient of the k^{th} mode
- B Chord of the body
- *c* Damping coefficient of the heave dof
- c_{α} Damping coefficient of the pitch dof
- D Depth of the body
- dt Time step
- dt_{PIV} Time step of the Tr-PIV measurements
- f Frequency of LCO
- f_0 Frequency at zero airspeed
- f_s Shedding frequency of the Karman Vortices
- $F_y(t)$ Aerodynamic vertical force
- $H_i(\omega)$ *ith* FRF in the input-output (C)POD based model

- I_{α} Polar moment of inertia
- k Structural stiffness of the heave dof
- k_{α} Structural stiffness of the pitch dof
- m Mass of the heave dof
- $M_a(t)$ Aerodynamic moment
- $Q_i(\omega)$ i^{th} generalized coordinate in the frequency domain
- $q_i(t)$ (C)POD generalized coordinates
- r Position of the aerodynamic center
- T_c Convection time of the MIV
- $u^*(x, y, t)$ (C)POD reconstructed horizontal airspeed
- V_c Convection velocity of the MIV
- V_{∞} Free stream airspeed
- Y Amplitude of LCO (heave dof)
- y_0 Initial structural displacement
- $Z(\omega)$ Cylinder displacement in the frequency domain
- **C** Damping matrix
- $\mathbf{f}_{\mathbf{a}}(V_{\infty}, \dot{\mathbf{x}}(t), \mathbf{x}(t))$ Unsteady aerodynamic force vector
- $\mathbf{f}_{\mathbf{e}}(t)$ External force vector
- **K** Stiffness matrix
- M Mass matrix

Greek Symbols

- α' Total angle of attack
- α_s Static angle of attack
- $\Delta \alpha$ Peak to peak amplitude of oscillation
- λ (C)POD eigenvalues
- u Kinematic viscosity
- $\phi_i(x, y)$ (C)POD mode shape
- ϕ_L Phase shift of the lift force

ρ	Air density	\mathbf{U}_∞	Free stream velocity vector
ξ_0	Damping at zero airspeed	Ω	Variation of angular velocity of the
Non-o	limensional numbers		body
C_D	Aerodynamic drag coefficient	\overline{ds}	Mean panel length
C_L	Aerodynamic lift coefficient	σ_c	Radius of the blob vortex
C_L^{max}	Peak value of the lift coefficient	A_b	Surface of the body
C_M	Aerodynamic moment coefficient	d	Distance between two vortices
Re	Reynolds number	ds_i	Length of panel i
S_G	Skop-Griffin number	dt	Time step
Sc	Scruton number	k_1	Numerical parameter related to dt
St	Strouhal number	k_2	Numerical parameter related to σ_c
U_r	Reduced airspeed	p_i^k	Pressure at the k^{th} time step at panel i
DVM notations			Velocity of a blob vortex at the k^{th} time step
$lpha_k$	Pitch position	\mathbf{x}_w^k	Position of a blob vortex at the k^{th} time
ω	Vorticity vector		step
δ^i_r	Releasing distance		
\dot{lpha}_k	Pitching velocity		
η	Random vector		
Г	Circulation of a blob vortex		
γ	Surface vortex sheet		
Γ_0	Initial amount of circulation		
Γ^k_{far}	Sum of the circulation of all the blob vortices at far-field at the k^{th} time step		
Γ^k_{in}	Sum of the circulation of all the vortical particles entering the body at the k^{th} time step		
Γ_i	Circulation of a blob vortex		
Γ^k_{wake}	Sum of the circulation of all the blob vortices at the k^{th} time step		
Ψ	Stream function		

Normal vector to the surface

 \mathbf{n}

List of Acronyms

AOA	Angle Of Attack
\mathbf{CFD}	Computational Fluid Dynamics
CPOD	Common-base Proper Orthogonal Decomposition
DOF	Degree Of Freedom
\mathbf{DVM}	Discrete Vortex Method
EIE	Extraneously Induced Excitation
\mathbf{FRF}	Frequency Response Function
HSTF	High Speed Torsional Flutter
HSG	High Speed Galloping
IIE	Instability Induced Excitation
\mathbf{KV}	Karman Vortex
LCO	Limit Cycle Oscillation
LSCF	Least Squares Complex Frequency Domain
LSTF	Low Speed Torsional Flutter
\mathbf{LSG}	Low Speed Galloping
MIE	Motion Induced Excitation
\mathbf{PIV}	Particle Image Velocimetry
POD	Proper Orthogonal Decomposition
\mathbf{PSD}	Power Spectral Density
STFT	Short Time Fourier Transform
TrPIV	Time-resolved Particle Image Velocimetry
VIV	Vortex Induced Vibration
VrTF	Velocity-restricted Torsional Flutter

Chapter 1

Introduction

1.1 Motivation

Bridges, tall buildings, chimneys, towers, stadiums, ... many civil engineering structures must be able to withstand atmospheric wind forces [1]. Because of their multiple primary uses and the corresponding constraints, these structures are intrinsically not adapted to deal with the effect of a flow around them. In fact they combine two important and critical characteristics:

- 1. *Flexibility*, due to their large dimensions and the use of new materials reducing their weight.
- 2. Bluff body characteristics, despite the design tendency to smooth their shapes.

These two ingredients constitute the basis of the field of **bluff body aeroelasticity**, which is the main concern of this doctoral thesis.

Collar [2] defined aeroelasticity as the interaction of the inertial, elastic and aerodynamic forces, which is depicted in the 'Collar triangle' in figure 1.1. From a mathematical point of view, aeroelasticity is represented in the form

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}_{\mathbf{a}}(V_{\infty}, \dot{\mathbf{x}}(t), \mathbf{x}(t)) + \mathbf{f}_{\mathbf{e}}(t)$$
(1.1)

where **M**, **C** and **K** represent the dynamic properties of the system in terms of mass, damping and stiffness respectively. The right hand side terms of equation 1.1 are of central importance in the field of aeroelasticity. The term $\mathbf{f}_{\mathbf{a}}(V_{\infty}, \dot{\mathbf{x}}(t), \mathbf{x}(t))$ denotes the aerodynamic loads (forces and moments) applied on the structure, depending on the airspeed V_{∞} and the motion of the structure. The term $\mathbf{f}_{\mathbf{e}}(t)$ denotes external forces that are independent of the structural motion, such as the aerodynamic forces due to the turbulent components of the wind.



Figure 1.1: Collar's aeroelastic triangle

The first aeroelastic considerations have been applied to the aeronautical domain, where light and flexible structures are subjected to high airspeeds. The primary aeroelastic considerations were static, i.e. the wing must be stiff enough to ensure that the structural restoring forces counterbalance the aerodynamic forces (see the bottom edge of Collar's triangle). Through the years, the aeroelastic instabilities followed the evolution of the technical progress in aircraft design: higher flight speed and more complex aeronautical structures (composite materials, automatic control systems, etc). Nowadays the field of aeroelasticity is oriented towards the study of nonlinear phenomena [3]. According to Dowell et al. [4], two sources of nonlinearity can be observed in aeroelastic systems: structural nonlinearities (e.g. freeplay, friction, material) or aerodynamic nonlinearities (e.g. flow separation, shocks).

This thesis is oriented towards the second type of nonlinearity, which is of paramount importance for bluff bodies since the flow around them is characterized by flow separation and re-attachment. Hence, the motion-dependent term $\mathbf{f}_{\mathbf{a}}(V_{\infty}, \dot{\mathbf{x}}(t), \mathbf{x}(t))$ in equation 1.1 models the nonlinear aeroelastic forces induced on a linear structure.

The notion of bluff body stems from the type of flow around it and in its wake. A bluff body is characterized by a separation of the flow on its surface and the dimension of the resulting wake is usually of the same order of magnitude as the body itself. Most of the drag force of a bluff body is due to the pressure component, while friction forces have less importance. A bluff body geometry is defined as opposed to a streamlined body, which is characterized by smooth and attached flow conditions, such as the flow over a wing at low incidence. The unsteady aerodynamics around a bluff body are dictated by the complete Navier-Stokes equations and no simplified mathematical form is usually possible. This is the reason why the term $\mathbf{f}_{\mathbf{a}}(V_{\infty}, \dot{\mathbf{x}}(t), \mathbf{x}(t))$ is complex to understand and quantify.



Figure 1.2: Da Vinci sketch of a hydrodynamic flow (reproduced from [5])

The sketch presented in figure 1.2 shows water flows behind rectangular obstacles. It is part of several drawings, sketched by Leonardo Da Vinci more than 500 years ago and accompanied with the following comments⁽¹⁾:

"Observe the motion of the surface of the water, which resembles that of hair, which has two motions, of which one is caused by the weight of the hair, the other by the direction of the curls; thus the water has eddying motions, one part of which is due to the principal current, the other to the random and reverse motion. [...]

[...] So moving water strives to maintain the course pursuant to the power which occasions it and, if it finds an obstacle in its path, completes the span of the course it has commenced by a circular and revolving movement. [...]

[...] The small eddies are almost numberless, and large things are rotated only by large eddies and not by small ones, and small things are turned by both small eddies and large."

These comments of Da Vinci, based on a scientific observation, are probably the first descriptions of the concept of turbulence. The description of the swirling motion of water behind a bluff body is the earliest reference to the process of vortex shedding [7]. Furthermore, Da Vinci clearly highlights the simultaneous occurrence of large energetic

⁽¹⁾translation from Ugo Piomelli, University of Maryland (source [6])

vortices and small turbulent eddies. This scale effect in the unsteady flow-fields around bluff bodies represents the main difficulty in the modelling of such flows.

More recently, Anatol Roshko (1993), an important contributor to the science of bluff body aerodynamics, stated: "The problem of bluff body flow remains entirely in the empirical, descriptive realm of knowledge" [8]. The complexity of the phenomenon increases even more when the flow-induced motion of a bluff body is taken into account. Hence, despite the huge advances in the domain of Computation Fluid Dynamics (CFD), the simulation of the flow around oscillating/flexible bluff bodies remains a challenge in terms of CPU time and validation of the results [9], even for simple static bluff geometries⁽¹⁾. For that reason, most of the research efforts dealing with this problem are still experimental, using wind or water tunnels [10]. The experimental measurements and observations are often used in order to validate/calibrate the numerical codes. Note that the same approach is retained in this thesis, where several experimental aeroelastic systems are investigated. The experimental measurements are then used as benchmarks for comparison with the results of a numerical tool developed in the scope of the thesis.

A third important characteristic of wind effects on civil engineering structures concerns the *turbulent* content of the wind itself. Indeed, the atmospheric boundary layer is naturally turbulent and a lot of research has been dedicated to the characterization of its turbulent content [11, 12]. This stochastic aerodynamic force appears in equation 1.1 through the term $\mathbf{f}_{\mathbf{e}}(t)$. As stated before, this excitation force does not depend on the motion of the structure and is usually taken into account in finite elements models of structures at the design stage. It lies outside the scope of this thesis to discuss this aspect of the aeroelastic behaviour of structures, which is treated extensively in several research works [13–15]. Nevertheless, although the study of the atmospheric wind and the calculation of the corresponding structural responses are not treated here, it is important to state that the turbulent oncoming flow-field has an effect on the flow around the bluff body and, hence, on its aeroelastic behaviour. In fact, it is not possible to decouple completely the effects of $\mathbf{f}_{\mathbf{e}}(t)$ and $\mathbf{f}_{\mathbf{a}}(V_{\infty}, \dot{\mathbf{x}}(t), \mathbf{x}(t))$ in the equation of motion. This important point is discussed in the next section, which deals with separated flow-fields. In order to minimize this effect, all wind tunnel experiments presented in this thesis have been performed with a careful control of the quality of the oncoming free-stream. Nonetheless, the impact of free-stream turbulence is discussed when necessary.

Most civil engineering structures have a three-dimensional character. Indeed, the flowfield around high buildings and towers can be roughly considered as two-dimensional at mid-height but it is not the case at the bottom where the airspeed is zero or at the top

⁽¹⁾See for example the Benchmark on the Aerodynamics of a Rectangular 5:1 Cylinder (BARC) at http://www.aniv-iawe.org/barc/

where 3D effects are important. The situation is equivalent for a long bridge: the flow is quasi-2D everywhere except at the extremities where the bridge in contact with the ground. Furthermore, three-dimensionality is clearly inherent to turbulent flows, such as atmospheric flows. This point is treated in textbooks on both aerodynamics and civil engineering structures (see for example [12]). Nevertheless, most of the research dealing with aeroelastic systems (in aeronautic and bluff bodies applications) relies on the study of two-dimensional models [3]. This is justified by the fact that some flows retain a certain bi-dimensionality, at least to a first approximation, around a major part of the body. The ability of 2D flow models to represent correctly the behaviour of 3D structures confirms their validity. Similarly to the effect of turbulence discussed above, three-dimensionality is usually viewed as an additional parameter in some applications. All the experimental aeroelastic systems of this thesis are studied on the basis of two-dimensional models. This allows to understand the main characteristics of the flow-field and its relation with the eventual aeroelastic instability.

1.2 Aerodynamics of separated airflows

As presented in the previous section, civil engineering structures are classified as bluff bodies. The flow separation and re-attachment characterizing this type of body is a source of excitation that may lead to aeroelastic instabilities. It is then of importance to introduce and describe these aerodynamic concepts. First we introduce the concept of the boundary layer and its separation over static bodies and extend it to the oscillating body case. Then, the aeroelastic phenomena occurring in the case of a flexible bluff body are presented, classified and reviewed.

1.2.1 Static body

The mechanism of flow separation is closely related to the behaviour of the boundary layer around the body. The latter is dominated by the viscosity of the fluid, which is one of the main physical properties of any fluid. It is a measure of the capability of the fluid to resist to shear forces. The relative importance between the inertial terms and the viscous terms in the governing Navier-Stokes equations is expressed by the Reynolds number, which is a key parameter in the domain of fluid mechanics. The Reynolds number is defined as

$$Re = \frac{V_{\infty}L}{\nu} \tag{1.2}$$

where V_{∞} denotes the free-stream velocity, L is a reference length of the system and ν is the kinematic viscosity of the fluid.



Figure 1.3: Airfoil at low angle of attack - regions of the flow (reproduced from [16])

The domain of low speed aerodynamics and more specifically the aerodynamics of civil engineering applications is characterized by large Reynolds numbers: typically starting at 10^4 for electrical conductors or suspension cables, up to 10^8 in the case of bridge decks. In this range of Reynolds number, the flow is said to be potential at a certain distance from the boundaries of the flow domain [17]. Figure 1.3 shows the region where viscosity plays an important role: the boundary layer, in the case of an airfoil at low angle of attack. The notion of potential flow was borrowed from the mathematical potential theory (see for example [18]) and introduced with success in the domain of fluid mechanics in order to represent the velocity fields. Potential flow theory is applicable in the case of incompressible inviscid flows. Mathematically, the curl of the velocity vector is equal to zero in a potential flow. In the case of streamlined shapes, such as airfoils at low angle of attack, the thickness of the laminar boundary layer is very small and can be approximated by Prandtl 's boundary layer theory [19]. At the sharp trailing edge of an airfoil, the flow separates smoothly from the surface and a thin wake is created [20] (see point *S* in figure 1.4).



Figure 1.4: Airfoil at low angle of attack - narrow wake at the trailing edge (reproduced from [16])

F.M. White [21] summarized perfectly the concept of potential flow by writing: 'When a flow is both frictionless and irrotational, pleasant things happen'. As shown through this thesis, the two assumptions of F.M. White are rarely satisfied: if the angle of attack of the airfoil is higher than the static stall angle, separation of the flow-field occurs before reaching the trailing edge. Nevertheless, despite the limited applicability of potential flow theory, we show in chapter 5 that it can be used, with some modifications, in order to model the separated airflows around bluff bodies.



Figure 1.5: Flow separation over a streamlined geometry (reproduced from [16])

Flow separation over static streamlined bodies can be explained by the adverse pressure gradient that develops in the chordwise direction, from the leading edge to the trailing edge: the flow decelerates, leading to high pressure values, according to Bernoulli's principle. At a certain point (the separation point), the velocity profile is inverted (as shown in figure 1.5) and the thickness of the boundary layer increases abruptly [22].

The behaviour of the boundary layer is strongly influenced by the level of turbulence of the flow. This turbulence has two possible origins:

- The unsteadiness of the oncoming flow-field, which is often denoted as the *oncoming turbulence* because it is generally caused by far-field factors (e.g. the atmospheric turbulence in the earth's boundary layer).
- The energy transfers in the boundary layer itself, which are characterized by the flow regimes on the basis of the Reynolds number: laminar or turbulent regime. For each flow, a critical Reynolds number marks the distinction between the two

regimes. For example, in the case of a smooth flat plate, the laminar boundary layer becomes unstable and turbulent for $Re \approx 5 \times 10^5$ (where the characteristic length L of equation 1.2 corresponds to the distance from the leading edge of the flat plate, to the location where transition occurs). For a smooth circular cylinder, the critical Reynolds value for turbulence in the wake is approximately equal to 300 (cfr figure 1.8 below).

A turbulent boundary layer contains more energy than a laminar one, hence the fluid resists more to the adverse pressure gradient and separation occurs later. This effect of the flow regime on the behaviour of the boundary layer is usually demonstrated in student textbooks through the example of the golf ball: dimples are present on its surface in order to increase the roughness, yielding a turbulent boundary layer. Through this trick, flow separation occurs further downstream, reducing the size of the wake and in turn the pressure drag.

The situation is different in the case of bluff bodies with sharped corners, where the separation points are fixed to the sharp edges of the body. The high curvature of the shape creates a strong adverse gradient, which is added to the decelerating effect of the skin friction, resulting in a free unstable shear layer downstream the sharp edge of the body.

Once separation occurs, the shear layer either re-attaches further downstream on the surface of the body, or not. In the case of a static bluff body, two factors are important:

- 1. An aerodynamic factor, describing the turbulent characteristic of the boundary layer and measured by the Reynolds number.
- 2. A geometric factor, characterizing the bluffness of the body (B/D), where B and D denote the chord and the depth of the body respectively.

As stated before, the range of interest of the Reynolds number starts around 10^4 in the domain of the aerodynamics of civil engineering structures. As a consequence, only turbulent flows are considered in this work and the difference between high and moderate turbulent flow is highlighted. A complete overview of the Reynolds effect can be found in references [13, 23, 24].

The effects of the two factors cited above are illustrated in figure 1.6, where flow separation is sketched for two different bluff bodies: a square cylinder (B/D = 1) and a rectangular cylinder (B/D = 2). The curved lines represent the position of the shear layer. Because of the unstable nature of the shear layers, e.g. due to the presence of small turbulent eddies, these lines show a mean position of the shear layer.

In figure 1.6(a), the smooth and turbulent flows do not re-attach to the surface of the body, and the body lies in its own wake. Due to the short width of the body, even a



Figure 1.6: Flow separation on bluff bodies: smooth flow (plain lines), turbulent flow (dashed lines)

turbulent shear layer cannot re-attach. In figure 1.6(b), the smooth flow separated from the leading edge does not re-attach to the surface of the body, similarly to the square cylinder of figure 1.6(a). On the contrary, the turbulent flow separates at the leading edge but re-attaches on the surface of the body. In this case the attached flow in the rear part of the body will separate, for the second time, on the sharp trailing edge. It is obvious that the resulting aerodynamic forces and moment are complex and it is useless to try to use classic aerodynamic concepts, such as thin airfoil theory [25], to model them. The situation becomes even more complex when the motion of the structure is considered, as presented below in section 1.2.2.

Note that the modification of the sharpness of bluff bodies into rounded corners or streamlined aerodynamic appendages is a good and economical method to mitigate the flow separation and the resulting aeroelastic instabilities [26, 27]. Another important parameter is the angle of attack of the body, relative to the oncoming flow-field: if the angle of attack is large enough, the shape of the flow changes radically, as shown in figure 1.7: in this case the upper shear layer does not re-attach to the surface [28]. In terms on nonlinear analysis, it can be argued that the angle of attack corresponds to a bifurcation parameter, similarly to the mean velocity or the turbulent features of the oncoming flow-field [29].

Finally, it is obvious that an airfoil at a high angle of attack can be considered as a bluff body [30]. Indeed, the term 'bluff body shedding' is commonly used in research dealing with oscillating airfoils [30,31]. Although it is interesting to consider the similarity between the flow over a bluff body and a streamlined body at off-design conditions, i.e. above the stall angle where standard aircrafts rarely operate. Nevertheless, as shown later in this introductory chapter, the corresponding aeroelastic phenomena are different for streamlined and non-streamlined bodies.



Figure 1.7: Bluff body at a non-zero angle of attack

Strouhal number

The shear layers, separated from the body form a wake which features a certain kind of regularity, as observed at the end of the 19^{th} century by Strouhal [32]. Strouhal pointed out the apparition of a vortex-shedding phenomenon, which follow the relation

$$St = \frac{f_s D}{V_{\infty}} \tag{1.3}$$

where St denotes the Strouhal number. In this expression, D denotes the dimension of the body perpendicular to the oncoming flow-field and f_s corresponds to the frequency of the vortex shedding process.

As presented above, the behaviour of the shear layers depends on the level of turbulence in the flow-field (through the Reynolds number) and on the geometric factor B/D. Nevertheless the relative importance of the these parameters depends on to the type of bluff body, i.e. sharp edged or smooth edged bluff body:

(i) In the case of **bluff bodies with sharp edges**, the effect of the ratio B/D is significant. Okajima [33] performed systematic experimental measurements of the Strouhal number and its dependence on Re for rectangular cylinders with different aspect ratios (B/D). He concluded that rectangles with B/D = 2 and 3 are strongly influenced by the Reynolds number. This is due to the possibility of re-attachment, which is a function of the level of turbulence, as schematically depicted in figure 1.6(b). On the other hand, square sections and sections with B/D = 4, showed a relative insensitivity to Re. In the latter case, the flow is re-attached on the surface and the shedding process occurs at the trailing edge of the rectangle.

The same type of experimental study is proposed for flat plates by Chen et al. [34], who observed that the effect of Re is marked for small angles of attack (0° - 10°), while

no effect is reported between $10^{\circ} - 90^{\circ}$. Finally, Schewe and Larsen [35] carried out an experimental study of the Reynolds effect on the flow around a bluff bridge deck (the Great Belt East Bridge). They reported a difference of 18% on the measurement of St between $Re = 5 \times 10^5$ and $Re = 10^6$ and concluded that slender bodies with sharp edges, such as bridge box girders, may suffer from more pronounced Reynolds effects.

(ii) In the case of **circular cylinders**, the situation is equivalent to the streamlined case as far as the position of the separation point varies with the Reynolds number. Nevertheless, unlike the case of the airfoil, for high Reynolds values, the flow-field around a cylinder will inevitably separate, because the pressure gradient becomes very high in the rear part of the cylinder. The main characteristics of the flow in the wake of a static cylinder are sketched in figure 1.8. Below a threshold value of Re (Re < 40), no alternate shedding is observed. In a certain range or Reynolds numbers, 40 < Re < 150, the shedding process is said to be laminar. It corresponds in fact to the well known Von Karman vortex street. Although these types of flow are not of direct interest in the present work where much higher Reynolds numbers are considered, they are presented for completeness. Below Re = 150, the flow is dominated by viscous effects and large laminar vortical structures are created and shed. With increasing Re, the inertia forces control the phenomenon: the vortex shedding is still present, up to $Re = 10^7$, but it is altered by small scale turbulent structures [36]. It is interesting to note that for $3 \times 10^5 < Re < 3.5 \times 10^6$, the transition from laminar to turbulent regime is total and the wake appears narrower and disorganized. If the Reynolds is further increased, the turbulent vortex shedding is re-established.

The variation of the Strouhal number with the Reynolds number is plotted in figure 1.9 for a circular cylinder. As stated above, a large peak occurs around $Re = 10^6$ for the smooth surface, while it is more constant over the entire Re range for the rough surface. The roughness increases the level of turbulence in the boundary level, which contains more energy, delaying separation (similarly to the golf ball). The recent experimental work by Zan [37] investigates the flow over a cylinder at Reynolds numbers up to 7 million. He shows that the level of turbulence of the oncoming flow has a strong influence on the Strouhal number. Another comprehensive study of the relation between St and Re numbers was carried out by Schewe [38].



Figure 1.8: Circular cylinder: Different flow regimes (reproduced from [39])



Figure 1.9: Circular cylinder: Strouhal vs Reynolds (reproduced from Techet [40])

Figure 1.10 presents an interesting summary of the sources of unsteadiness in the flowfield around a static bluff body. It was proposed in the review work by Ahsan [41] on bluff body aerodynamics and aeroelasticity. This figure shows a qualitative comparison of the frequency content of the different sources of unsteadiness in the flow-field. The three subplots in the bottom part of the figure correspond to the oncoming turbulence, the flow separation (and re-attachment) and the vortex shedding process respectively. From these subplots, it is clear that three distinct bands of frequency characterize the aerodynamic system. As shown in the next section, when the dynamics of the flexible system are added, important interactions between these different sources of unsteadiness can occur.



Figure 1.10: Schematic flow around a bluff body: oncoming turbulence (a), flow separation (b) and vortex shedding (c)

1.2.2 Oscillating body

Up to this point, the mechanisms of flow separation and re-attachment have been presented for static airfoils and bluff bodies. Before going into the core of this thesis, i.e. the fluid-structure interaction, the discussion is extended to oscillating bodies.

The flow-field around an oscillating bluff body changes radically compared to the static configuration. Many investigations have been carried out in the past to characterize this effect using *imposed motion tests* [42–44]. This type of test, also denoted *forced oscillations test*, allows to study the unsteady aerodynamics around a body through pressure and flow measurements when the model is driven by a servo-motor apparatus. It must be distinguished from the *free vibration tests*, where the dynamics of the structure are reproduced in the wind tunnel model and which allow aeroelastic investigations. In this case, the feedback of the flow on the structure is modeled and the complete fluid/structure phenomenon is studied [45]. Note that all the experiments presented in this thesis are free vibration tests.

Reduced airspeed

When dealing with an oscillating body, it is necessary to introduce an additional nondimensional variable, the reduced velocity, U_r :

$$U_r = \frac{V_\infty}{fB} \tag{1.4}$$

where f denotes the frequency of oscillation of the structure and B is the width (or chord) of the body. In the case of forced oscillations, f corresponds to the frequency of the imposed motion. When free vibration tests are carried out, f is the frequency of the body's motion, which can vary with airspeed, V_{∞} , as shown in chapters 3 and 4.

It is obvious that expressions 1.3 and 1.4 are similar, but the characteristic lengths (D and B) used in the two definitions are different. The reason for this choice is historical: Strouhal was interested in the cross-flow phenomena in the wake of a circular cylinder [32], hence he proposed a non-dimensional number based on the cross-flow characteristic dimension, which is of course equal to the chordwise dimension in the case of a circular cylinder. On the other hand, the concept of reduced velocity was first introduced in the field of aeronautics, in the modelling of the flutter of wings. On the basis of the work by Theodorsen [25], and others, the chord length, B, was naturally selected.

The reduced velocity gives important informations about the scales of the phenomena involved when a body oscillates in a fluid:

- A large U_r corresponds to slow motion of the body compared to the flow velocity: the fluid has the time to adapt to the motion of the body, which is the basis of the quasi-steady theory presented and used in the following chapters of this thesis.
- A small U_r corresponds to a high frequency motion, leading to unsteady aerodynamic phenomena due to both the body oscillations and the vortex shedding process.

It is interesting to shed light through a qualitative analysis to the main characteristics of the flow-field around an oscillating body. As presented in the previous section, the morphology of the separated flow around a bluff body depends on the Reynolds number and a geometric factor, B/D. It is proposed here to limit the discussion to large Re(turbulent flows) and to describe the effect of the motion, through U_r , on (i) a circular cylinder oscillating in heave and (ii) a body undergoing pitching oscillations. Because of the complexity of this type of flow and the multiple effects of the parameters, this discussion is limited to several important features in the two systems cited above. In each case the aerodynamic phenomenon is presented, followed by the introduction of the underlying aeroelastic instability. More complete reviews of the effect of the motion on the aerodynamics of bluff bodies can be found in the literature [45, 46].

(i) Heaving cylinder

Early studies of the effect of the motion on the shedding process around a cylinder forced to oscillate were performed by Koopman [43] and Griffin [47]. Williamson and Roshko [48] studied the effect of the amplitude and frequency of oscillation on the wake of an oscillating cylinder. They presented a complete analysis of the wake for different amplitudes and frequencies of the forced heaving motion. Figure 1.11 shows the mapping of the different patterns, or modes, identified as a function of these two parameters. The horizontal axis of this figure has the dimension of a reduced velocity, as defined by equation 1.4 and the vertical axis is simply the non-dimensional amplitude of the imposed motion. The periodic vortex patterns consist of single vortices (S) or vortex pairs (P), leading to combinations such as 2S (classical Karman Vortices), 2P or P+S. The critical curve corresponds to the case where the shedding frequency is equal to the heaving frequency. For small amplitude motions, i.e. $A/D \approx 0$, the critical curve intersects the horizontal axis at $U_r = \lambda/D = 5$, which is in agreement with the static Strouhal number $St = 1/U_r = 0.2$, as shown in figure 1.9 in the range $200 < Re < 10^{5(1)}$.

From these observations and the comparison with figure 1.8, it is obvious that the flow-field around the cylinder and the resulting aerodynamic forces are more complicated in the presence of oscillations. For example, Facchinetti [49] showed that the dependence of the lift force with the amplitude of oscillation is nonlinear: the static lift coefficient is multiplied by a factor of 2.5 when the amplitude of oscillation reaches $0.5 \times D$.

If the cylinder is elastically mounted, the resulting aeroelastic phenomenon corresponding to the shedding of Karman vortices is commonly referred to as Vortex Induced Vibration (VIV). This phenomenon is presented in the next section and studied through wind tunnel experiments in chapter 2. The instability occurs when the shedding frequency f_s matches the structural frequency f and the critical airspeed is calculated via the Strouhal relation 1.3, i.e. $V_{crit} = \frac{fD}{St}$. The lock-in, or synchronization phenomenon is a well known example of a case where the body oscillations control the vortex shedding process, even when the airspeed is increased beyond V_{crit} [50].

 $^{^{(1)}300 &}lt; Re < 1000$ in the study of Williamson and Roshko [48]



Figure 1.11: Effect of the heaving motion on the wake of a circular cylinder (reproduced from Williamson and Roshko [48])

(ii) Pitching body

The characteristics of the flow-field around a pitching body are introduced using the example of a pitching airfoil. As presented in the introduction, streamlined bodies such as airfoils can be considered as bluff bodies when large angles of attack are considered. This is of course true for the static airfoil above its static stall angle, but it is especially interesting in the case of a pitching airfoil. The separation of the flow is followed by a re-attachment phase when the pitching amplitude reaches smaller values [51, 52]. This unsteady aerodynamic phenomenon was observed in aerospace applications in the early 1940's by Victory [53], where it was termed dynamic stall.

Many other research works have been carried out on dynamic stall in applications such as airfoils [42, 54], propeller blades [55] or helicopter blades [56–58]. This phenomenon is purely aerodynamic, in the sense that the airfoil's motion is imposed, usually through imposed motion tests in a wind tunnel. Fukushima and Dadone [59] studied dynamic stall for purely pitching and purely heaving oscillations, highlighting the important difference in terms of onset of the stall, due to the collapse in the leading edge pressures and its chordwise progression. Ericsson, Lars and Reding presented an extensive collection of experimental investigations of the dynamic stall phenomenon [60–64]. The main results of these works concern the delay of the stall phenomenon due to the pitching motion of the airfoil. They developed extended quasi-steady models, taking into account the delayed separation through a leading edge jet effect [65].

McCroskey [51] identified two important stall regimes: light and deep stall. Figure 1.12 shows the main characteristics of the above-mentioned regimes. Light stall is characterized by a strong interaction between viscous and inviscid effects. The separation originates at the trailing edge and travels towards the leading edge (it may stop before reaching it, depending on the flow and motion characteristics). Deep stall is dominated by viscous effects only. A large vortex is shed at the leading edge and convects downstream, while a small vortex is created by the pitching motion of the sharp trailing edge (see sketch (b) in figure 1.12). The resulting aerodynamic forces and moment are strongly nonlinear with respect to a significant phase delay with the motion. When coupled to a flexible structure such as the wing of an aircraft, the dynamic stall phenomenon leads to an aeroelastic instability called stall flutter [65–68].



Figure 1.12: Stall regimes of the dynamic stall phenomenon (reproduced from [51])

In the case of bluff bodies, especially in civil engineering applications where structures are not designed to move, there is no point to deal with imposed motion as in the case of helicopter or wind turbine blades [57,69]. Hence, the notion of dynamic stall is not reported in the literature about bluff body aerodynamics. Nevertheless, a torsional instability is also experienced on bluff structures, through the denomination of torsional flutter. The flow phenomenon leading to this aeroelastic instability is similar to the deep stall regime discussed in figure 1.12, where the flow separation starts at the leading edge of the body. The behaviour of the flow-field over one cycle of oscillation is sketched in figure 1.13. It is observed from that figure that a large vortical structure is shed at the leading edge and convects downstream while moving away from the surface of the body. This large vortex is identified as the source of the torsional flutter of bluff bodies and its behaviour is extensively analyzed in chapter 4.



Figure 1.13: Flow separation and re-attachment over a pitching rectangular bluff body

1.3 Classification of aeroelastic instabilities for bluff bodies

Many authors have investigated the aeroelastic phenomena of flexible bluff bodies. Among them one finds the aerospace research and industrial communities with many applications cited in the previous sections. The energy industry is also concerned by bluff bodies, e.g. the vibration of tubes in heat exchangers, hydrodynamic problems in ducts and open channels flows or the galloping instabilities of iced electrical power lines. The civil engineering community (bridge decks, towers, cable stayed bridges, pylons, ...) is also widely confronted to flexible structures lying in the atmospheric boundary layer.

Aerodynamic and aeroelastic terminology varies according to the application area and scientific background of researchers. For this reason, and despite several complete review articles on this subject [45, 70–72], it is not always easy to clearly classify the different types of aeroelastic instabilities of bluff bodies.

Simiu and Scanlan [12] proposed a general list of aeroelastic effects in bridge engineering, based on three global categories: vortex-shedding, galloping and flutter. The flutter class is further subdivided into classical two degree-of-freedom (dof) flutter, torsional flutter (with attached flow), panel flutter (typically applied to bridge elements) and single dof flutter. The latter type treats separated flows for vertical flexion and torsion dof. This list, retained by Dowell et al. [3], has the advantage of being exhaustive but, on the other hand, it contains no real classification of the phenomena. In this thesis different types of instabilities are studied with emphasis on the cause of the phenomena observed. Hence, the aeroelastic phenomena listed by Scanlan are classified according to their fundamental causes and major characteristics. Such a classification is also proposed in the general book by Naudascher and Rockwell [72] dealing with flow-induced vibrations. Three different basic elements are defined in the scope of fluid/structure interactions: the *body oscillator*, which consists of a structure with one or more degrees of freedom. This system is governed by the physical laws of structural dynamics [73]. The second element is the *fluid oscillator*, corresponding to the unsteady flow-field around the body oscillator. It is governed by the complete Navier-Stokes equations. The third type of basic elements are the *sources of excitation*, which correspond in fact to the link between the two first elements. Three types of excitations are defined:

- Extraneously Induced Excitation (EIE): independent of any fluid instability due to the structure or its motion, e.g. turbulence in the oncoming flow-field or influence of the wake of an obstacle located upstream.
- Instability Induced Excitation (IIE): flow instability due to the presence of the structure, e.g. alternate vortex shedding (in the sense of Von Karman), or buffeting loads.
- Motion Induced Excitation (MIE): fluctuating forces due to the motion of the structure. It leads to self-excited phenomena where energy transfers from the flow-field to the structure, e.g. flutter.

As stated in section 1.1, the body oscillator is assumed to be linear in the present work and the resulting nonlinear behaviour of the system comes from the nonlinear fluid oscillator (flow separation and re-attachment). Furthermore, since the present applications deal with the aeroelasticity of bluff bodies in civil engineering, it is possible to reduce the complexity of the classification of Naudascher and Rockwell by omitting the concept of fluid oscillator where acoustic effects are relevant. As far as civil engineering problems are concerned, it is justified to limit the discussion to incompressible, adiabatic, and high Reynolds flows. The resulting classification is based on the three sources of excitation defined by Naudascher and Rockwell, similarly to the choice of Hémon [74]. It is shown in figure 1.14, where the principal aeroelastic phenomena are categorized.

The intersections between the different types of excitation are intentional: they highlight the possible simultaneous occurrence of several sources of excitation in a system. This situation is common in any wind engineering application where a structure lies in the turbulent atmospheric boundary layer. In the case of high buildings, VIV occurs in a turbulent flow, hence the combination of EIE with IIE is obtained. The combination


Figure 1.14: Classification of the aeroelastic instabilities of bluff-bodies

EIE+MIE occurs for a bridge deck or an iced electrical power line, where galloping or flutter takes place in the atmospheric boundary layer. Scanlan studied the flutter stability of bridge sections at vortex lock-in (MIE+IIE) [75] or in a turbulent flow (MIE+EIE) [76]. Matsumoto presented a study of the occurrence of IIE and MIE leading to VIV, followed by torsional flutter which led to the destruction of the Tacoma Narrows Bridge [77]. Another study by Matsumoto investigates the VIV and its effect on the torsional flutter (IIE+MIE) in the case of a rectangular cylinder [78]. In chapter 4, this combination of IIE+MIE is also observed in the study of a bridge section and a rectangular cylinder. In fact it is difficult to uncouple the turbulent excitation from the other aeroelastic phenomena (as stated in section 1.1).

All the wind tunnel experiments presented in this thesis are carried out without the addition of turbulence in the oncoming flow-field. The turbulent intensity of the flow inside the test section is under 0.15%, which is very small compared to the levels reached in wind engineering applications (5%-20%). Nevertheless it is not possible to exclude completely EIE because turbulence induced by the supporting structure is reported in chapters 3 and 4. Generally speaking, the aeroelastic investigations presented in this thesis are designed to observe each type of instability separately, when possible.

Figure 1.15 shows a representation of the typical responses of an aeroelastic system to the three different types of excitation discussed above. This plot is the compilation of the schematic representations proposed in different works or books dealing with aeroelastic systems [4, 74, 79–81]. This type of plot, showing the variation of the amplitude of the response of a system as a function of a parameter is commonly denoted a bifurcation diagram in the field of nonlinear analysis. This term is retained in the present thesis because of its wide use in the scope of nonlinear aeroelasticity [4]. It is also common



Figure 1.15: Classification of the aeroelastic instabilities of bluff bodies - typical responses

to use the concept of Limit Cycle Oscillation (LCO), which can occur as the result of a Hopf bifurcation⁽¹⁾. A LCO can be defined as a constant amplitude motion around an equilibrium position, and can only occur in nonlinear systems. Different types of aeroelastic LCO are investigated in detail in chapters 3 and 4.

EIE-type excitation can lead to limited amplitude motion of the structure, increasing continuously but slowly with airspeed. IIE-type excitations generally lead to velocityrestricted responses, characterized by small amplitude oscillations.

The case of MIE is more complex because three different types of responses can be observed. First a divergent motion such as coupled flutter can occur, represented as a vertical line at a critical airspeed. Below this airspeed the response is stable and any initial perturbation applied to the system decays slowly to zero. On the contrary, at the critical airspeed and above, the system is divergent and the response amplitude increases exponentially, leading to structural failure. The second type of MIE is the result of a subcritical Hopf bifurcation. In this case the bifurcation diagram features two limit cycles: a stable (plain line) and an unstable (dashed line). An hysteretic loop is observed, i.e. at an airspeed lower than the critical value, if the system is sufficiently perturbed from its equilibrium position it can jump to the stable limit cycle branch. If the airspeed is further reduced, the oscillations continue down to a threshold airspeed (termed the fold airspeed in the rest of the thesis). The third type of MIE is the result of a supercritical Hopf bifurcation. It is similar to the divergent response to MIE in the sense that no instability occurs below the critical airspeed. Nevertheless, above that value, the resulting motion

⁽¹⁾Dowell et al. [4] refer poetically to LCO as 'a first stop on the road to chaos'

corresponds to LCOs whose amplitude increases with airspeed. Note that Dowell et al. [4] denote the LCOs resulting from a supercritical Hopf as 'good LCO', while LCOs resulting from a subcritical Hopf are referred to 'bad LCO'. This discrimination is justified by the fact that the occurrence of good LCO can be predicted using linear theory while bad LCO occurs without warning, if the system is sufficiently excited.

The three main aeroelastic instabilities highlighted in bold in figure 1.14 are investigated in this thesis. A review of each phenomenon is presented in the next sections.

1.3.1 Vortex induced vibration

As stated in section 1.2.2, vortices are shed behind bluff bodies over a large range of Reynolds numbers. This shedding process leads to periodic aerodynamic forces and can cause resonance of the structure if the shedding frequency matches one of its modal frequencies. The resulting vibration is denoted Vortex Induced Vibrations (VIV), and included in the Instability Induced Excitation (IIE) category defined in the previous section.

In his review paper about VIV, Nakamura [82] states that: 'vortex excitation of bluff body is perhaps the most common but most complicated of all bluff body flutter because of the resonant interaction between vortex shedding and body oscillation'. This introductory remark about VIV clearly summarizes the difficulty to understand the fluid-structure interaction of bluff bodies. Furthermore it is worth noting that Nakamura uses the term 'flutter', while in the present work it was decided to differentiate VIV from the other types of flutter.

Scanlan [70] discussed the two different types of structure that are prone to undergo VIV: bluff bodies without aft-body and slender bluff bodies. The former case will experience VIV, while the second can undergo both VIV and galloping. Galloping is discussed in the next section.

The case of **bluff bodies without aft-body** was highlighted in the previous section where the effect of the heaving motion on the vortex shedding process was discussed. Classical examples of such bodies are: circular and square cylinders, D sections and more generally all the bodies for which $B/D \leq 1$. The flow separates at the sharp edges or further downstream in the case of a cylinder (depending on Re). Due to the low B/D value, it does not re-attach onto the surface of the body. The investigations of Williamson and Roshko showed that different patterns can be observed in the wake of the cylinder [48], so that each pattern corresponds to a specific aerodynamic loading, which in turn is responsible for the excitation of the elastically supported cylinder.

A significant amount of research has been dedicated to the modelling of VIV [49,83,84].



Figure 1.16: VIV of the Great-Belt Bridge (reproduced from [26])

Despite the apparent simplicity of the system, many different types of model have been proposed, ranging from a harmonically excited linear oscillator to nonlinear equations (Van der Pol oscillator) coupling the flow to the structure [85].

In the case of **slender bluff bodies**, it was shown that the flow separating at the leading edge can re-attach downstream and the vortex shedding process takes place at the trailing edge. A recent example was observed on the Great-Belt Bridge, in Denmark, which experienced VIV during its construction [26,86]. The solution used to mitigate these vibrations was the installation of aerodynamic guide vanes, attached to the lower corners of the bridge girder. These add-ons, shown in figure 1.16, change the shedding process and hence the Strouhal number of the bridge section. The bottom plot of figure 1.16 shows the root mean square of the heaving displacement, before and after the installation of the guide vanes. This solution suppressed completely the instability.

Figure 1.17 presents different bridge deck configurations with the corresponding heave oscillation amplitudes. It is observed that the addition of fairings on both sides of the deck reduces significantly the amplitudes. Hence the mitigation of this aeroelastic instability can be achieved through small aerodynamic modifications of the shape of the deck, without any modification of the dynamic features of the structure (damping devices, added masses, etc.).



Figure 1.17: VIV of bridge sections (reproduced from [86])

As pointed out by Morgenthal [87], VIV is a self-limited phenomenon: even in the absence of structural damping, the amplitude of oscillations always remains limited. Furthermore, the range of airspeeds where VIV occurs is also limited around the airspeed leading to the matching of the shedding frequency to the structural frequency (see bottom plot of figure 1.16). Because of these two characteristics VIV is usually non-critical, so that the phenomenon is often only taken into account for fatigue and comfort considerations.

The main three parameters of VIV are the airspeed V_{∞} , the oscillating mass m and the structural damping of the system δ [48]. Skop and Griffin [50] introduced a nondimensional parameter characterizing the maximum displacement at lock-in, the so-called Skop-Griffin number, defined as $S_G = 4\pi S t^2 \frac{m\delta}{\rho D^2} = 4\pi S t^2 S c$. In this expression, St and Sc are the Strouhal and Scruton numbers respectively, ρ is the flow density and D is a characteristic length of the body. This combined mass-damping parameter includes both dynamic and aerodynamic parameters of the system and is used as an estimate of the likelihood of a structure to undergo VIV: above $S_G = 5$, no risk of VIV is expected according to Skop and Griffin [50]. This simple criterion is widely used in design and construction codes, such as the Eurocodes (see section 1.3.4).

1.3.2 Galloping

Galloping is a single degree of freedom vibration in a direction normal to the oncoming flow-field. It was first observed and described in the case of iced transmission lines by Davison and Den Hartog [88, 89]. Davison referred to 'dancing vibrations' to describe these large amplitude oscillations. The phenomenon of galloping of rectangular cylinders has received attention since the beginning of the 1960's, with the works by Parkinson [90], Novak [91,92] and others. The terminology was found 'rather appropriate' by Parkinson because of the visual impression given by vibrating transmission lines, reminiscent of a galloping horse. The notation of *across-wind galloping* was also given to the phenomenon due to the direction of the observed motion. In this thesis, the term galloping is retained, in agreement with most of the recent literature on the subject. As shown in figure 1.14, this instability is associated to MIE excitation, i.e. the aerodynamic excitation results from the separation and re-attachment of the flow, due to the motion of the structure.

Many investigations, starting with Parkinson [90], have centered on the understanding of the fundamental mechanism leading to galloping oscillations [81,93]. Paidoussis [81] defined galloping as a velocity-dependent, damping-controlled instability. Parkinson [94] compiled several studies, resulting in a comprehensive characterization of the flow mechanisms as a function of the ratio B/D. An important result of his work is reproduced in figure 1.18, where the ratio B/D is denoted by d/h. In this figure, the drag coefficient C_D , the Strouhal number St and the maximum non-dimensional oscillation amplitude $(Y/U)_{max}^{(1)}$ are presented as a function of the geometric parameter B/D. Two values of this parameter lead to jumps in the curves of C_D and St:

- $(B/D)_{crit} = 0.62$ a peak is observed in the curve of C_D . Below this value, the shear layer that separates from the leading edge does not interact with the aff-body, similarly to figure 1.6(a). Figure 1.18 does not show the lift coefficient C_L , but it can be stated from [29] that its shape is similar to that of C_D : below $(B/D)_{crit}$, the slope of $C_L(\alpha)$ is positive and it becomes negative above that critical value. Note that the discussion based on the slope of the aerodynamic coefficients corresponds to the quasi-steady approach, which is capable of predicting the occurrence of galloping. This approach is discussed in more detail in the rest of this section.
- $(B/D)_{attach} = 2.8$ (or more generally $2.5 < (B/D)_{attach} < 3.0$) a steep increase of the Strouhal number is observed, in fact St is almost doubled. This abrupt change is due to the re-attachment of the shear layer on the top and bottom surfaces of the bluff body. The re-attached flow separates then a second time at the trailing edge of

⁽¹⁾In this ratio, Y denotes the non-dimensional amplitude of oscillation y/D and U is the reduced airspeed $U = V_{\infty}/\omega D$

the body, as shown in figure 1.6(b). The subscript 'attach' is used here to highlight the important change in the flow pattern. The partial re-attachment of the flow is found responsible for the self-limiting amplitudes of the galloping oscillations [95].

The notion of critical depth ratio $(B/D)_{crit}$ was also used by Brooks and Smith [96, 97], who reported hard-oscillator galloping for B/D < 0.75 and soft-oscillator for 0.75 < B/D < 3.0. The flow re-attachment for $B/D > (B/D)_{attach}$ provides a pressure loading on the aft-body which cancels out the small heaving disturbances. For that reason, rectangular cylinders with $B/D > (B/D)_{attach}$ are stable regarding the galloping instability, as shown by the curve of $(Y/U)_{max}$ in figure 1.18. It is worth noting that above $(B/D)_{attach}$, another type of aerodynamic excitation occurs: Motion Induced Vortices, which are related to the torsional instability, presented in the next section.



Figure 1.18: Characteristics of the flow-field around a rectangular cylinder Effect of the aft-body (d/h ratio) (reproduced from [72])

From these findings, it can be stated that galloping occurs for bluff bodies with $(B/D)_{crit} < B/D < (B/D)_{attach}$ because the separated flow-field leads to a negative slope of the lift coefficient, $\frac{\partial C_L}{\partial \alpha} < 0$. This aerodynamic force, associated with the velocity of the heaving dof, leads to negative damping above a certain critical airspeed. This statement is in agreement with the Paidoussis's description of the galloping phenomenon

and corresponds to the quasi-steady theory, which is valid when the reduced airspeed is high (as discussed in section 1.2.2). This theory is extensively developed and used in the scope of this thesis in order to compare its predictions to the experimental measurements. Given the high reduced airspeed assumption, the heaving instability described here is commonly termed *High Speed Galloping* (HSG).

On the other hand, it was observed by Nakamura and Matsukawa [98] that a negative aerodynamic damping can occur in the case $B/D < (B/D)_{crit}$, at low reduced velocities, i.e. below the airspeed leading to VIV. This heaving instability was naturally denoted *Low Speed Galloping* (LSG). Figure 1.19 shows an idealized diagram of the heaving response of a bluff body, where LSG, HSG and VIV responses are represented. The first observation concerns the different types of response of LSG and HSG. The former is a velocity-restricted and occurs over a limited range of airspeeds, while HSG leads to larger amplitudes that continuously increase with airspeed. In the low speed range, the quasisteady theory mentioned above is not valid because the separated shear layers and the motion of the body strongly interact: a clearly unsteady phenomenon takes place. Similarly to the case where $B/D > (B/D)_{attach}$, discussed above, the source of excitation is a Motion Induced Vortex.



Figure 1.19: Heaving instabilities: LSG - VIV - HSG

As stated above, the quasi-steady theory was successfully used in order to estimate the critical airspeed of HSG. The initial Den Hartog criterion [89] was frequently revisited in order to take into account additional characteristics of the system, such as the damping variation with amplitude or the nonlinearity of the quasi-steady force coefficients. Parkinson [90] and Novak [91,92] proposed nonlinear polynomials for the vertical force coefficient of square and rectangular cylinders. They highlighted the hysteritic loop in the bifurcation diagram of the galloping system. More recent works by Ng et al. [99] proposed high order polynomial curve fits for a square cylinder. Hortmanns and Ruscheweyh [100] developed a method to calculate the galloping amplitudes above the critical airspeed, in order to explain the destruction of a crane. Ziller et al. [101] proposed an new approach for determining the onset airspeed by taking into account the nonlinearity of the aerodynamic damping characteristics. They highlighted the importance of the turbulence intensity of the oncoming flow in the calculation of the galloping response of a structure. Oudheusden et al. [102] currently investigate the validity of the quasi-steady theory in modelling the galloping phenomenon. Quantitative flow visualization is carried out around a square cylinder at incidence in order to observe the main characteristics of the flow, with an emphasis on flow re-attachment, which is obviously the driving phenomenon of the instability.

Other papers present experimental and numerical investigations of the galloping instability for bridge sections [103]. In chapter 3, a complete investigation of the galloping response of a bridge deck is presented and a polynomial force coefficient is proposed with a discussion of the validity of the linear and nonlinear quasi-steady approaches.

1.3.3 Torsional flutter

The torsional flutter phenomenon is an aeroelastic instability in the pitch degree of freedom of the body. As stated in section 1.2.2, the aerodynamic source of excitation shows similarities with the dynamic stall phenomenon studied in aerospace structures such as wings, compressor blades of gas turbines or helicopter and wind turbine's blades [57,69]. It has also been named *torsional galloping* and *rotational galloping*, due to the single dof feature of the phenomenon, but this notation is gradually supplanted by torsional flutter because the aerodynamic excitation is different from that of the galloping phenomenon.

Amongst civil engineering applications, structures with sharped edges and an aft-body (i.e. a sufficient B/D ratio) are prone to undergo torsional flutter oscillations. Hence, bridge decks are usually studied in order to avoid this phenomenon [104]. It is worth mentioning that torsional flutter has been found responsible for the destruction of the Tacoma Narrows bridge in 1940 [77, 105, 106]. Nevertheless, in the years following the collapse of the bridge, the accident was wrongly attributed to VIV [107]. This explanation is still widely used in textbooks. Figure 1.20 shows four snapshots of the historical movie of the bridge's failure. It is reported that the deck underwent large torsional oscillations (up to 35° !), during several minutes before its failure.

While many investigations have been carried out on galloping, the torsional flutter of bluff bodies remains less well understood. Washizu et. al [108] investigated the interaction



Figure 1.20: Tacoma Narrows collapse, 1940

between vortex excitation and torsional flutter and highlighted the inability of quasisteady theory to predict the occurrence of torsional flutter. Furthermore they pointedout that different types of flutter can affect rectangular cylinders with the pitching axis at the center: soft torsional flutter if 2.5 < B/D < 5.5 and hard flutter for B/D <2.5. The notion of critical depth ratio $(B/D)_{crit}$, presented in the scope of the galloping instability, is also retained in the case of torsional flutter. Nakamura [109] defined two types of rectangular cylinders, depending on the value of B/D. He defined a critical ratio $(B/D)_{crit} = 2.8$, which corresponds to the limit of applicability of the quasi-steady theory to model torsional flutter: for $B/D < (B/D)_{crit}$, the criterion $\frac{\partial C_M}{\partial \alpha} < 0$ is often used to assess the torsional stability of the structure, but the criterion is not universally applicable if $B/D > (B/D)_{crit}$. It is worth noting that the value of $(B/D)_{crit}$ is in fact equal to $(B/D)_{attach}$ of the galloping case (see figure 1.18). The re-attachment of the flow leads to another type of aerodynamic excitation: Motion Induced Vortices. These large vortical patterns are identified as the cause of the torsional flutter oscillations in chapter 4. Oudheusden proposed a new approach to adapt the quasi-steady theory to the torsional motion of a body in a flow-field [110]. Note that the limit of the theory of van Oudheusden corresponds to the galloping phenomenon.

Additional research by Nakamura [111] investigated the aerodynamic mechanism of

torsional flutter. He pointed out that the memory effect (wake of the body) is responsible for the onset of torsional flutter, as for airfoils. Nevertheless his work highlights some essential differences between the stall flutter of an airfoil and the torsional flutter of a bluff body: the former can occur only under very limited conditions, that is for large reduced airspeed and a limited range of positions of the pitching axis. On the other hand, torsional flutter of a bluff body is observed for wide ranges of reduced airspeeds and positions of the pitching axis.

Matsumoto et al. [93,112,113] carried out extensive experimental investigations of the torsional flutter of rectangular and H-shaped cylinders. Most of their work is based on the use of the Scanlan flutter derivatives (A_i^*, H_i^*) [114] to model torsional flutter. Matsumoto [113] defined four types of flutter: velocity-resticted torsional flutter (VrTF), low speed torsional flutter (LSTF), high speed torsional flutter (HSTF) and coupled flutter. The latter corresponds to the classical two dof flutter and is briefly presented below. The three other types of torsional flutter are characterized by their aerodynamic source of excitation:

- VrTF: where complete flow separation occurs. It affects bluff structures, characterized by B/D < 1.
- LSTF: caused by the convection of a Motion Induced Vortex along the surface of the prism. This type of torsional flutter affects slender bodies.
- HSTF: caused by an unsteady local separation bubble on the upper or lower surface of the body.

Recent work by Matsumoto [78] dealing with the torsional motion of a rectangular cylinder with B/D = 4 draws interesting conclusions about the interference effects between torsional flutter and VIV (MIE+IIE, in figure 1.14). Matsumoto showed that torsional flutter is triggered by the Motion-Induced Vortex while Karman vortices can delay the apparition of torsional flutter. Note that this interference effect between Karman Vortices and torsional flutter is proposed as an explanation of the ruin of the Tacoma Narrows bridge [77].

Coupled flutter

In contrast to the single dof instabilities presented above, coupled flutter involves the pitch and heave dofs of the structure. It was not investigated experimentally during the course of the present research, but it was studied in the case of the viaduct of Millau in previous work by the author [115]. The main characteristics of coupled flutter are briefly presented here for completeness.

Coupled flutter is also referred to as classical flutter in reference to the flutter phenomenon of airfoils [25]. For a wing, the flutter is characterized as a linear aeroelastic phenomenon, because it occurs under attached flow conditions (small angle of attack). The resulting divergent oscillations lead to the destruction of the structure. Many different methods have been developed over the last century in order to calculate the flutter airspeed [116, 117]. Simple aerodynamic models, such as panels methods, are used at the aircraft design stage to obtain preliminary estimates of the aerodynamic forces $\mathbf{f}_{\mathbf{a}}(V_{\infty}, \dot{\mathbf{x}}(t), \mathbf{x}(t))$.

Suspended bridges are also prone to coupled flutter because they usually contain bending and torsion modes whose frequencies are low and close to each other. Modern suspension bridge design favours streamlined deck sections: box girders are supplanting truss girder structures [118].

On the basis of the resemblance between wings and modern bridge decks, it is tempting to try to apply an analytical approach to the study of the flutter behaviour of bluff bodies. Nevertheless, this effort is frequently vain, because of the type of flow over the bridge deck. Despite the streamlined geometries of modern decks, flow separation is always present, especially on the upper part of the deck: aerodynamic appendages, such as security rails, wind-shields and of course vehicles are the main causes of flow separation. Matsumoto [78] states that coupled flutter is caused by an unsteady local separation bubble at the leading edge of the bridge, similarly to HSTF presented in the previous section.

Matsumoto and his research associates [119] proposed a complete list of two dof (pitch and heave) instabilities, which is an extension of the four types listed in the previous section dealing with torsional flutter. Taylor [120] studied the timing of vortices in the case of two dof rectangular sections. Many research works dealing with the aeroelastic stability of bridge decks are based on the identifications of the Scanlan flutter derivatives [114, 119, 121]. It is worth noting that the use of the Scanlan flutter derivatives allows the detection of aeroelastic instabilities but does not really explain their physical mechanism [29]. In fact these aerodynamic coefficients demonstrate that every aeroelastic study of a civil engineering system is unique, in the sense that the estimated Scanlan derivatives are valid only for the tested configuration.

1.3.4 Comments on EuroCode EN1991

After presenting the main aeroelastic phenomena affecting bluff bodies through a classification adapted to civil engineering structures, it is interesting to discuss how these phenomena are treated in practice. Of particular interest is the approach specified by EuroCode EN1991-1-4 [122], which is the design standard used by every European design office in the field of civil engineering. Note that this standard is still under development by the member countries of the European Committee for Standardization. Consequently, each country is still using partially its national standard, which is included as an appendix to the Euro-Code, with priority to the latter. This temporary confusion should be resolved over the next few years by the introduction of a monolithic document applicable to all EU members. The discussion in this section concerns the EuroCode EN1991-1-4 itself, as well as the Belgian National Appendices (ANB 03-002-1:1988 [123] and ANB 03-002-2:1988 [124]).

The EuroCode standard establishes the design rules for structures where wind considerations are required. It deals principally with the effects of the mean wind and the atmospheric turbulence on static structures. Dynamic effects of the wind are considered under the following classification:

- Vortex shedding, corresponding to the VIV defined above, with the additional consideration of the ovalization of shells and assembled cylinders. Note that no reference is made to the lock-in effect discussed previously. Vortex shedding is taken into account in the calculation of the life time of the structure due to fatigue damage. Two different methods for the calculation of the vertical displacements of the structure undergoing VIV are proposed. The possibility of mitigation of VIV is suggested, without providing a clear solution.
- 2. Galloping, in agreement with the present definition of galloping, i.e. a self-excited instability along the heave dof. Note that assemblies and wake galloping effects are also taken into account.
- 3. Divergence and flutter:
 - (a) Divergence, corresponding to the well-known static aeroelastic phenomenon.
 - (b) Stall flutter, equivalent to the torsional flutter term defined earlier (pure torsion).
 - (c) Classical flutter, which occurs by frequency matching of the torsional mode to the bending mode. It corresponds to the couple flutter defined previously.

These phenomena are considered in the standards on the basis of three criteria. If at least one of the criteria is not satisfied, the structure is said to be safe regarding divergence and flutter. These criteria concern: the geometry of the section, the position of the torsional axis and the value of the lowest modal frequencies of the structure. The second and third phenomena in the list above are considered as critical and an estimate of the critical airspeed is suggested for each of them. All these estimates rely on tabulated values and their validity is limited. For several specific problems, the document suggests wind tunnel experiments or expert consultations. Such a comment is shown in figure 1.21, which presents a page from the norm (in french). The sentence at the bottom of the page (surrounded by a rectangle) states that if the critical airspeed of VIV, V_{crit} , is too close to the critical airspeed of galloping, V_{CG} (example of IIE+MIE), it is recommended to consult a specialist. Furthermore, the EuroCode is not applicable for viaducts and suspension bridges with a span dimension higher than 200m. In fact all the aeroelastic considerations of the EuroCode appear in an appendix, which is presented as 'informative'.

These examples show the necessity of wind tunnel experiments in order to guarantee accurate and safe analysis of the effects of the wind on flexible structures. The need for dedicated investigations is due to the complexity of the unsteady aerodynamics around civil engineering structures, which are nearly always unique to each project (in terms of geometry, atmospheric conditions, dynamic properties, ...).

On the basis of this important observation, there is a clear need for the development of new analysis techniques for the investigation of (some) aeroelastic phenomena, in order to enhance the understanding of the main characteristics of these complex engineering systems.

(E.19)

E.2.2 Vitesse du vent de déclenchement du galop

La vitesse de vent déclenchant le galop, v_{CG}, est donnée par l'expression (E.18) suivante :

$$v_{\rm CG} = \frac{2 \cdot Sc}{a_{\rm G}} \cdot n_{\rm 1,y} \cdot b \tag{E.18}$$

où

- Sc est le nombre de Scruton tel que défini en E.1.3.3 (1)
- n_{1,y} est la fréquence du mode fondamental perpendiculaire au vent de la construction; les valeurs approchées de n_{1,y} sont données en F.2
- b est la largeur telle que définie dans le Tableau E.7
- a_G est le coefficient d'instabilité en galop (Tableau E.7) ; si ce coefficient n'est pas connu, la valeur a_G = 10 peut être utilisée

(2) Il convient de s'assurer que :

$$v_{CG} > 1,25 \cdot v_{m}$$

où

v_m est la vitesse moyenne du vent telle que définie dans l'expression (4.3) et calculée à la hauteur où le phénomène de galop est attendu, c'est-à-dire au point où l'amplitude d'oscillation devrait être maximale

(3) Lorsque la vitesse critique du détachement tourbillonnaire v_{crit} est proche de la vitesse du vent déclenchant le galop v_m :

$$0.7 < \frac{v_{CG}}{v_{crit}} < 1.5$$
 (E.20)

des effets d'interaction entre le détachement tourbillonnaire et le phénomène de galop sont susceptibles de se produire. Dans ce cas, il est recommandé de consulter un spécialiste.

Figure 1.21: Part of the Euro-Code EN1991-1-4 April 2005 [122]

1.4 Objectives of the thesis

The discussion presented above highlights the need to investigate in more details the aeroelastic responses of bluff bodies. The objective of this doctoral thesis is to develop and apply appropriate techniques to the analysis of such systems. Extensive experimental studies are carried out in the wind tunnel, focusing in particular on the relationship between the unsteady aerodynamics (measured using Time-resolved Particle Image Velocimetry) and the aeroelastic stability of the system. Furthermore, an aerodynamic/aeroelastic simulation tool based on the Discrete Vortex Method is developed. This 2D unsteady aerodynamic solver is capable of calculating the flow-field and forces acting on any type of bluff body, under any kinematic condition: static, imposed motion or free to oscillate. The concept of Proper Orthogonal Decomposition is employed in order to reduce the amount of information in the measured data, extracting only the main features of the observed phenomena. This technique is also used to compare the numerical simulation results to the experimental measurements.

Because of the large number of civil engineering applications, it is obviously impossible or even absurd to investigate them all. The choice of the aeroelastic systems studied in this thesis is motivated by the desire to propose a progression from basic bluff body shapes to generic civil engineering applications. The understanding of the fundamental phenomena is facilitated when simple and documented bluff body geometries are first considered. As a consequence, the selected test cases and applications are a circular cylinder, a rectangular cylinder (B/D = 4) and a generic bridge section, representative of a modern bridge. For each application, the fluid/structure instability is examined in depth using the analysis tools mentioned above.

1.5 Summary by chapter

The main characteristics of the aerodynamics around static and oscillating bluff structures are presented in **Chapter 1**. The discussion particularly concerned with aeroelastic systems, i.e. elastically supported bluff bodies. An adapted classification of the different aeroelastic phenomena is presented on the basis of the source of the aerodynamic excitation leading to instability. The following chapters of this thesis are organized as follows:

Chapter 2

The Vortex Induced Vibrations of a circular cylinder are studied through the analysis of the unsteady velocity field in the wake of the structure. Flow measurements are carried out using Time resolved Particle Image Velocimetry (Tr-PIV). The resulting unsteady velocity fields are analyzed in time and space using the concepts of the Proper Orthogonal Decomposition (POD) and its extension, the Common-base POD (CPOD). It is demonstrated that CPOD enhances the interpretation and the comparison of the unsteady flow-fields as a function of airspeed. Another original analysis technique is developed within the scope of the POD and CPOD methodologies, dealing with the identification of the structural damping of linear dynamic systems.

Chapter 3

The galloping phenomenon of a generic bridge deck is investigated experimentally. It is shown that the Hopf bifurcation is subcritical and followed by two folds. The quasi-steady theory is applied in order to estimate the critical airspeed of the aeroelastic system. It is demonstrated that the first and third order quasi-steady approaches fail to predict the critical airspeed observed experimentally. Hence, it is proposed to model the aerodynamic force coefficient using a 5th order polynomial in terms of the vertical velocity of the structure. It is shown that the results of the time simulations are in good agreement with the experimental heaving displacements. The resulting model is compared with the galloping responses of a square cylinder and a rectangular cylinder (B/D = 2) using the concept of universal galloping curves.

Chapter 4

The experimental study of the torsional flutter phenomenon is carried out for two types of bluff body: a generic bridge section (the one from chapter 3) and a rectangular cylinder (B/D = 4). These two sections are free to oscillate around their pitching axis, which lies at the mid-chord. For each aeroelastic system the complete bifurcation diagram is identified experimentally, leading to a subcritical behaviour for both systems. The investigation of the fundamental cause of the torsional flutter is carried out for the rectangular cylinder. A quantitative analysis of the Tr-PIV measurements is performed, using the CPOD method developed in chapter 2. The examination of the outputs of the CPOD analysis shows that the generation and convection of the Motion Induced Vortex (MIV), from the leading edge to the trailing edge of the rectangle is the fundamental cause of the torsional flutter phenomenon. Thanks to the capability of CPOD to compare different sets of data using a single basis, the role of the MIV is discussed for two different values of the airspeed. Finally, different methods to estimate the critical airspeed of torsional flutter are investigated: quasi-steady approach and empirical simplified models. It is concluded that because of their simplicity, these methods must be used with care because they do not necessarily represent the high complexity of the aerodynamics around a bluff body oscillating in pitch.

Chapter 5

This chapter presents the development and the evaluation of a numerical tool modelling the aeroelastic behaviour of 2D bluff bodies. The aerodynamic solver is based on the Discrete Vortex Method (DVM), which is a Lagrangian approach where vortical particles shed from the surface of a body are tracked through time simulations. The main advantage of this method is its grid-free nature, which is well adapted to deal with oscillating bodies in a flow-field. The aerodynamic solver is coupled to a linear structural model in order to obtain an aeroelastic simulation tool. The aerodynamic and aeroelastic systems of the previous chapters are selected in order to demonstrate the capabilities of the numerical tool: the oscillating cylinder (chapter 2), the 4:1: rectangular cylinder (chapter 4) and the static bridge section (chapter 3). Quantitative comparisons based on the CPOD technique and qualitative comparisons show very good agreement between the experimental measurements and the numerical results.

Chapter 6

A brief summary of the main results are presented, focussing on the most important conclusions and contributions of the thesis. Suggestions for future work are made.

Chapter 2

Orthogonal decomposition of aeroelastic responses

2.1 Introduction

This chapter summarizes the method of Proper Orthogonal Decomposition (POD) [11, 125] and introduces its specific application to the case of aeroelastic systems. The unsteady flow-field in the wake of a flexible cylinder and the corresponding structural vibrations are measured using Time-resolved Particle Image Velocimetry (TrPIV). The quantitative analysis of these experimental data is carried out using the POD technique.

Furthermore, an extension of POD, denoted Common-base POD (CPOD) [126], is adapted to the study of two types of systems: aeroelastic systems and linear dynamic systems. The first type of system deals with aeroelastic data sets composed of a structure oscillating in an flow-field. The effect of airspeed can be modelled through the unique orthogonal basis used by the CPOD approach. The second type of system concerns the development of a novel technique for the identification of the damping of linear system from response data using CPOD [127]. The principle of the method is presented and applied to an experimental system.

The objective of this chapter is to introduce the principles and the theoretical basis of the POD and CPOD techniques and to demonstrate them through preliminary applications. These techniques will be used in the following chapters for quantitative analysis or validation of numerical simulations.

2.2 Proper Orthogonal Decomposition

The modal decomposition of unsteady flow-fields was proposed in the 1990s by several authors, e.g. Hall [128] or Dowell [129]. Proper Orthogonal Decomposition (POD) is one method that can be used in order to perform this modal decomposition; it became popular for aerodynamics research in the 2000s, starting with the work of Tang et al. [130], although it was first proposed for use in fluid dynamics in the 1960s by Lumley [11]. The POD technique has been used to decompose several types of aerodynamic flows, such as the flow behind a disk [131], the flow past a delta wing [132], the unsteady flow impinging on an aircraft tail behind a delta wing [133], the unsteady flow around a F-16 fighter configuration [134] and others.

It should be noted that there are two types of POD research being carried out at the moment. The first focuses on the decomposition of flow-fields observed in experiments in order to better understand the flow mechanisms and physics underlying these flows. The second type of research concerns the Reduced Order Modelling of unsteady Computational Fluid Dynamic (CFD) simulations or even, CFD/CSD (Computational Structural Dynamics) simulations, in order to produce simplified but representative models that can be used in practical applications such as aircraft design.

The work of interest here is of the first type, i.e. the experimental work. It is usually combined with TrPIV measurements [135], although there are examples of other instrumentation being used, such as hot wire rakes [131, 136]. The limitation of all research works published on the subject is that the models around which the flow-field is measured are always static or rotating at constant velocity. Additionally, only one source of flow unsteadiness is ever considered.

The objective of the present work is to extend the methodology of the application of POD to experimental flow-fields with a particular focus on two specific aspects:

- 1. Extension to oscillating models: the source of the unsteadiness is then the motion of the model as well as any unsteadiness due to flow separation.
- 2. Interaction between different sources of unsteadiness: the mechanisms of how the modes generated by one source of unsteadiness interact with the modes generated by the other are analyzed; in particular, the goal is to differentiate between the structural and aerodynamic sources of unsteadiness.

The basic principle of POD is the creation of a mathematical model of an unsteady flow that decouples the spatial from the temporal variations. To illustrate this, we consider the TrPIV measurement data in the wake of a circular cylinder from section 2.3 The outputs of the TrPIV measurement consists of M snapshots of a 2D vectorial flowfield u(x, y, t) and v(x, y, t), at times t_1, \ldots, t_M . Figure 2.1 shows four unsteady velocity fields measured in the wake of a cylinder at an airspeed of 13.9 m/s. The flow direction is right to left and the time between each snapshot is equal to 2 ms. The alternate shedding of vortices and their convection downstream appear clearly in the figure. As shown in the next section, the cylinder oscillates at this airspeed due to the shedding of vortices. The unsteady fields are measured in the wake of the cylinder, excluding the latter due to the limited size of the PIV window. Note that TrPIV measurements with an oscillating body inside the observation window are carried out in chapter 4.



Figure 2.1: Tr-PIV measurements in the wake of a cylinder at $V_{\infty} = 13.9 \ m/s$. Flow from right to left

The velocity vectors are available on a spatial grid of size $n_y \times n_x$, i.e. n_y gridpoints in the y direction with spacing δy and n_x in the x direction with spacing δx . Therefore, u(x, y, t) and v(x, y, t), are available in discrete form, i.e. in the form of $n_y \times n_x \times M$ real arrays. It is worth noting that several interesting pieces of information can be extracted from these time resolved velocity fields, such as the size of the recirculation area, the frequency content of the phenomenon or the spacing between consecutive vortices. Nevertheless, the POD analysis presented here allows a deeper quantitative investigation of the phenomenon.

The POD technique applied to 2D unsteady flow-fields consists in the decomposition

of u(x, y, t) and v(x, y, t) into the form

$$u(x, y, t) = \bar{u}(x, y) + u'(x, y, t) = \bar{u}(x, y) + \sum_{i=1}^{M} q_i(t)\phi_{u,i}(x, y)$$

$$v(x, y, t) = \bar{v}(x, y) + v'(x, y, t) = \bar{v}(x, y) + \sum_{i=1}^{M} q_i(t)\phi_{v,i}(x, y)$$

(2.1)

where $\bar{u}(x, y)$ and $\bar{v}(x, y)$ are obtained by time averaging the flow-field over M time instances, while u'(x, y, t) and v'(x, y, t) are time-dependent fluctuations from the mean. These fluctuations are decomposed using M mode shapes $\phi_{u,i}(x, y)$, $\phi_{v,i}(x, y)$ and M generalized coordinates $q_i(t)$. For a reduced order model, the number of modes, $n \ll M$, is to be chosen as a compromise between model simplicity and model accuracy. The main feature of the POD technique is to extract the most energetic modes that capture most of the unsteady flow energy.

The unsteady velocity components are obtained simply from

$$u'(x, y, t) = u(x, y, t) - \bar{u}(x, y)$$

$$v'(x, y, t) = v(x, y, t) - \bar{v}(x, y)$$
(2.2)

where the time-averaged $(\bar{u}(x,y), \bar{v}(x,y))$ is obtained from

$$\bar{u}(x,y) = \frac{1}{M} \sum_{i=1}^{M} u(x,y,t_i) \text{ and } \bar{v}(x,y) = \frac{1}{M} \sum_{i=1}^{M} v(x,y,t_i)$$

The POD technique is based on the auto-correlation matrix $\mathbf{D}(t_1, t_2)$, of the total energy in the flow at every instance in time. For a continuous flow,

$$D(t_1, t_2) = \frac{1}{M} \int \int \left(u'(x, y, t_1) u'(x, y, t_2) + v'(x, y, t_1) v'(x, y, t_2) \right) dxdy$$
(2.3)

For a discrete flow, the integrals become summations. Using trapezoidal integration,

$$D_{i,j} = \frac{1}{M} \sum_{k=1}^{n_y - 1} (G_{i,j,k} + G_{i,j,k+1}) \delta y/2$$
(2.4)

where $D_{i,j}$ is the element in the *i*th line and *j*th column of **D**, $G_{i,j,k} = \sum_{l=1}^{n_x-1} (F_{i,j,k,l} + F_{i,j,k,l+1}) \delta x/2$, $F_{i,j,k,l} = (u'_{k,l,i}u'_{k,l,j} + v'_{k,l,i}v'_{k,l,j})$ and the notation $u'_{k,l,i}$ is shorthand for $u'(x_k, y_l, t_i)$. Higher order integration schemes can also be used.

The Proper Orthogonal Decomposition process requires the solution of the eigenvalue

problem

$$\mathbf{DA} = \lambda \mathbf{A} \tag{2.5}$$

where **A** is the matrix of eigenvectors of the **D** matrix and λ are its eigenvalues. If the eigenvectors are normalized in the form $\mathbf{a}_i/\sqrt{\lambda_i M}$, where \mathbf{a}_i is the *i*th column of **A**, then they will form an orthonormal basis. The mode shapes $\phi_{u,i}$ and $\phi_{v,i}$ can then be constructed from

$$\phi_{u,i}(x,y) = \frac{1}{\sqrt{\lambda_i M}} \sum_{m=1}^{M} u'(x,y,t_m) a_{m,i}$$

$$\phi_{v,i}(x,y) = \frac{1}{\sqrt{\lambda_i M}} \sum_{m=1}^{M} v'(x,y,t_m) a_{m,i}$$
(2.6)

where $a_{m,i}$ is the m^{th} element of the i^{th} eigenvector of **D**. The mode shapes are only functions of space but can be used to describe the unsteady flow-field when combined with the generalized coordinates $q_i(t)$, which can be obtained from

$$q_i(t) = \int \int \left(u'(x, y, t) \phi_{u,i}(x, y) + v'(x, y, t) \phi_{v,i}(x, y) \right) dxdy$$
(2.7)

or from the discrete version of this equation.

There are M eigenvalues and hence M sets of mode shapes and generalized coordinates. However, the aim of POD is to create a reduced order model, using only the first n modes that contain most of the fluctuating flow energy. To this end, the quantity $\lambda_i / \sum_{j=1}^n \lambda_j$ can be inspected, assuming that λ_i is ordered from highest to lowest eigenvalue. If the first n eigenvalues that have ratios higher than, say, 10% are chosen, then a model with n modes will be created.

Finally, the *n*-mode approximation of the complete velocity field can be reconstructed from the *n* retained modes using equation 2.1

$$u^{*}(x, y, t) = \bar{u}(x, y) + \sum_{i=1}^{n} q_{i}(t)\phi_{u,i}(x, y)$$
$$v^{*}(x, y, t) = \bar{v}(x, y) + \sum_{i=1}^{n} q_{i}(t)\phi_{v,i}(x, y)$$

These expressions are also used in Reduced Order Modelling (ROM), as discussed in the beginning of the section. Note that these reconstructed velocity fields can be alternatively used as filtered data in order to remove the measurement noise and simplify the analysis of the main characteristics of the phenomenon [137].

2.3 Analysis of the wake of an oscillating cylinder

The POD procedure described above is applied (with some modifications) to experimentally measured unsteady flows behind a circular cylinder at conditions both near to and far from resonance.

2.3.1 Vortex Induced Vibrations

As stated in the previous chapter, Vortex Induced Vibrations (VIV) are due to the matching of the vortex shedding frequency to one of the modal frequencies of the structure. In the case of circular cylinders, the shedding process is strongly influenced by the level of turbulence of the flow-field, that is the Reynolds number (see figure 1.9 in chapter 1).

The different values of airspeed tested during experiments range from 10 m/s to 20 m/s, i.e. *Re* varies between 20,000 and 50,000. The flow-field is thus characterized by a transition to turbulence located in the shear layer [36]. In other words, the boundary layer on the cylinder's surface is expected to be laminar. After separation, there is periodic ejection of vortices in the wake, as in the case of laminar vortex shedding, but the shear layer causing this ejection is transitional, giving rise to small turbulent eddies.

2.3.2 Experimental setup

A circular cylinder of 36 mm diameter and 1.32 m span is placed in the wind tunnel, supported at its mid-span point near the middle of the test section (see figure 2.2). The cylinder is made of an aluminum tube painted matt black. It is rigidly supported and the frequency of its first symmetric bending mode is equal to 70 Hz. Therefore, it is expected that when the frequency of the Von Karman vortex street behind the cylinder matches the first bending frequency, the free ends of the cylinder will oscillate quite visibly. Away from resonance, the cylinder will be static. This setup is ideal for the purposes of the present investigation, as it allows the examination of the unsteady flow behind both a static and an oscillating structure.

PIV system setup

The PIV system is presented in Appendix B. For this set of experiments, the size of the observation window is approximately equal to 10 cm width by 6cm height and the acquisition frequency is set to 1 kHz. The laser sheet is placed on the side of the cylinder nearest to the working section's observation window and aligned with the airflow, so as to illuminate a 2D section of flow around the cylinder. The laser sheet position can be seen in figure 2.2 and the observation window is schematically represented in figure 2.3.



Figure 2.2: Laser sheet illuminating a 2D section of the flow around a cylinder. Flow from right to left. The cylinder is supported by a pylon on its center point

Notice that the centre of the laser sheet lies aft of the cylinder. The very sharp shadow under the cylinder is also worth noting.

Figure 2.4 shows a snapshot of the illuminated particles around the cylinder, where the flow comes from the right of the picture. It can be seen that the laser illuminates the aft upper section of the cylinder itself (white arc) as well as seeded particles on the upper surface of the cylinder and in the wake. Obviously, there are no illuminated particles in the shaded area under the cylinder.

The high speed camera is synchronized to the double laser pulse in order to take two snapshots at a very short time interval, typically 1-1000 μ s. This time interval, denoted time between pulses, is set to 50 μ s during the experiments presented here. This choice is related to the range of airspeeds tested in the wind tunnel and chosen according to the experience.

A region of interest (ROI) is defined in the snapshots: it corresponds to the area where enough light is present in order to track accurately the particles. This ROI is further divided into subregions of 32×32 pixels. The motion of the particles inside each subregion of the first snapshot is correlated to the second snapshot. The aim of the analysis is to quantify the most probable displacement of the particles inside each subregion between photo 1 and photo 2. Thus, a velocity vector is placed in the centre of each subregion, representing the global motion of the particles inside the subregion. The entire process is carried out by means of the Dynamics Studio PIV software. A calibrated dimension is required in the snapshot, in order to convert the units from pixel/s to m/s. In this case, the characteristic length is the cylinder diameter.



Figure 2.3: Sketch of the PIV set-up. The cylinder is clamped on its right side. Two PIV windows are shown as black patterns

The end result of the TrPIV data reduction process is a velocity vector field calculated at each instance in time for which the visualization took place, e.g. as shown in figure 2.1. The image correlation process sometimes leads to the calculation of bad vectors, denoted outliers; these are detected according to a selected threshold [138]. The outliers, once detected, can be replaced by interpolating between the neighboring vectors. It is important to point out that this reconstruction must be performed carefully: abusive use of mathematical reconstruction may lead to the loss of the physical characteristics of the measured phenomenon [139].

TrPIV visualizations for the circular cylinder are carried out at airspeeds from 10 to 20 m/s, at a sampling frequency of 1 kHz and sampling times from 0.1 s to 4 s. The recovered unsteady vector fields have a resolution of 24 gridpoints in the *y*-direction and 41 in the *x*-direction. Therefore, the sizes of the *u* and *v* matrices range from $24 \times 41 \times 100$ to $24 \times 41 \times 4000$.

2.3.3 Flow frequency variation with airspeed

In order to verify that the TrPIV system and POD decomposition analysis are performing correctly, a large number of TrPIV measurements are carried out at airspeeds between 4 m/s and 26 m/s. The flow frequencies recovered by the POD method for all these mea-



Figure 2.4: Illuminated particles around the cylinder. Flow from right to left and the cylinder appears in the right of the photograph

surements are then compared to the theoretical frequencies, assuming that the cylinder has a Strouhal number of 0.2 (see figure 1.9 in chapter 1). This comparison can be seen in figure 2.5, where the experimentally estimated frequencies are plotted as stars with error bars and the theoretical frequency is plotted as a dashed sloped line. The error bars represent the frequency increment, which equals the sampling frequency divided by the number of time measurements M and whose value is 2.02 Hz. The theoretical frequencies are plotted as a dashed line.

Figure 2.5 shows that the frequencies estimated from the decomposed TrPIV measurements are in good agreement with the theoretical predictions. It can be concluded that both the instrumentation and the POD analysis are correctly operated. The two vertical lines in the figure represent the airspeed values between which significant cylinder vibration amplitudes are observed. Indeed, resonance phenomena are observed at airspeeds between 13.5 m/s and 18.0 m/s, corresponding to vortex shedding frequencies of 70-105 Hz. The lock-in phenomenon, whereby the flow frequency adapts itself to the structure's natural frequency throughout the resonance airspeed range, is not evident in this data. The reason for this absence of lock-in is that the measurements used for constructing figure 2.5 are taken close to the cylinder's midpoint, as seen in figure 2.2. At this location the amplitudes of the vertical vibration are small and have no impact on the shedding process. Therefore, the frequency of the latter follows the linear Strouhal relation (see equation 1.3).



Figure 2.5: Comparison of estimated frequencies and theoretical frequencies. Vertical dashed lines representing the interval of airspeed where VIV is observed. The oblique dashed line corresponds to the linear Strouhal relation. Error bars represent the frequency increment (2.02 Hz)

2.3.4 POD analysis

At the end of the PIV data treatment, a set of u(x, y, t) and v(x, y, t) matrices are obtained for each tested airspeed. These matrices are then analyzed using the POD technique. Sample results from four airspeeds are presented and discussed in this section. These are labeled as:

- Test 1: Free-stream airspeed of 18.8 m/s, sampling frequency of 1 kHz, sampling time of 0.298 s, PIV laser sheet at 0.2 m from the cylinder's midpoint.
- Test 2: Free-stream airspeed of 13 m/s, sampling frequency of 1 kHz, sampling time of 0.099 s , PIV laser sheet at 0.2 m from the cylinder's midpoint.
- Test 3: Free-stream airspeed of 13.9 m/s, sampling frequency of 1 kHz, sampling time of 0.099 s, PIV laser sheet at 0.2 m from the cylinder's midpoint.
- Test 4: Free-stream airspeed of 14.8 m/s, sampling frequency of 1 kHz, sampling time of 0.099 s, PIV laser sheet at 0.4 m from the cylinder's midpoint

Test 1 is used as the reference test, as it lies very far from aero-structural resonance and, therefore, there is negligible cylinder movement. Test 4 lies right on resonance and the cylinder vibrates significantly at the PIV measurement position. For all the tests, the first step in the POD procedure is to define the region of interest, so as not to include in the POD calculations the velocity vectors under the cylinder, which are not observable. The region of interest is therefore limited to areas just downstream of the cylinder.



Figure 2.6: Mean flow vectors for Test 1



Figure 2.7: Eigenvalue ratios for the first 20 eigenvalues for Test 1

Test 1

The next step in the POD procedure for Test 1 is to calculate the mean flow. This calculation involves the time averaging of the u(x, y, t) and v(x, y, t) matrices, leading to the mean flow shown in figure 2.6. It can be seen that the mean flow consist of an area of slow recirculating flow located just behind the cylinder and the free-stream away from

the cylinder. In other words, the mean flow can be seen as the steady wake, to which an unsteady wake is superimposed.

Once the mean flow is subtracted from u(x, y, t) and v(x, y, t), the unsteady vector fields u'(x, y, t) and v'(x, y, t) are obtained and Proper Orthogonal Decomposition is applied. The eigenvalue ratios $\lambda_i / \sum_{j=1}^M \lambda_j$ obtained for the first 20 modes are shown in figure 2.7. It can be seen that only the first two eigenvalues have significant contributions to the total energy in the unsteady flow, of 35% and 25% respectively. All higher eigenvalues have contributions of less than 5% and can therefore be neglected.



Figure 2.8: The first two mode shapes for Test 1

The mode shapes associated with the two retained eigenvalues are shown in figure 2.8, plotted as filled contour plots where the colors black and white corresponds to a low and high value, respectively. Subfigure 2.8(a) plots the values of $\phi_u(x, y)$ (left) and $\phi_v(x, y)$ (right) for mode 1 as a filled contour plot. Subfigure 2.8(b) depicts the same information for mode 2. The horizontal distance between a maximum and a minimum grows from 30mm to 40mm with distance downstream. The vertical distance increases with downstream distance from 30mm to 46mm. If the mode shapes are assumed to be periodic, i.e. repeatable further downstream, then the two modes are separated by 1/4 of a cycle.

Figure 2.9 shows the variation in time of the two retained generalized coordinates. It can be seen that the two signals have the same fundamental frequency of 107.5 Hz. Furthermore, they are both subjected to a beating phenomenon, with the response amplitudes dropping momentarily at around 0.12 s and again at around 0.3 s. This beating demonstrates that the flow is quasi-periodic, with significant variations in amplitude occurring momentarily. This quasi-periodic nature is justified by the fact that the flow is transitional in the shear layer and there are small turbulent eddies absorbing some of the flow energy. Free-stream turbulence may also be responsible for this beating phenomenon.



Figure 2.9: Variation in time of the two retained generalized coordinates, Test 1

In fact, the two mode shapes of figure 2.8 are the dominant modes and can be viewed as 'laminar' modes. The modes that have been neglected can be viewed as 'turbulent' modes, which contain little energy over the complete time history but can momentarily absorb energy from the dominant modes. This is exactly what happens in the case of this test. At time indices of 0.12 s and 0.3 s, the response amplitude of modes 1 and 2 drops significantly; simultaneously the response amplitude of mode 3 increases visibly, as shown in figure 2.10(a), which plots the variation of generalized coordinate $q_3(t)$ with time. It can be seen that the maxima of this mode occur at 0.12 s and 0.3 s.

The mode shape for mode 3 can be seen in figure 2.10(b). This mode shape is significantly noisier than the mode shapes of the first two modes, shown in figure 2.8. This noisiness is consistent with the hypothesis that mode 3 represents turbulent flow energy. It should be mentioned that higher modes do not demonstrate any clear increases in



Figure 2.10: The third mode for Test 1

amplitude at 0.12 s and 0.3 s. Therefore, a complete model of the flow of Test 1 will contain:

- two modes if only the laminar flow is of interest
- three modes if it is desired to account for some of the energy momentarily lost by the laminar modes

Test 2

Test 2 is similar to Test 1 in the sense that resonance is not occurring yet; however, the condition is much closer to resonance than Test 1 and the amplitude of vibration of the cylinder is small but noticeable. Under these circumstances, the effect of evaluating a mean flow and subtracting it from the total vector field must be revised. More precisely, as the mean flow corresponds to the steady wake behind the cylinder, if the cylinder

oscillates by a significant amount, then the concept of a steady wake is no longer valid and the mean flow should not be evaluated. In other words, while it is still possible to calculate values for \bar{u} and \bar{v} over the entire time history, these values will not be the same over a different time history. In essence, the movement of the cylinder is an excitation force that is applied to the fluid; it will respond at the excitation frequency and at higher harmonics but there will be no response component with zero frequency.

Nevertheless, if the amplitude of oscillation is very small, then evaluating and subtracting the mean flow will not cause large errors in the POD procedure. For Test 2, the POD method is applied twice, the first time after subtracting the mean flow and the second after subtracting only the wind tunnel free-stream, V_{∞} . In other words, in the first application the POD analysis is carried out on u' and v' using equations 2.3 to 2.7, while the second application is carried out on $u - V_{\infty}$ and v, such that

$$D(t_1, t_2) = \frac{1}{M} \int \int \left((u(x, y, t_1) - V_\infty) (u(x, y, t_2) - V_\infty) + v(x, y, t_1) v(x, y, t_2) \right) dxdy$$
(2.8)

with

$$\phi_{u,i}(x,y) = \frac{1}{\sqrt{\lambda_i M}} \sum_{m=1}^M (u(x,y,t_m) - V_\infty) a_{m,i}$$

$$\phi_{v,i}(x,y) = \frac{1}{\sqrt{\lambda_i M}} \sum_{m=1}^M v(x,y,t_m) a_{m,i}$$

and

$$q_i(t) = \int \int \left((u(x, y, t) - V_\infty) \phi_{u,i}(x, y) + v(x, y, t) \phi_{v,i}(x, y) \right) dxdy$$
(2.9)

For the first application, the number of retained modes is 2, i.e. only the laminar unsteady flow modes are found to be significant; the resulting mode shapes are qualitatively similar to those obtained for Test 1 (figure 2.8). For the second application, the number of retained modes is 3. The first mode represents the mean flow while the other two modes represented the laminar unsteady flow and are very similar to the modes of the first application and, consequently, of figure 2.8. It is interesting to compare the mean flow subtracted from the data in the first application with the first mode obtained from the second application. Figure 2.11 shows contour plots of the first mode $\phi_{u,1}$ and $\phi_{u,2}$ evaluated from the application of POD to $u - V_{\infty}$ and v (top two plots) and of the mean flow components \bar{u} and \bar{v} (bottom plots). It can be seen that the two sets of contour plots are very similar. Therefore, the POD procedure described by equations 2.8 to 2.9 will calculate the mean flow as the most energetic mode.

This is quite an interesting result because it suggests that there is no need to subtract



Figure 2.11: The first mode shape for Test 2 (top) compared to the mean flow (bottom)

the mean flow. If there is a steady component to the flow, then it will be identified automatically as the first mode. If there is no steady component, then equations 2.8 to 2.9 should be used anyway. Figure 2.12 shows the time response of the three resulting generalized coordinates. It can be seen that the generalized coordinate of the first mode is nearly constant with a value of around 600, while the coordinates of the other two modes oscillate around zero. It is clear that the POD procedure can differentiate between steady and unsteady responses.

Test 3

For Test 3 the POD procedure is carried out only using equations 2.8 to 2.9. As mentioned before, Test 3 gives rise to significant amplitudes of cylinder vibration as the flow frequency is very close to a natural frequency of the cylinder. Again, three modes are retained, one representing a mean flow and two representing the laminar oscillating flow.



Figure 2.12: Variation in time of the three retained generalized coordinates, Test 2



Figure 2.13: Variation in time of the three retained generalized coordinates, Test 3

The response amplitudes of the three generalized coordinates are significantly higher than in the case of Test 2, as seen in figure 2.13. Even the generalized coordinate of mode 1 is far more unsteady, although its variation in time is not periodic. This aperiodic variation suggests that there is a component of the cylinder vibration in the flow data but it is of the same order as the turbulent disturbances and/or experimental error. This is indeed the case, as the measurement point is close to the cylinder's midpoint (see figure 2.3), therefore the local vibration amplitude is small, of the order of less than 1 mm.

Test 4

The final test is carried out at a slightly higher airspeed but, crucially, the PIV laser sheet is positioned at a span-wise location further away from the cylinder's midpoint than for the other three tests (see figure 2.3). At this particular span-wise position, the cylinder


Figure 2.14: Variation in time of the three retained generalized coordinates, Test 4

is vibrating significantly, with an amplitude of nearly 2 mm and a frequency of 70.5 Hz.

The POD procedure is again applied using equations 2.8 to 2.9, i.e. without subtracting the mean flow. The resulting mode shapes are similar to those obtained during Test 3. The resulting generalized coordinates are plotted on figure 2.14. It can be clearly seen that the response of the first generalized coordinate is now much more oscillatory than in the case of figure 2.13. Furthermore, despite the randomness of this response, there is a periodic component at a frequency close to that of the oscillating modes.

The observations of Test 4 suggest that POD can decompose flow-fields that feature unsteadiness due to the presence of both alternate shedding of vortices and structural motion of the model. However, for this decomposition to be successful, the response amplitude of the structure must be significantly higher than the size of the turbulent eddies. In such cases, straightforward POD will result in generalized coordinates that contain the response frequencies of both the shedding process and the structural motion. Figure 2.15 shows Power Spectral Densities (PSD) of the $q_i(t)$ and z(t) signals, z(t) being the vertical displacement time history of the cylinder at the PIV measurement position. The PSDs are calculated using the Welch method, with a Hamming window 512 samples long and 50% overlap. The cylinder's displacement response clearly contains only one frequency component at 70.5 Hz. The generalized coordinates feature two frequency components, one at 70.5 Hz and a stronger one at 85.9 Hz. It can be inferred that 70.5 Hz is the structural response frequency while 85.9 Hz is the vortex shedding frequency. The Strouhal frequency at the Test 4 airspeed is 82.2 Hz if a Strouhal number of 0.2 is assumed (see section 2.3.3), i.e. quite close to 85.9 Hz.

The generalized coordinates can be seen as the responses in time of the fluid due to both flow unsteadiness and cylinder motion. Furthermore, the cylinder motion can be seen as an external excitation acting on the fluid. Therefore, it possible to set up an



Figure 2.15: Power Spectral Density plots of the generalized coordinates and the cylinder displacement

input-output POD model, whereby the input is the cylinder motion and the outputs are the generalized coordinates. Frequency Response Functions (FRF) can then be created of the form

$$H_i(\omega) = \frac{Q_i(\omega)}{Z(\omega)} \tag{2.10}$$

where H_i is the i^{th} FRF, Q_i is the i^{th} generalized coordinate in the frequency domain, Z is the cylinder displacement in the frequency domain and ω is the radial frequency. Such FRFs can be estimated using a Welch-type windowed approach and involve cross and auto-correlations of the outputs and input.

Figure 2.16 shows the FRFs estimated for the first three modes of Test 4. It can be seen that the main frequency component of all FRFs is the vortex shedding frequency at 85.9 Hz, in the frequency range between 50 and 100 Hz.

2.3.5 Summary

The feasibility of applying Proper Orthogonal Decomposition to experimentally measured flows around vibrating structures has been demonstrated. It has been shown that this type of decomposition analysis can provide some very interesting data about the observed flows, such as the dominant mode shapes and frequencies. Furthermore, it is shown that structural vibrations can be detected by the POD procedure applied on PIV flow visualization data using an output-only approach.

By considering the cylinder structural response as a forcing function, it is possible to create input-output POD models, whereby the generalized coordinates can be obtained from Frequency Response Functions relating the cylinder displacement response



Figure 2.16: FRFs between the generalized coordinates and the cylinder displacement

to the generalized coordinates themselves. It is shown that such FRFs feature two main frequency components, the mean flow frequency (i.e. 0 Hz) and the vortex shedding frequency. Therefore, they are independent of the structural response frequency.

2.4 Common-base POD

The Common-base Proper Orthogonal Decomposition (CPOD) approach is an extension of the classical POD method. It was proposed by Kriegseis [126] in order to analyze TrPIV measurements of a flow-field. In that work, CPOD enabled the quantitative comparison of the characteristics of the flow-field for different positions of a plasma actuator.

The general principle of CPOD is to assemble in a global matrix several sets of data corresponding to different values of a parameter of the system. Then the POD analysis of this global matrix is carried out and the resulting mode shapes and generalized coordinates represent the initial sets of data in an optimal form. The advantage is the uniqueness of the modes shapes (the common basis), on which the generalized coordinates corresponding to each value of the parameter can be compared.

This concept is especially well adapted to aeroelastic systems, where the key parameter is the airspeed. Hence the CPOD analysis is carried out on the unsteady velocity fields presented in section 2.3. A quantitative analysis of TrPIV measurements corresponding to different airspeeds is proposed. In addition, the method is extended to linear dynamic systems where CPOD is used in order to identify the damping.

2.4.1 CPOD of aeroelastic systems

In the case of aeroelastic systems, the global matrix, denoted U contains several sets of data corresponding to different free-stream velocity V_{∞} . The data of interest are the TrPIV measurements analyzed in the previous section using the POD approach.

The global matrix **U** takes the form:

$$\mathbf{U} = \begin{bmatrix} \mathbf{u}^{(1)} & \mathbf{u}^{(2)} \\ V_{\infty}^{(1)} & \overline{V_{\infty}^{(2)}} & \cdots & \mathbf{u}^{(p)} \\ V_{\infty}^{(p)} & \cdots & \overline{V_{\infty}^{(Np)}} \end{bmatrix}$$

where $\mathbf{u}^{(p)}$ is an abbreviation for $u^{(p)}(x_i, y_i, t_k)$, denoting the horizontal velocity component. The superscript p denotes the dataset corresponding to the p^{th} free-stream velocity, $V_{\infty}^{(p)}$, with $p = 1, \ldots, N_p$. A similar global matrix \mathbf{V} is assembled for the vertical velocity component $\mathbf{v}^{(p)} = v^{(p)}(x_i, y_i, t_k)$:

$$\mathbf{V} = \left[\begin{array}{cccc} \frac{\mathbf{v}^{(1)}}{V_{\infty}^{(1)}} & \frac{\mathbf{v}^{(2)}}{V_{\infty}^{(2)}} & \dots & \frac{\mathbf{v}^{(p)}}{V_{\infty}^{(p)}} & \dots & \frac{\mathbf{v}^{(N_p)}}{V_{\infty}^{(N_p)}} \end{array}\right]$$

Each set of data $u^{(p)}(x, y, t)$ and $v^{(p)}(x, y, t)$ is divided by the corresponding free-stream velocity $V_{\infty}^{(p)}$ in order to build a global matrix with data of the same order of magnitude. This division is equivalent to the subtraction of the mean velocity components (equation 2.2) or the free-stream velocity (equation 2.8) in the case of the POD analysis.

The velocity vectors, $u^{(p)}(x, y, t)$ and $v^{(p)}(x, y, t)$, are available in the form of $n_y \times n_x \times M^{(p)}$ real arrays, where $M^{(p)}$ denotes the number of snapshots measured at the p^{th} free-stream velocity. Note that it is easier to set $M^{(p)}$ constant for all tested airspeeds (e.g. $M^{(p)} = 100$ for $p = 1, \ldots, N_p$) but it can vary without restriction. The size of the global matrices **U** and **V** is $n_y \times n_x \times \sum_{p=1}^{N_p} M^{(p)}$. If the last dimension of these matrices is denoted $M^* = N_p \times \sum_{p=1}^{N_p} M^{(p)}$, the final size of the global matrices is $n_y \times n_x \times M^*$ and the CPOD method is equivalent to the POD, where M (equation 2.1) is replaced by M^* .

Then, similarly to POD, CPOD is based on the eigenvalue decomposition of a co-

variance matrix built from the global matrices **U** and **V** (equations 2.3 to 2.7). The unique vector basis resulting from this eigen-decomposition is optimal in representing the N_p different unsteady velocity fields taken into account.

The CPOD technique expresses the 2D flow-field $u^{(p)}(x, y, t)$ and $v^{(p)}(x, y, t)$ measured at a value of airspeed $V_{\infty}^{(p)}$ in the form

$$u^{(p)}(x,y,t) = \sum_{i=1}^{M^{(p)}} q_i^{(p)}(t) \Phi_{u,i}(x,y) \qquad v^{(p)}(x,y,t) = \sum_{i=1}^{M^{(p)}} q_i^{(p)}(t) \Phi_{v,i}(x,y)$$
(2.11)

where $\Phi_{u,i}(x, y)$, $\Phi_{v,i}(x, y)$ are the unique basis mode shapes and $q_i^{(p)}(t)$ represent the time dependence of the velocity fields of the p^{th} parameter (airspeed) in the i^{th} mode shape. There are M^* eigenvalues and hence M^* sets of CPOD modes and generalized coordinates. Since the decomposition is optimal, the first n modes, with $n \ll M^*$, contain most of the fluctuating flow energy and are sufficient to reconstruct accurately the initial velocity fields.

CPOD analysis of the wake of an oscillating cylinder

An application of the CPOD method presented above is proposed in this section. The analysis is based on the TrPIV measurements of Test 1 and 4 presented in section 2.3, with the objective to show the capability of the CPOD technique to highlight the differences in the aerodynamics between the two airspeeds.

Recall that VIV are observed for Test 4, when the airspeed is set to 14.8 m/s. Furthermore, the position of the PIV window was set to 0.4 m from the cylinder's center point, hence the vibration of the body was observed in the POD outputs (see figure 2.14).

Because of the motion of the cylinder in the case of Test 4, the mean value of the velocity components is not subtracted from the PIV measurements. This is justified by the discussion of the section 2.3, where it is stated that the mean flow around an oscillating body is not consistent with the mean flow around a static body. This is the reason why the mean velocity does not appear in equations 2.11.

The size of the PIV grid is equal to $n_y \times n_x = 24 \times 41$ (unchanged from the previous PIV measurements). The frequency of acquisition is equal to 1 kHz and the length of measurement is set to 0.3s for each airspeed, leading to $M^{(1)} = M^{(2)} = 300$. Two airspeeds are selected, i.e. $N_p = 2$ and the size of the global matrices **U** and **V** is $24 \times 41 \times 600$.

Figure 2.17 shows the first four CPOD mode shapes corresponding to the abovementioned matrices, in the form of contour plots, as presented before for the POD modes. The first mode shape (figures 2.17(a) and 2.17(b)) corresponds to the mean flow, as shown in figure 2.6. Furthermore, the shapes of modes 2 and 3 are obviously identical to the



Figure 2.17: CPOD modes - contour plots

laminar modes (figure 2.8). Finally, the fourth mode shape of figure 2.17 corresponds to the third mode shown in figure 2.10. The difference between the numbering of the POD analysis and the present CPOD analysis is of course due to the fact that the mean flow is subtracted from the PIV measurements in the former case, while it is retained in the present analysis.

Another interpretation can be proposed by plotting the mode shapes in vector form. Figure 2.18 shows plots of the four modes, where the vectors' horizontal component is taken as $\phi_{u,i}(x, y)$ and the vertical one as $\phi_{v,i}(x, y)$. The four mode shapes can be described as

- Mode 1 (figure 2.18(a)) consists of two areas of recirculation behind the top and the bottom of the cylinder. Between them, they cause an area of flow towards the cylinder with a height approximately equal to the cylinder's diameter. This mode shape is the mean flow, as stated above (see figure 2.6).
- Mode 2 (figure 2.18(b)) consists of a large vortex, positioned approximately one diameter behind the cylinder and centered on the cylinder's centerline.
- Mode 3 (figure 2.18(c)) consists of two counter-rotating vortices at approximately half a diameter (the larger) and one diameter (the smaller) behind the cylinder. They are both centered on the cylinder's centerline.



Figure 2.18: CPOD modes - vector plots

• Mode 4 (figure 2.18(d)) consists of strongly recirculating flow towards the cylinder, surrounded by two counter-rotating vortices.

The generalized coordinates of these mode shapes are shown, together with their frequency content, in figures 2.19 to 2.22. The top plot of each figure corresponds to the generalized coordinate $q_i(t)$ and the bottom plot is the corresponding Power Spectral Density in the frequency range 0-200 Hz.

It is observed in figure 2.19 that $q_1(t)$ has a non zero mean, in accordance with its interpretation as the mean flow. Furthermore, the frequency content varies with airspeed: at 18.8 m/s, where no vibration are observed, the only important frequency peak occurs at 107.4 Hz (dashed line in the bottom plot of figure 2.19). It corresponds to the vortex shedding, in agreement with a Strouhal number equal to 0.2 ($f_s = \frac{18.8 \times 0.2}{0.036} = 104.4$ Hz). At 14.8 m/s, VIV are observed and appear in the PIV measurements: a first peak, is observed at 70.5 Hz, which is the natural frequency of the first bending mode of the



Figure 2.19: Generalized coordinates $q_1(t)$ (top) and their PSD's (bottom)

cylinder and a second peak, corresponding to the vortex shedding frequency occurs at 87.3 Hz (the theoretical value of the vortex shedding frequency for a Strouhal of 0.2 being 82.2 Hz at this airspeed).

The second and third modes behave similarly to those from the POD analysis: only the shedding frequency appears at 18.8 m/s, while the structural motion frequency component is added at 14.8 m/s (see figures 2.20 and 2.21).



Figure 2.20: Generalized coordinates $q_2(t)$ (top) and their PSD's (bottom)

The amplitude of the generalized coordinate of mode 4 is low (figure 2.22). In addition its frequency content is broad, despite three small peaks in the case of 14.8 m/s, at 70.5 Hz, 97 Hz and 157 Hz. The first of these frequency is the natural frequency of the cylinder but the other two are not related to the vortex shedding process. As stated previously, this mode is considered to be a turbulent mode.



Figure 2.21: Generalized coordinates $q_3(t)$ (top) and their PSD's (bottom)



Figure 2.22: Generalized coordinates $q_4(t)$ (top) and their PSD's (bottom)

From the present analysis, it can be argued that the CPOD technique shows the same capabilities as the POD method with the advantage of the uniqueness of its modal basis: the comparison between two states of the flow-field is performed through the analysis of the generalized coordinates only. As far as Reduced Order Models are concerned, this approach is more robust than a single POD analysis based on one state of the flow-field. The counter-part of this robustness is of course the accuracy, which is naturally lower than a single, devoted, POD analysis of an observation.

2.4.2 Damping identification for dynamic systems using CPOD

This section presents a novel application of the POD and CPOD techniques to the domain of structural vibrations. The method was developed in the course of this thesis, in parallel to the aeroelastic applications presented in this chapter and later in chapter 4. During the last decade the Proper Orthogonal Decomposition (POD) method has been extensively applied to vibrations problems. Feeney and Kappagantu [140] and Kerschen and Golinval [141] presented a physical and geometrical interpretation of the Proper Orthogonal Modes. Despite the current high level of maturity of modal analysis methods, the identification of damping in vibrating systems remains an important challenge [142].

The technique is briefly presented here, starting from the POD concept and then extended to the CPOD approach. It results from a mathematical development of the co-variance matrix of the free response of the degrees of freedom of the system. The mathematical derivations are presented in Appendix C, while this section presents the principal ideas of the method. A benchmark application, concerning the identification of structural damping, is used to demonstrate the technique. Despite the wind-off characteristic of the benchmark, the method is used in the next chapters to identify the total damping of aeroelastic systems, i.e. the sum of the structural damping and the aerodynamic damping.

In the scope of dynamic systems, the unsteady velocity fields of the previous section are replaced by the free responses of the system, denoted $\mathbf{y}(x,t)$. The POD consists in the spatio-temporal decomposition:

$$\mathbf{y}(x,t) = \sum_{k=1}^{N} q_k(t) \Phi_k(x)$$
(2.12)

where $q_k(t)$ and $\Phi_k(x)$ are the generalized coordinates and the corresponding mode shapes, as defined for equation 2.1. The maximum number of POD modes that can be identified is denoted by N. The upper limit of N is set by the number of geometrical positions where the response of the system is known. The co-variance matrix **D** is defined as

$$\mathbf{D} = \frac{1}{M} \sum_{k=0}^{M} \mathbf{y}(k\Delta t) \mathbf{y}^{T}(k\Delta t)$$

where M is the number of time steps in the signal $\mathbf{y}(x, t)$. The method consists in relating the properties of \mathbf{D} to the modal characteristics of the system. This is carried out through the development of the product $\mathbf{y}(k\Delta t)\mathbf{y}(k\Delta t)^T$ in Appendix C.

Consider the equation of motion of a linear dynamic system:

$$M\ddot{y}+C\dot{y}+Ky=0$$

The free response of this system can be written in the discrete form: $\mathbf{y}(k\Delta t) = \mathbf{Q}e^{\mathbf{A}k\Delta t}\mathbf{x}(0)$, where $\mathbf{x}(0) = [\dot{\mathbf{y}}(0)^T \ \mathbf{y}(0)^T]^T$ corresponds to the initial condition applied to the system. The matrix \mathbf{A} in this expression corresponds to the damping of the modes of the system. The mathematical developments of Appendix C show that the diagonal terms a_k of \mathbf{A} can be approximated through the POD analysis as being equal to

$$a_k \approx -\frac{z_{kk}\Delta t}{4n\lambda_k} \tag{2.13}$$

where z_{kk} is the diagonal term of matrix $\mathbf{Z} = \mathbf{\Phi}^{-1} \mathbf{y}(0) \mathbf{y}^T(0) (\mathbf{\Phi}^{-1})^T$. The terms λ_k and $\mathbf{\Phi}$ denote the POD eigenvalue and the mode shapes matrix respectively.

This expression is extended to the CPOD approach, in order to increase the robustness of the technique, by taking into account several responses of the system. In the case of vibration problems, the parameter which characterizes the different sets of data is the definition of the initial conditions of the system. Hence, a global matrix \mathbf{Y} is defined as

$$\mathbf{Y} = \left[\begin{array}{ccc} \mathbf{y}^1 & \mathbf{y}^2 & \dots & \mathbf{y}^{(p)}_0 & \dots & \mathbf{y}^{N_p}_0 \end{array} \right]$$

where the notations of the previous section are used, i.e. $\mathbf{y}^{(p)}$ is a short-cut for $\mathbf{y}^{(p)}(x,t)$, denoting the free response of the system to the p^{th} initial condition. Note that the division by $y_0^{(p)}$ is performed in order to obtain the same order of magnitude for all data sets, as previously stated. Considering N_p initial conditions, the co-variance matrix **D** becomes

$$\mathbf{D} = \frac{1}{N \times n} \mathbf{Y} \mathbf{Y}^T = \frac{1}{N \times n} \sum_{k=0}^n \sum_{p=1}^N \mathbf{y}_p(k\Delta t) \mathbf{y}_p^T(k\Delta t)$$

Each free response of the system can be expressed as $\mathbf{y}_p(k\Delta t) = \mathbf{Q}e^{\mathbf{A}k\Delta t}\mathbf{x}_p(0)$ and it is shown in Appendix C that the same relation is found for the diagonal term of matrix **A** (equation 2.13):

$$a_k \approx -\frac{z'_{kk}\Delta t}{4n\lambda'_k} \tag{2.14}$$

where the prime denotes the quantities calculated from the CPOD decomposition.

The difference between expressions 2.13 and 2.14 lies in the origin of z_{kk} , λ_k , z'_{kk} and λ'_k . Because the CPOD results (z'_{kk} and λ'_k) are based on several structural responses, they are more robust to the uncertainty in the mode shapes and it is more likely that all the modes of interest are excited. Furthermore it can be shown that the method is less sensitive to noise in the responses.

Experimental application

This section presents an experimental application of the method to a simple structure. The resulting damping estimates are compared to those obtained from the Least Squares Complex Frequency Domain technique (LSCF) [143]. The tested structure consists of a triangular cantilevered flat plate made of aluminium sheet with a thickness of 1mm and represents a flexible Delta wing [144]. The root of the wing is clamped to the floor, as shown in figure 2.23.



Figure 2.23: Photograph and sketch of the experimental setup

Four PCB accelerometers with a frequency range between 0 and 300 Hz are fixed on the Delta wing in order to measure its structural response. The positions of the accelerometers are defined in the drawing of figure 2.23. The structural responses are sampled at 1 kHz and data sequences of 20000 samples are recorded.

The excitation of the structure is provided through hammer impact at three different positions on the accelerometer-free side of the wing: at the locations of accelerometers 1 and 2 and at a central location shown as a cross in figure 2.23. The impacts are repeated five times at each excitation position, resulting in a set of 15 structural responses (N = 4 and $N_p = 15$).

Damping identification using the present CPOD method is carried out and the re-

sulting damping estimates compared to the predictions of the LSCF technique. Both methods are applied in a polyreference context, i.e. by taking into account all excitations and responses simultaneously. The data are low-pass filtered below 30 Hz for the CPOD approach and band-pass filtered between 1 Hz and 30 Hz for the LSCF approach. For the latter, a maximum model order of 16 is used and only poles with damping ratios between 0.1% and 3% are included in the stabilisation diagram. For the CPOD technique only the first 10s worth of response data are used.

Modes	1	2	3	4
CPOD	1.95%	0.30%	0.16%	0.79%
LSCF	1.72%	0.40%	0.13%	0.80%

Table 2.1: Comparison of the damping estimations

The estimated damping ratios for the first four modes are presented in table 2.1. It is clear that there is very good agreement between the damping estimates obtained from the two methods. The clear advantage of the CPOD approach is that it does not require a stabilization diagram. On the other hand, the maximum number of modes that can be estimated is equal to the number of locations where the responses are measured. In the present case, the method cannot estimate more than four modal dampings. Nevertheless, this limitation is not very important as the number of sensors can be chosen equal to the number of modes of interest.

2.5 Chapter summary

This chapter discusses the concept of orthogonal decompositions with specific attention to aeroelastic systems. The Proper Orthogonal Decomposition (POD) method is presented and applied to the analysis of the wake of a flexible cylinder undergoing VIV. It is shown that the method is suitable for extracting the dominant mode shapes and the corresponding frequencies and enhances physical interpretation. The most energetic structures are identified as 'laminar modes', while the others are classified as 'turbulent modes'. The latter are responsible for the random variations in the wake due to the small turbulent eddies, typical of the Reynolds number range reached during experiments $(2 \times 10^4 - 5 \times 10^5)$. In addition the possibility to create input-output models, based on the POD analysis is established.

From these interesting results, the extension of POD to Common-base POD (CPOD) is presented. This technique consists in assembling several sets of data from a system,

corresponding to different values of a parameter. The advantage of CPOD, compared to POD, is that it results in a unique basis of mode shapes, in which each set of data is decomposed.

The CPOD analysis is applied to the TrPIV measurements in the wake of the cylinder, under both static and oscillating conditions. In this application, the parameter is the airspeed, which is responsible for the occurrence of vortex shedding yielding to Vortex Induced Vibrations (VIV). It is shown that the analysis of the VIV is made easier, through the use of a unique set of mode shapes resulting from the CPOD. The comparison is based on the generalized coordinates only and the effect of the motion of the cylinder is well captured by the common-base decomposition.

Another innovative application of POD/CPOD is presented. It concerns the identification of the damping of linear dynamic systems using CPOD. The method is presented and demonstrated through an application to data from an experimental system. The comparison to the results of the LSCF method shows very good estimates of the damping coefficient.

As a concluding remark, despite the introductory nature of this chapter, the VIV of a circular cylinder are observed and analyzed quantitatively. Important observations are made concerning the manner of applying orthogonal decompositions to unsteady flows, especially in the case where a body oscillates. The present understanding will be used to a large extent in the next chapters, in order to analyze the mechanisms of flow separation and re-attachment (chapter 4), but also to carry out validations of numerical simulations (chapter 5).

Chapter 3

Galloping of a bridge deck

3.1 Introduction

This chapter presents the experimental investigation of the galloping instability of a generic bridge section. The complete aeroelastic behaviour of the bridge is identified through wind tunnel experiments. According to the classification of the aeroelastic phenomena presented in section 1.3, galloping is part of the MIE, i.e. the source of excitation leading to the galloping oscillations is due to the motion of the structure.

The phenomenon of galloping of bluff sections has been investigated by a large number of researchers, using both basic shapes, such as rectangles and representative structures, such as bridge decks. Since the seminal work by Parkinson and his co-workers [79, 145], there have been many studies addressing various issues, such as aerodynamic modelling [103, 146], multiple degree-of-freedom structures [147], oscillation control or suppression [148, 149] and various case studies [150–152]. Nevertheless, few attempts have been made to advance the theoretical understanding of the phenomenon further than the quasi-steady theory proposed by Parkinson and used many times since then in several variants. In particular, the complete bifurcation behaviour of a system undergoing galloping oscillations has rarely been addressed, except in the work of Van Oudheusden [153] and Vio [154].

In this chapter, estimates of the critical airspeed of galloping, based on the first order quasi-steady theory are compared to the experimental data. Then, a higher order polynomial model is introduced to propose an improved quasi-steady model of galloping. The identification of the model is based on the experimental dynamic measurements. A first order Harmonic Balance approach is used to identify the coefficients of the polynomial.

3.2 Experimental set-up

Experiments are carried out in the aeronautical test section of the wind tunnel of the University of Liège (see Appendix A). A generic bridge deck section is installed inside the test section by means of a support structure, which can be used for static and dynamic tests.

3.2.1 Generic bridge section

The generic bridge model is a section type model measuring 0.317 m wide by 1.2 m long. The bridge deck is built using an additive manufacturing technique named stereolithography. The resulting white structure shown in figure 3.1 is highly rigid and the sharp edges of the bridge are well reproduced. The surface of the model is relatively rough, which leads to turbulent flow regime (high Reynolds number), in agreement with the real civil engineering structures (see section 1.2).



Figure 3.1: Experimental set-up: general view of the generic bridge section

The deck is comprised of a trapezoidal beam supporting the deck, which is composed of two double traffic lanes. Windscreens can be added to both sides of the bridge, i.e. at the leading edge and trailing edge. The equivalent porosity of these screens is 42%, hence their effect on the aerodynamic shape is considerable. In addition, an acoustic panel can be fixed on one side of the deck section. This barrier is impermeable to air and is modeled by an aluminum plate of 30mm height and 2mm thickness, along the whole span of the deck. These add-ons, usually used on bridge's decks, represent aerodynamic appendages that create a strong asymmetry in the shape of the bridge. They are shown in figure 3.2. Note that safety barriers present on the top of the deck are slightly off-centre.



Figure 3.2: Experimental set-up: acoustic panel (left), wind shield (right)

A square aluminum axis passes through the plane of symmetry of the bridge deck. The center of this axis lies 2.7cm below the upper surface. A sketch of the bridge section with the principal dimensions is presented in figure 3.3. The above mentioned axis is shown as a dashed square. End-plates are placed at each spanwise extremity of the model to approximate a two-dimensional flow around the structure (see figure 3.4).



Figure 3.3: Experimental set-up: main dimensions of the generic bridge deck



Figure 3.4: Experimental set-up: generic bridge section with end plates

3.2.2 Supporting structure

The model is supported by means of a rigid framework that is placed inside the wind tunnel's test section. The structure allows both static and dynamic tests on the bridge model. Its design was based on the experimental apparatus proposed by Sarkar [155]. Two pictures of the structure supporting the bridge deck are shown in figure 3.5.

Figure 3.6 shows pictures of the force and torque sensors used during the static aerodynamic tests. The upper photograph depicts the vertical and horizontal force sensors. The black blocks appearing on this picture correspond to the pneumatic bushings, which ensure a frictionless measurement of the forces. The bottom picture shows the torque sensor, connected to the torsional axis passing through the bridge model and supported on each side by a ball bearing.



Figure 3.5: Experimental set-up: supporting structure inside the test section



Figure 3.6: Experimental set-up: static load sensors (force up and torque bottom)

3.2.3 Suspension system

The rigid bridge deck is suspended in the supporting structure using extension springs. The latter are secured to symmetric arms attached to the deck's central axis, as shown in figure 3.7. The bridge is free to vibrate along its six degrees of freedom. The windscreens and acoustic barrier are present on the generic bridge deck in the tests described in this chapter.

The suspension apparatus consists of 16 linear extension springs with a stiffness of 755N/m each, attached to two suspension arms linked to the model. They are shown in figure 3.7. The springs are in tension in order to avoid over compression and possible shocks. The deflections of the springs are limited to their linear range leading to a globally linear behaviour of the complete structure.

The motion of the model is measured using four capacitive accelerometers placed on the horizontal suspension arms. PCB accelerometers are used, characterized by an operating frequency range from 0 to 300 Hz and an acceleration range of ± 50 g. The acquisition frequency is set to 1 kHz and signals are filtered through a bandpass filter, limiting the frequency content between 2 Hz and 20 Hz.



Figure 3.7: Experimental set-up: suspension apparatus

3.3 Wind-off dynamic tests

Vibration tests at zero airspeed are performed in order to identify the modal parameters of the suspended bridge deck. The structure is excited by different initial conditions along its pitching, heaving and rolling degrees of freedom. Figure 3.8 shows the time signal and the frequency content of the first accelerometer when the bridge deck is excited vertically.



Figure 3.8: Response of the structure at zero airspeed

Three modes appear clearly in the frequency plot: heaving at 4.65 Hz, pitching at 7.0 Hz and rolling at 8.52 Hz. The present work focuses on the heaving mode, which is the only mode that participates in the galloping oscillations. The equation of the vertical motion is

$$m\ddot{y} + c\dot{y} + ky = F_y^{ext} \tag{3.1}$$

where m, c and k denote respectively the mass, damping coefficient and structural stiffness of the system. Their values are identified experimentally and expressed per unit length in table 3.1. The term F_y^{ext} on the right hand side of equation 3.1 corresponds to the external forces per unit length applied to the heave dof of the system.

$$\frac{m \, [kg/m]}{11.9} \frac{c \, [kg/s/m]}{9.0} \frac{k \, [N/m^2]}{10066.7}$$

Table 3.1: Structural parameters of the bridge model

3.4 Static wind-on tests

Figure 3.9 shows the evolution of the lift and drag coefficients with the static angle of attack α_s according to their definitions:

$$C_{L} = \frac{F_{L}}{1/2\rho V_{\infty}^{2}B} \qquad \qquad C_{D} = \frac{F_{D}}{1/2\rho V_{\infty}^{2}B} \qquad (3.2)$$

where B denotes the chord length of the model. The airspeed, V_{∞} , is set to 15 m/s and the angle of attack varies between -15° and 30° . The lift and drag forces are measured using the static force sensors shown in figure 3.6. The notations F_L and F_D express the lift and drag forces per unit length. The moment coefficient is also measured but it is not presented here because this section is dedicated to the vertical motion of the bridge only.

The slope of the lift curve, $\frac{\partial C_L}{\partial \alpha}\Big|_{\alpha_s}$ is positive for angles of attack between -15° and 4°. It becomes negative in the range of 4° to 24° and has its minimum negative value around 20°.



Figure 3.9: Lift and drag coefficients of the bridge section

Note that at $\alpha_s = 19^\circ$, the lift coefficient is approximately equal to 0.5. If one considers an airspeed of 20 m/s and the measured stiffness of the springs, the vertical displacement y_0 due to the static aerodynamic lift is less than 4 mm.

3.5 Dynamic wind-on tests

Dynamic tests are carried out to investigate the galloping behaviour of the bridge section for $\alpha_s = 19^{\circ}$. Note that additional qualitative experiments have been performed at different angles of attack (0°, 5°, 10° and 15°), but no galloping instability was observed.

The aeroelastic system is excited in order to study its bifurcation behaviour. The excitations considered here can be classified as follows

- Turbulence excitation caused by the wind tunnel free-stream turbulence, the wake behind the support structure or the vibrations of the end-plates. Remark that this type of perturbation is equivalent to the turbulent excitation tests carried out on a rectangle cylinder free to oscillate around its pitching dof (section 4.3).
- External excitation taking the form of vertical displacements applied to the bridge deck by the operator via a string attached to the bridge's central axis.

In both cases the excitations are not measured but the analysis of the resulting response allows to draw conclusions about the nonlinear behaviour of the system. At each airspeed, the acquisition of the accelerometer signals was launched before applying any controlled perturbation. The measured accelerations were then processed in order to estimate the vibration amplitude of the bridge.

The recovered displacement response amplitude is presented in figure 3.10 in the form of a bifurcation diagram. The diagram shows the evolution of the amplitude of the vertical motion with airspeed. The airspeed is varied in the wind tunnel between 6 m/s and 17.8 m/s. Three types of responses were observed: decaying motion, intermittent and stable LCOs.

Decaying motion was observed within the entire range of airspeeds. Figure 3.11 shows the vertical displacement response to an external excitation of the system from the static equilibrium position applied by the operator. The measurement corresponds to an airspeed of 9.9 m/s. When the motion dies out, the noisy response is due to the weak turbulence of the oncoming flow-field. Note that the system is very lightly damped since more than 30 seconds are necessary for it to reach the equilibrium position.

Intermittent LCO responses is denoted by INT LCO in the bifurcation diagram (figure 3.10). Figure 3.12 shows an example of the occurrence of intermittent LCOs for 11.4 m/s. Three sections of the response are picked out with rectangles. The first high amplitude event is the result of an external excitation applied by the operator while the second and the third are self-excited. In both cases, the high amplitude response is limited



Figure 3.10: Bifurcation diagram of the bridge deck



Figure 3.11: Time response: decaying motion at 9.9 m/s

in time and dies out after 5 - 10 seconds. The amplitude of this type of response is small compared to that of the stable LCOs and constant with the airspeed.

Finally, large amplitude stable LCOs are observed for airspeeds between 10 m/s and 15 m/s. The amplitude of the LCOs increases with airspeed, the peak to peak oscillation amplitude increasing from 0.014 m at 10 m/s to 0.042 m at 14 m/s. Figure 3.13 shows the response of the system to external excitation when the airspeed is 13.1 m/s. The amplitude of oscillation increases slowly during the first 40 seconds and is stabilized to the value of 0.032 m (peak to peak). Despite subsequent small amplitude variations, this type of response is stable, staying on the limit cycle for as long as the airspeed is held constant.

For airspeeds above 14 m/s, even larger amplitude LCOs are observed. They are



Figure 3.12: Time response : intermittent LCO at 11.4 m/s

similar to the stable LCOs that occur between 10 m/s and 14 m/s. Between 14.0 m/s and 14.2 m/s the LCO amplitude jumps abruptly. At higher airspeeds the oscillations reach dangerously high amplitudes (peak to peak amplitude equal to 0.07 m). For this reason, no dynamic tests were carried out at airspeeds higher than 15.1 m/s.



Figure 3.13: Time response : stable LCO at 13.1 m/s

The frequency content of the responses of the system was measured over the entire range of airspeeds. It was found that the frequency remains unchanged and equal to around 4.6 Hz at all airspeeds. This observation is in accordance with the quasi-steady theory, which states that galloping is a damping driven phenomenon. The complete quasisteady approach is presented in the next section.

The dynamic results presented above can be summarized as follows:

- In the airspeed range between 6 m/s and 17.8 m/s the bridge will remain in its equilibrium position unless excited by an external factor, either controlled or uncontrolled.
- At airspeeds lower than 10 m/s, if any excitation is applied, the bridge will oscillate for a while before returning to its equilibrium position.
- At airspeeds higher than 10 m/s, if a high amplitude excitation is applied, the bridge will start undergoing limit cycle oscillations whose amplitude increases with

airspeed. If a low amplitude excitation is applied, the bridge will return to its equilibrium position or undergo intermittent LCOs.

• At 14.2 m/s the stable LCO amplitude jumps to higher values. At this airspeed, different LCO amplitudes of 0.02 m and 0.027 m were measured on separate tests.

In a large class of nonlinear systems undergoing such bifurcations, including many aeroelastic systems [4], the unstable limit cycle originates at a subcritical Hopf point. This point appears at the coordinates (5,0) in figure 3.14, which shows an idealized representation of the experimental results presented in figure 3.10. This figure depicts the occurrence of one Hopf and three fold bifurcations. The notion of Hopf and fold is defined later in this thesis, in the discussion of the torsional stall flutter oscillations, where the nonlinear behaviour is less complex (see section 'Stability concepts' on page 116).



Figure 3.14: Fold and Hopf bifurcations

The Hopf point was sought but was not found during the course of the present experiments. At a subcritical Hopf point the unstable limit cycle disappears, the stable fixed point becomes unstable and the only stable attractor is a large amplitude stable limit cycle (see figure 3.14). As the LCO amplitude was already dangerously high at 15 m/s, it was decided not to test at higher airspeeds for safety reasons.

The intermittent LCO behaviour is due to the existence of a weakly unstable limit cycle at all airspeeds tested above 10 m/s. This cycle has a small amplitude and pushes

the system's response away but at a very slow rate. Hence, when the system is excited towards the cycle, it oscillates around it before returning back to the equilibrium position.

The next section presents the quasi-steady theory based on the static measurements presented in section 3.4. These estimations are compared with the dynamic behaviour observed and discussed in the section.

3.6 Quasi-steady model

According to the general linear theory, an instability occurs when the total damping in the system vanishes. In the case of an aeroelastic system, the aerodynamic damping becomes negative and the structural damping can not dissipate the energy transferred by the flow-field to the system. Den Hartog showed that a fairly good estimation of the onset velocity of the galloping instability can be obtained using the linear quasi-steady theory [89].

The typical quasi-steady galloping model uses the variation of the static aerodynamic coefficient with angle of attack in order to predict the unsteady aerodynamic loads. The theory is developed in Appendix D. The resulting Den Hartog's criterion states that the system can undergo galloping oscillations, when starting from rest, if the quasi-steady quantity A_1 is positive, i.e.

$$A_1 = -\left(\left.\frac{\partial C_L}{\partial \alpha}\right|_{\alpha_s} + C_D(\alpha_s)\right) > 0 \tag{3.3}$$

and the linearized quasi-steady equation of motion is

$$m\ddot{y} + c\dot{y} + ky = 1/2\rho V_{\infty}^2 B C_{F_y} \tag{3.4}$$

The aerodynamic force coefficient C_{F_y} is expressed for the oscillating body around the static position α_s as

$$C_{F_y} = A_1 \frac{\dot{y}}{V_\infty} \tag{3.5}$$

The corresponding critical airspeed is

$$V_{crit} = \frac{2c}{\rho B A_1} \tag{3.6}$$

Validity of the quasi-steady theory

The quasi-steady theory is based on the assumption that the aerodynamic forces acting on an oscillating body are equal to the instantaneous force evaluated at each positions of the static body. In this case, the effect of the wake of the body, also known as memory effect is neglected. Different criteria have been proposed in order to guarantee the applicability of the theory [81]:

• Fung [156] stated that the time for a disturbance to convect downstream from the body T_f must be much smaller than the characteristic time of oscillation T_h . The convection time is defined as $T_f = D/V_{\infty}$, where D is the characteristic length of the vertical motion, while the oscillation time is given by $T_h = 1/f_h$, where f_h is the frequency of the heaving motion. Fung's criterion states that

$$\frac{T_h}{T_f} = \frac{V_\infty}{f_h D} > 10$$

From the experimental measurements, one finds $f_h = 4.65$ Hz and for an airspeed $V_{\infty} = 15$ m/s, with D = 0.1 m, the ratio T_h/T_f is equal to 30 and Fung's quasi-steady criterion is respected.

• Blevins [157] proposed a criterion based on the vortex shedding process: he states that the frequency of shed vortices (according to the Strouhal relation) must be at least twice the heaving frequency f_h . Hence the quasi-steady criterion is expressed as

$$\frac{f_s}{f_h} = \frac{\frac{StV_{\infty}}{D}}{f_h} = \frac{StV_{\infty}}{Df_h} > 2$$

An estimation of the Strouhal number of the generic bridge deck can be found in chapter 4: St = 0.15. Considering an airspeed of $V_{\infty} = 15$ m/s, one finds $f_s/f_h = 4.8$ and the criterion of Blevins is respected.

• Bearman et al. [158] proposed a more restrictive criterion, based on the study of a square cylinders:

$$\frac{V_{\infty}}{f_h D} > 30$$

Note that this criterion is respected, as shown in the discussion of Fung's criterion.

An additional requirement was pointed out by van Oudheusden [110]. He stated that it is necessary to be able to define a steady situation, which is aerodynamically equivalent to the unsteady situation'. This additional condition is respected in the case of galloping, as shown in figure D.2 in Appendix D.

According to the fulfillment of the criteria listed above, it is concluded that the quasisteady theory is applicable in the case of the vertical motion of the bridge section presented in this chapter. Figure 3.15 shows the variation of A_1 with the static angle of attack α_s . It is based on the static measurements presented in section 3.4 and the definition of A_1 in equation 3.3, where the calculation of the slope of the lift coefficient $\frac{\partial C_L}{\partial \alpha}\Big|_{\alpha_s}$ is calculated by forward difference. The value of A_1 is negative for α_s lower then 4°, hence no galloping oscillations should be encountered if no important perturbation is applied to the bridge deck. For α_s between 4° and 16° the value of A_1 is positive and rather constant with the angle of attack. For angles of attack in the range from 15° to 21°, the shape of A_1 shows a sharp peak and its value is equal to 5.32 for $\alpha_s = 19^\circ$.



Figure 3.15: A_1 coefficient of the generic bridge deck

According to table 3.1, the critical airspeed V_{crit} for $\alpha_s = 19^{\circ}$ is 8.7 m/s. This critical airspeed is reported by a circle in figure 3.16, showing the bifurcation diagram identified during dynamical experiments, where only the positive amplitudes of the vertical oscillations are shown. It is observed that the quasi-steady estimation of the critical airspeed of the galloping phenomenon is a conservative estimation of the critical velocity: no oscillations where measured during experiments for that airspeed.

It is now possible to explain why no galloping instability was observed during experiments for static angles of attack equal to 0° , 5° , 10° and 15° :

- At 0°, A₁ is negative and Den Hartog's criterion states that no galloping instability is possible.
- At 5° and 10°, A_1 is positive but small values are measured. According to equation 3.6, the critical airspeeds are equal to 31.9 m/s and 30.5 m/s for 5° and 10° respectively. These airspeeds are not tested experimentally for safety reasons.
- At 15°, A_1 is equal to zero, hence an infinite critical airspeed is foreseen by the quasi-steady equation 3.6.

As the maximum airspeed tested during experiments was 16.5 m/s the above mentioned critical airspeed were not reached and no galloping instability is observed for these configurations.



Figure 3.16: Bifurcation diagram of the bridge deck

It can be argued that the linearized quasi-steady theory gives a good conservative estimate of the onset velocity of the galloping phenomenon. This justifies the choice of Den Hartog's criterion in the majority of the construction norms taking into account the effects of the wind on structures (e.g. Eurocode [122]).

Nevertheless, this linear approach does not give any information about the post-critical behaviour of the bridge deck. For airspeeds higher than the critical airspeed, the total damping is negative and the response predicted by the underlying linear system is divergent (e.g. classical linear flutter).

However, LCOs with stable amplitudes are observed during experiments (section 3.5). These LCOs find their origin in the nonlinear effect of the aerodynamic loads, due to the complex phenomena of separation and re-attachment of the flow on the oscillating body. These nonlinear effects can be introduced in a quasi-steady model by using higher order polynomials. It is proposed here to increase the order of the polynomial representing the vertical aerodynamic force based on the quasi-steady coefficients.

Figure 3.17 plots the C_{F_y} coefficient defined in the static case by equation D.2 and recalled here for convenience:

$$C_{F_y} = -\left[C_L(\alpha') + C_D(\alpha')\tan\alpha\right]\sec\alpha \tag{3.7}$$

The horizontal axis in this figure is the static angle α_s in the range between -0.26 and 0.52 radian. When considering the oscillating body in the quasi-steady theory, the ratio $\alpha = \tan^{-1}(\dot{y}/V_{\infty})$ is defined as the relative angle of attack. The range of values reached by α when undergoing LCOs are measured during experiments: it varies between -0.1



Figure 3.17: C_{F_y} coefficient

rad and 0.1 rad around the static position $\alpha_s = 19^\circ = 0.33$ rad. Hence the interesting part of the curve of $C_{F_y}(\alpha)$ lies in the rectangle drawn in figure 3.17. For small α , the approximation $\alpha = tan^{-1}(\dot{y}/V_{\infty}) \approx \dot{y}/V_{\infty}$ holds.

Figure 3.18 shows a zoom on the rectangle of figure 3.17, zeroed on the static angle α_s and where $\alpha = \dot{y}/V_{\infty}$ varies between -0.1 and 0.1. Hence the notation $\alpha' = \alpha_s + \alpha$ of Appendix D becomes $\alpha' = \alpha$, i.e. $\alpha_s = 0$.

This figure expresses the variation of the vertical force due to the heaving oscillation around the static angle of attack, under the quasi-steady assumption. The dashed curve in figure 3.18 represents the coefficient C_{F_y} of the underlying linear system (equation 3.5). It is obvious from that figure that the linear model is valid for small values of the ratio \dot{y}/V_{∞} but it fails to reproduce the nonlinear shape of C_{F_y} for larger values.



Figure 3.18: C_{F_y} coefficient - zoom between $\alpha = [-0.1 \ 0.1]$

The vertical force coefficient using a third order polynomial in terms of α is:

$$C_{fy} = -\left[\left(\frac{\partial C_L}{\partial \alpha}\Big|_{\alpha_s} + C_D(\alpha_s)\right)\frac{\dot{y}}{V_{\infty}} + \left(\frac{1}{2}\frac{\partial^2 C_L}{\partial \alpha^2}\Big|_{\alpha_s} + \frac{1}{2}\frac{\partial C_L}{\partial \alpha}\Big|_{\alpha_s} + \frac{\partial C_D}{\partial \alpha}\Big|_{\alpha_s} + C_D(\alpha_s)\right)\left(\frac{\dot{y}}{V_{\infty}}\right)^2 + \frac{1}{2}\left(\frac{1}{3}\frac{\partial^3 C_L}{\partial \alpha^3}\Big|_{\alpha_s} + \frac{1}{2}\frac{\partial^2 C_L}{\partial \alpha^2}\Big|_{\alpha_s} + \frac{\partial^2 C_D}{\partial \alpha^2}\Big|_{\alpha_s} + \frac{\partial C_D}{\partial \alpha}\Big|_{\alpha_s}\right)\left(\frac{\dot{y}}{V_{\infty}}\right)^3\right]$$
(3.8)

Unfortunately, the higher order derivatives of the lift and drag curves cannot be obtained accurately from differentiating the measured data, i.e. directly from figure 3.9. Each differentiation increases the errors, leading to very poor estimates for the second and third order derivatives. One option is to use again polynomial curve fits for the lift and drag curves; these can be differentiated analytically. It is proposed to use a fifth order Taylor expansion of the nonlinear coefficients $C_L(\alpha)$ and $C_D(\alpha)$ (see Appendix F). The resulting polynomials are:

$$C_L(\alpha) = 0.417 - 6.49\alpha - 26.62\alpha^2 + 137.73\alpha^3 + 198.43\alpha^4 - 411.47\alpha^4$$

$$C_D(\alpha) = 0.49 - 1.11\alpha - 8.3\alpha^2 + 15.47\alpha^3 + 66.50\alpha^4 - 64.80\alpha^5$$

The constant coefficients appearing in the parenthesis of equation 3.8 are calculated using the above mentioned polynomials of the aerodynamic coefficients $C_L(\alpha)$ and $C_D(\alpha)$. They are presented in table 3.2 and equation 3.8 becomes

$$C_{fy} = 6.0 \frac{\dot{y}}{V_{\infty}} + 30.5 \left(\frac{\dot{y}}{V_{\infty}}\right)^2 - 115.6 \left(\frac{\dot{y}}{V_{\infty}}\right)^3$$
(3.9)

$$\frac{C_L|_{\alpha_s}}{\partial \alpha} \left|_{\alpha_s} \frac{\partial^2 C_L}{\partial \alpha^2} \right|_{\alpha_s} \frac{\partial^3 C_L}{\partial \alpha^3} \left|_{\alpha_s} C_D|_{\alpha_s} \frac{\partial C_D}{\partial \alpha} \right|_{\alpha_s} \frac{\partial^2 C_D}{\partial \alpha^2} \right|_{\alpha_s}}{0.417 - 6.49 - 53.24 826.4 0.490 - 1.11 - 16.60}$$

Table 3.2: Aerodynamic coefficients and their derivatives around $\alpha_s = 19^{\circ}$

It is observed that the first coefficient in equation 3.9 is equal to 6.0, while its value, based on the experimental measurements is equal to $A_1 = 5.32$ in the Den Hartog's criterion (equation 3.3). This is due to the fact that the lift and drag coefficients are approximated using a fifth order polynomial in table 3.2. This difference highlights the strong sensitivity of the estimation of the critical airspeed based on the quasi-steady approach.

Another important source of error when using expression 3.6 is the direct proportionality of the critical airspeed to the structural damping coefficient c. Indeed the value of the damping can be difficult to estimate for a civil engineering structure where different materials and operational conditions are involved [159,160]. Hence the error on the critical airspeed estimate is directly proportional to the error on the damping identification.



Figure 3.19: C_{F_y} coefficient - Third order fitting

The cubic fitting of the vertical force coefficient resulting from equation 3.8 is shown in figure 3.19. It is obvious that this polynomial, based on these coefficients is not capable to fit the measurement points. The benefit of using this polynomial is relatively low compared to the first order estimation (equation 3.5).

Time simulations are performed using a 5^{th} order Runge-Kutta scheme to solve the equation of motion 3.4 with the nonlinear force coefficient 3.8. The numerical results are presented in the bifurcation diagram in figure 3.20.

As expected from the negative sign of the coefficient of the third order term in the polynomial approximation of C_{F_y} (equation 3.9), the resulting bifurcation is super-critical: the equilibrium position is stable until 7.7 m/s. Then it becomes unstable and any perturbation from that position leads to LCOs whose amplitudes are denoted by white circles in the present figure. It is obvious that the predicted LCO amplitudes are significantly over-estimated.

The white circle on the horizontal axis corresponds to the first estimation of the critical velocity, $V_c = 8.7 \text{ m/s}$, using the experimental measurements to calculate A_1 (figure 3.15). The difference between the two critical airspeeds is due to the polynomial fitting of equation 3.9, as discussed above.



Figure 3.20: Bifurcation diagram - Third order polynomial

In conclusion, despite the simplicity of the present approach based on the static aerodynamic coefficient and the attempt to improve it by increasing the order of the model, it is concluded that the form of the polynomial 3.8 is not adapted to reproduce the correct amplitudes of the LCOs and the post-critical behaviour observed experimentally.

3.7 Empirical models

Up to this point, the classical quasi-steady approach, based on the static aerodynamic coefficients has been presented. Another approach is proposed here: higher order polynomials are used to model the vertical force but the identification of the coefficients is based on the experimental dynamic measurements (presented in section 3.5) and not on the static measurements.

Parkinson initially proposed a fifth order polynomial to model the instability mechanism of square cylinder free to oscillate vertically [90]. Later he extended the approximation up to the seventh order in order to allow the reproduction of the hysteretic phenomenon (see figure 3.14) [145]:

$$C_{Fy}^{p}(\dot{y}, V_{\infty}) = A_{1} \frac{\dot{y}}{V_{\infty}} + A_{3} \left(\frac{\dot{y}}{V_{\infty}}\right)^{3} + A_{5} \left(\frac{\dot{y}}{V_{\infty}}\right)^{5} + A_{7} \left(\frac{\dot{y}}{V_{\infty}}\right)^{7}$$
(3.10)

Novak proposed a modified version of the model of Parkinson, where he introduced a quadratic term in the power expansion of the nonlinear expression of the vertical force
[161]:

$$C_{Fy}^{n}(\dot{y}, V_{\infty}) = C_{Fy}^{p}(\dot{y}, V_{\infty}) + A_{2} \left(\frac{\dot{y}}{V_{\infty}}\right)^{2} \frac{\dot{y}}{|\dot{y}|}$$

$$= A_{1} \frac{\dot{y}}{V_{\infty}} + A_{2} \left(\frac{\dot{y}}{V_{\infty}}\right)^{2} \frac{\dot{y}}{|\dot{y}|} + A_{3} \left(\frac{\dot{y}}{V_{\infty}}\right)^{3} + A_{5} \left(\frac{\dot{y}}{V_{\infty}}\right)^{5} + A_{7} \left(\frac{\dot{y}}{V_{\infty}}\right)^{7}$$
(3.11)

It is proposed to decrease the order of the polynomial and to retain all the odd and even exponents, by using the artifice of the absolute value $\frac{\dot{y}}{|\dot{y}|}$, similarly to the force coefficient proposed by Novak. Hence $C^a_{Fy}(\dot{y}, V_{\infty})$ is modelled by a 5th order polynomial of the form:

$$C_{Fy}^{a}(\dot{y}, V_{\infty}) = A_{1} \frac{\dot{y}}{V_{\infty}} + A_{2} \frac{\dot{y}|\dot{y}|}{V_{\infty}^{2}} + A_{3} \left(\frac{\dot{y}}{V_{\infty}}\right)^{3} + A_{4} \frac{\dot{y}^{3}|\dot{y}|}{V_{\infty}^{4}} + A_{5} \left(\frac{\dot{y}}{V_{\infty}}\right)^{5}$$
(3.12)

Unlike the previous third order polynomial, based on the aerodynamic coefficients and its derivatives, the A_n coefficients appearing in equations 3.10 to 3.12 are identified using the amplitude of the LCOs observed experimentally (figure 3.10).

3.7.1 Stability analysis

A preliminary stability analysis is carried out in order to justify the choice of the polynomial function (equation 3.12). It is shown here that the three bifurcations observed experimentally can be modelled: a subcritical Hopf bifurcation followed by two fold bifurcations (at 10.2 m/s and 14.2 m/s). Remember that the Hopf airspeed has not been reached during the wind tunnel experiments for safety reasons. Nevertheless, it will be shown that the proposed model allows a good estimation of the aeroelastic behaviour of the bridge section.

Using expression 3.12, the linear equation of motion 3.4 becomes nonlinear and the aeroelastic system is modelled by

$$m\ddot{y} + \left[(c - \frac{qA_1}{V_{\infty}})\dot{y} - \frac{qA_2}{V_{\infty}^2}\dot{y}|\dot{y}| - \frac{qA_3}{V_{\infty}^3}\dot{y}^3 - \frac{qA_4}{V_{\infty}^4}\dot{y}^3|\dot{y}| - \frac{qA_5}{V_{\infty}^5}\dot{y}^5 \right] + ky = 0$$
(3.13)

where q denotes $1/2\rho V_{\infty}^2 B$.

By differentiating equation 3.13 with respect to time, it can be written in the form of

the Lienar equation:

$$\begin{aligned} \ddot{y} + \frac{1}{m} \left[(c - \frac{qA_1}{V_{\infty}})\ddot{y} - \frac{qA_2}{V_{\infty}^2} (\ddot{y}|\dot{y}| + \dot{y}\text{sign}(\dot{y})\ddot{y}) - \frac{3qA_3}{V_{\infty}^3} \dot{y}^2 \ddot{y} \\ - \frac{qA_4}{V_{\infty}^4} \left(3\dot{y}^2 \ddot{y}|\dot{y}| + \dot{y}^3 \text{sign}(\dot{y})\ddot{y} \right) - \frac{5qA_5}{V_{\infty}^5} \dot{y}^4 \ddot{y} \right] + \frac{k}{m} \dot{y} = 0 \end{aligned}$$

Let $x = \dot{y}$, then

$$\ddot{x} + \frac{1}{m} \left[(c - \frac{qA_1}{V_{\infty}}) - \frac{qA_2}{V_{\infty}^2} (|x| + x \operatorname{sign}(x)) - \frac{3qA_3}{V_{\infty}^3} x^2 - \frac{qA_4}{V_{\infty}^4} \left(3x^2 |x| + x^3 \operatorname{sign}(x) \right) - \frac{5qA_5}{V_{\infty}^5} x^4 \right] \dot{x} + \frac{k}{m} x = 0$$
(3.14)

which has the Lienard form: $\ddot{x} + f(x)\dot{x} + \frac{k}{m}x = 0$. Defining $z = \dot{x} + F(x)$, where

$$F(x) = \int_0^x f(s)ds = \frac{1}{m} \left[(c - \frac{qA_1}{V_\infty})x - \frac{qA_2}{V_\infty^2}x|x| - \frac{qA_3}{V_\infty^3}x^3 - \frac{qA_4}{V_\infty^4}x^3|x| - \frac{qA_5}{V_\infty^5}x^5 \right]$$

Yields,

$$\begin{cases} \dot{x} = z - F(x) \\ \dot{z} = \ddot{x} + \dot{F}(x)\dot{x} = \ddot{x} + f(x)\dot{x} = -\frac{k}{m}x \end{cases}$$

i.e.

$$\begin{cases} \dot{x} = z - \frac{1}{m} \left[(c - \frac{qA_1}{V_{\infty}})x - \frac{qA_2}{V_{\infty}^2} x |x| - \frac{qA_3}{V_{\infty}^3} x^3 - \frac{qA_4}{V_{\infty}^4} x^3 |x| - \frac{qA_5}{V_{\infty}^5} x^5 \right] \\ \dot{z} = -\frac{k}{m} x \end{cases}$$

Now define a radius R as

$$R = \frac{1}{2}\left(\frac{k}{m}x^2 + z^2\right)$$

Differentiate with respect to time:

$$\dot{R} = \frac{k}{m}x\dot{x} + z\dot{z} = \frac{k}{m}x(z - F(x)) - z\frac{k}{m}x = -\frac{k}{m}xF(x)$$
(3.15)

The stability of the system can be established on the basis of equation 3.15:

- $\dot{R} < 0$ denotes a stable trajectory
- $\dot{R} > 0$ denotes an unstable trajectory

The stability of the system depends on the number of zeros and the sign of \dot{R} , expressed as a function of x, which has the dimensions of velocity $(x = \dot{y})$. A general stability assessment is performed on the basis of the nonlinear function:

$$\dot{R}(x) = 0 \to -x^2 \left[(c - \frac{qA_1}{V_{\infty}}) - \frac{qA_2}{V_{\infty}^2} |x| - \frac{qA_3}{V_{\infty}^3} x^2 - \frac{qA_4}{V_{\infty}^4} x^2 |x| - \frac{qA_5}{V_{\infty}^5} x^4 \right] = 0 \qquad (3.16)$$

The form of the polynomial depends on the signs of the A_n coefficients and on the airspeed V_{∞} . The polynomial is even and possesses two double roots at the equilibrium position, i.e. x = 0.

Figures 3.21 to 3.24 show the stability criterion R as a function of x. White areas correspond to stable regions while grey areas are unstable. For increasing airspeeds, these stable and unstable regions alternate, following the Pointcaré-Bendixson theorem.

Figure 3.21 corresponds to an airspeed of 8 m/s, where no LCO is observed: the value of the criterion is negative in the entire range of values of x. The only attractor is a stable focal point at the equilibrium position (fixed point). All response trajectories wind down towards the fixed point. The type of time response is shown in figure 3.11.



Figure 3.21: Stability diagram at 8 m/s

As mentioned in section 3.5, the dynamics of the system changes radically at airspeeds higher than 10 m/s: the system has to be excited in order to undergo galloping oscillations, otherwise it will remain stable. Such behaviour can be encountered in systems that feature concentric stable and unstable limit cycles [162].

Figure 3.22 shows the stability criterion for an airspeed equal to 12 m/s. Through the occurrence of a fold bifurcation, two limit cycles start to grow around the stable fixed point (x = 0): one unstable (grey area) and one stable (outer white area). At this airspeed, the trajectory x = 0 is stable: any trajectories starting inside the white region will wind down onto the fixed point. The time response corresponding to this trajectory is similar to the one shown in figure 3.11. Any trajectories originating inside the unstable limit cycle (grey area) will wind towards the fixed point or the stable limit cycle. Notice that the exact value of the origin of this trajectory is not available from this figure, which is only a qualitative investigation of the stability of the system with the nonlinear moment coefficient $C_{F_y}^a$ (equation 3.12).

Figure 3.23 corresponds to an airspeed of 14 m/s and represents the most complex



Figure 3.22: Stability diagram at 12 m/s

nonlinear behaviour of the system. Through a second fold bifurcation, a third LCO occurs in the dynamics of the system. Between -0.4 and 0.4 the system behaves in a manner similar to the 12 m/s case. Any trajectories originating inside an unstable limit cycle (grey area) will wind toward one of the surrounding stable regions. Hence, three possibilities arise: decay to the fixed point or tend to one of the stable LCOs.



Figure 3.23: Stability diagram at 14 m/s

When the airspeed reaches 16 m/s, the system's dynamics becomes identical to those at 12 m/s. The stability criterion is shown in figure 3.24. The fixed point at the equilibrium position is surrounded by a large unstable region. For perturbations inside this large unstable region, the system will either undergo large LCOs or decay to the equilibrium position.



Figure 3.24: Stability diagram at 16 m/s

If the airspeed is increased beyond the Hopf airspeed, the equilibrium position becomes unstable: the Hopf bifurcation airspeed is exceeded. Any perturbation will result in a sudden jump to a large amplitude LCO, as shown in figure 3.25.



Figure 3.25: Stability diagram above the Hopf airspeed

3.7.2 Identification of the A_n coefficients

As stated before, the A_n coefficients appearing in the force coefficient $C^a_{F_y}(\dot{y}, V_\infty)$ are identified using the measurements of the dynamic responses of the bridge deck. The equation of motion 3.13 may be written

$$m\ddot{y} + f(\dot{y}) + ky = 0 \tag{3.17}$$

where the term inside the brackets in equation 3.13, denoted by $f(\dot{y})$, is a nonlinear function that models the structural and aerodynamic damping of the system:

$$f(\dot{y}) = (c - \frac{qA_1}{V_{\infty}})\dot{y} - \frac{qA_2}{V_{\infty}^2}\dot{y}|\dot{y}| - \frac{qA_3}{V_{\infty}^3}\dot{y}^3 - \frac{qA_4}{V_{\infty}^4}\dot{y}^3|\dot{y}| - \frac{qA_5}{V_{\infty}^5}\dot{y}^5$$

It is proposed to linearize the expression for $f(\dot{y})$ using a first order Harmonic Balance. This corresponds to the approach presented by Parkinson [145] and Vio et al. [154]. In the latter work, the objective was the comparison of different numerical methods for predicting the bifurcation behaviour using the galloping model of Parkinson [145]. The linearized expression of $f(\dot{y})$ is given by

$$f(\dot{y}) = \frac{a_0}{2} + a_1 \cos(\omega t) + b_1 \sin(\omega t)$$

where a_0 , a_1 and b_1 coefficients are given by

$$a_0 = \frac{2}{T} \int_0^T f(\dot{y}) dt, \quad a_1 = \frac{2}{T} \int_0^T f(\dot{y}) \cos(\omega t) dt, \quad b_1 = \frac{2}{T} \int_0^T f(\dot{y}) \sin(\omega t) dt$$

The Harmonic Balance assumes that the system undergoes LCOs, hence the vertical position and velocity are respectively: $y(t) = Y \sin(\omega t)$ and $\dot{y}(t) = Y\omega \cos(\omega t)$, where ω is the radial frequency and Y is the amplitude of the LCO (positive values). The values of ω and Y have been measured experimentally in a range of airspeeds between 6 m/s and 18 m/s. It was verified that ω is constant over the entire range of airspeeds, while

the amplitude Y varies.

The Fourier coefficients a_0 , a_1 and b_1 become

$$a_0 = \frac{2}{T} \int_0^T f(Y\omega\cos(\omega t))dt$$
$$a_1 = \frac{2}{T} \int_0^T f(Y\omega\cos(\omega t))\cos(\omega t)dt$$
$$b_1 = \frac{2}{T} \int_0^T f(Y\omega\cos(\omega t))\sin(\omega t)dt$$

Noting that the expression of $f(\dot{y})$ is odd and that there is no offset, one obtains $a_0 = b_1 = 0$. After integration, a_1 is given by

$$a_1 = (c - \frac{qA_1}{V_{\infty}})\omega Y - \frac{8q\omega^2 A_2}{3\pi V_{\infty}^2}Y^2 - \frac{3q\omega^3 A_3}{4V_{\infty}^3}Y^3 - \frac{32q\omega^4 A_4}{15\pi V_{\infty}^4}Y^4 - \frac{5q\omega^5 A_5}{8V_{\infty}^5}Y^5$$

The first order Fourier expansion of the nonlinear function f(y) is then

$$\begin{split} f(\dot{y}) &\approx a_1 \cos(\omega t) \\ &\approx \left[c - \frac{qA_1}{V_\infty} - \frac{8q\omega A_2}{3\pi V_\infty^2} Y - \frac{3q\omega^2 A_3}{4V_\infty^3} Y^2 \right. \\ &\left. - \frac{32q\omega^3 A_4}{15\pi V_\infty^4} Y^3 - \frac{5q\omega^4 A_5}{8V_\infty^5} Y^4 \right] \underbrace{Y\omega \cos(\omega t)}_{\dot{y}(t)} \end{split}$$

This expression is replaced in the equation of motion 3.17, leading to

$$m\ddot{y} + C_{eq}\dot{y} + ky = 0 \tag{3.18}$$

where C_{eq} denotes the equivalent damping of the linear system:

$$C_{eq} = c - \frac{qA_1}{V_{\infty}} - \frac{8q\omega A_2}{3\pi V_{\infty}^2}Y - \frac{3q\omega^2 A_3}{4V_{\infty}^3}Y^2 - \frac{32q\omega^3 A_4}{15\pi V_{\infty}^4}Y^3 - \frac{5q\omega^4 A_5}{8V_{\infty}^5}Y^4$$
(3.19)

At different airspeeds V_{∞} and for different initial conditions, the system will undergo LCOs, which corresponds mathematically to the cancellation of the equivalent damping C_{eq} .

$$c - \frac{qA_1}{V_{\infty}} - \frac{8q\omega A_2}{3\pi V_{\infty}^2}Y - \frac{3q\omega^2 A_3}{4V_{\infty}^3}Y^2 - \frac{32q\omega^3 A_4}{15\pi V_{\infty}^4}Y^3 - \frac{5q\omega^4 A_5}{8V_{\infty}^5}Y^4 = 0$$
(3.20)

Experimental couples $(Y^{(k)}, V_{\infty}^{(k)})$, with $k = 1 \dots m$, are selected on the bifurcation diagram (circled dots in figure 3.26). They are selected on the two different types of motion leading to LCOs: intermittent low amplitude LCO and stable LCO. In addition

a threshold is applied on the selected experimental points: only couples characterized by an amplitude higher than 2 mm are selected in the identification of the A_n coefficients.

The coefficients are identified by fitting the 4th order polynomial (equation 3.20) to the experimental data using a least square method.



Figure 3.26: Experimental bifurcation plot

A similar method is used for the identification of the A_n coefficients appearing in the models of Parkinson and Novak. Equation 3.20 is adapted to the expressions of C_{Fy}^p and C_{Fy}^n (see expressions 3.10 and 3.11). The resulting A_n coefficients are shown in table 3.3 for the three models.

Model	A_1	A_2	A_3	A_4	A_5	A_7
5^{th} order	2.15	348.15	$-1.843e^4$	$3.468e^{5}$	$-2.191e^{6}$	-
Parkinson	3.58	-	586.09	-	$-5.659e^{5}$	$9.555e^{7}$
Novak	2.54	220.32	-7662.1	-	$1.901e^{6}$	$-2.034e^{8}$

Table 3.3: A_n coefficients of the polynomial models

3.7.3 Comparison between models

The different models of galloping presented in the previous sections are compared in this section. First, time simulations are carried out, resulting in a bifurcation diagram of the system for each model. Then a discussion about the form of the force coefficient C_{Fy} is presented.

Time simulations

Time simulations are performed using a 5^{th} order Runge-Kutta scheme. The maximum vertical displacements are computed for each airspeed between 6 m/s and 30 m/s and for different initial conditions. Figure 3.27 shows the bifurcation diagram corresponding to the experimental data, the third order polynomial based on the static aerodynamic measurements and the three higher orders models presented in the previous section.



Figure 3.27: Bifurcation: comparison between models and experiments

The following observations can be addressed:

- Parkinson's model is capable of reproducing the subcritical bifurcation but the quality of the model is poor: the intermittent LCO branch is nearly vertical, while experimental points are distributed horizontally.
- The fold airspeed of Parkinson's model is equal to 12.6 m/s, i.e. much higher than the experimental fold airspeed (10.2 m/s).
- Parkinson's model is not capable of capturing the second fold. On the contrary, the branch decreases strongly towards lower airspeeds.
- The third order polynomial based on the static aerodynamic measurements shows a supercritical behaviour, as stated in section 3.6. This model is the most conservative. The supercritical behaviour is hard, in the sense that the slope of the branch is high:

for an airspeed above the Hopf airspeed (7.7 m/s), the equilibrium position becomes unstable and, if excited, the system suddenly jumps from the equilibrium position to large LCO amplitudes.

- The 5th order model and Novak's model lead to good predictions of the intermittent and stable LCOs. Nevertheless, it must be pointed out that both models miss the experimental points between 10 m/s and 11 m/s.
- The Hopf airspeed, not identified experimentally for safety reasons, is equal to 21.7 m/s and 18.4 m/s for the 5th order and Novak models respectively.
- The two fold airspeeds, corresponding to each model (5th order and Novak), are nearly identical. The first fold takes place around 11.2 m/s, and the second around 14.3 m/s. These values are in agreement with the fold airspeeds observed experimentally: 10.2 m/s and 14.2 m/s.

The analysis of figure 3.27 shows that two polynomials (5^{th} order and Novak) reproduce the complete bifurcation behaviour observed experimentally. The discussion concerning the form of these polynomials is shown below.

Force coefficients C_{Fy}

Novak proposed an interesting analysis of the evolution of C_{Fy} with the non dimensional ratio $\alpha = \tan^{-1} \dot{y}/V_{\infty}$ in the case of prismatic structures [161]. He defined three types of response of the system regarding the value and the sign of A_1 and the resulting shape of the coefficient $C_{Fy}(\alpha)$. Figure 3.28 shows a reproduction of the work of Novak, proposed by Paidoussis [81]. It is preferred to the figure initially presented by Novak for its completeness. The left plots show the force coefficient as a function of α and the right plots are the corresponding response's amplitudes vs airspeed. A brief summary of the classification is presented below, then the response of the bridge is integrated into it.



Figure 3.28: Basic types of lateral force coefficients and corresponding galloping response (reproduced from the work of Novak in [81])

The first type of galloping response is characterized by $A_1 > 0$ and it corresponds to plots (a), (b) and (c) in figure 3.28. Among these curves, the difference appears in the

number of inflection points, according to the values and signs of the coefficients A_i with i > 2. Figure 3.28(a) corresponds to the third order model, based on the aerodynamic coefficients and its derivative (equation 3.8). Figures 3.28(b) and (c) show an hysteretic loop in the bifurcation plot. The former response corresponds to $A_2 > 0$ while the latter to $A_2 < 0$. It is observed from the right hand side plots that the response of the bridge deck is a combination of two response types: plots (b) and (c) of figure 3.28.

The second type of galloping response corresponds to the case where $A_1 < 0$, as shown in figure 3.28(d). This type of response is associated with aerodynamic shapes without after body, i.e. the ratio B/D < 1 as sketched in the right plot of figure 3.28(d). The flow, separated at the leading edge does not re-attach on the body, because of its bluff shape. In this case, Den Hartog's criterion fails to predict an unstable behaviour, but if the airspeed is higher than U_0 (see figure), a sufficient initial perturbation from the equilibrium position lead to stable LCOs.

Finally, the third case, defined by $A_1 = 0$ (not shown in figure 3.28), is a subclass of the second case described above and the remark about Den hartog's criterion holds. In this case, the slope of the force coefficient is zero for $\alpha = 0$.

Through the present classification, Novak demonstrated that Den Hartog's criterion is a condition sufficient but not necessary.



Figure 3.29: Force coefficient - C_{Fy}

The force coefficients C_{Fy} , corresponding to each models identified from the bridge's

motion are shown in figure 3.29. The first observation is the asymmetric shape of the static curve, compared to the data-driven model (5th order, Parkinson and Novak). This is due to the asymmetric geometry of the bridge deck section, contrary to the square and rectangular cylinders studied by Parkinson and Novak.

The shape of C_{Fy}^p , corresponding the Parkinson's model, is rather different from the other polynomial models. A steep increase is observed around ± 0.08 rad and the concavity of the curve is positive. This feature of C_{Fy}^p is responsible for the wrong subcritical behaviour observed in figure 3.27 for this model.

The force coefficients $C_{F_y}^a$ and $C_{F_y}^n$ are shown in figure 3.30 for the positive values of α , where three inflection points are shown as black dots. Luo et al. have demonstrated through flow visualization that the presence of inflection points in the curve of $C_{Fy}(\alpha)$ is due to the reattachment of the flow on the sides of the body [146]. Barrero et al. [163] studied the effect of the number of inflection points on the shape of the bifurcation diagram. They established that, as far as the qualitative behaviour of the response is concerned, the number of limit cycles is independent of the number of terms in the polynomial model of the force coefficient $C_{Fy}(\alpha)$. On the contrary, he showed that the behaviour is dependent on the reproduction of all the inflection points in the curve of C_{Fy} . This statement is confirmed in the case of the bridge deck section, when comparing the polynomials C_{Fy}^a and C_{Fy}^n (equations 3.12 and 3.11): despite a difference in the orders of these polynomials, the inflection points are reproduced and the resulting behaviours are equivalent.



Figure 3.30: Force coefficients - C_{Fy} - 5th order and Novak's models

From the observations of figures 3.28 and 3.30, it can be argued that the curve of $C_{F_y}^a(\alpha)$ has the characteristics of two sub-classes of the first type of response, amongst the three discussed by Novak:

- The curvature is positive at the origin, which is due to the positive value of A_1 and A_2 . It leads to the subcritical behaviour observed experimentally and corresponds to the plot (c) in figure 3.28.
- For larger values of α (> 0.03 in figure 3.30), the concavity of the force coefficient changes. This change of curvature occurs before the maximal value of C_{F_y} , similarly to figure 3.28(b).

It is interesting to note the relation between the form of C_{Fy} in figure 3.30 and the stability criterion discussed in figure 3.24. Indeed, the shape of the stability criterion, \dot{R} , for positive values of x, is similar to the shape of C_{Fy} . On the other hand, the shape of the C_{Fy} from the measurement of the aerodynamic coefficients (equation 3.7) is similar to figure 3.21 below the Hopf airspeed and figure 3.25 above.

In accordance with the observations addressed about the time simulations, it is argued that the 5th order polynomial is equivalent to the polynomial proposed by Novak. Hence, it is demonstrated that a fifth order polynomial is sufficient to model correctly the galloping behaviour of the present generic bridge section. In other words, by introducing a 4th order term, there is no need for Novak's 7th order term, which can lead to numerical problems.

3.8 Universal curves

This last section deals with the possibility of obtaining a universal galloping curve based on the non-dimensional form of the equation of motion, using a polynomial expression for $C_{F_y}^a$. Then it is made possible to compare the universal curves of the generic bridge deck with others geometries.

Adopting a suitable change of variables, the equation of motion 3.4 can be written as

$$\xi'' + 2\beta\xi' + \xi = 2nUC_{F\xi} \tag{3.21}$$

with the non-dimensional variables: $\xi = y/B$, $U = V_{\infty}/(\omega B)$, $\beta = \frac{c}{2m\omega}$, $n = \frac{\rho B^2}{4m}$ with $\omega = \sqrt{k/m}$. The prime symbol denotes differentiation with respect to the nondimensional time variable $\tau = \omega t$, t being the time expressed in seconds. $C_{F\xi}$ stands for the aerodynamic coefficient C_{Fy} , expressed as a polynomial in terms of ξ' .

Novak [164] introduced the concept of universal galloping curve through the two non-

dimensional variables $\bar{\xi}$ and \bar{U} :

$$\bar{\xi} = \xi \frac{n}{\beta} \qquad \qquad \bar{U} = U \frac{n}{\beta}$$

Using these variables, equation 3.21 becomes

$$\bar{\xi}'' + 2\beta\bar{\xi}' + \xi = 2\beta\bar{U}C_{F\xi}(\bar{\xi}') \tag{3.22}$$

The critical modified reduced velocity \overline{U}_{crit} corresponding to the first order approximation of $C_{F\xi}$ is

$$\bar{U}_{crit} = \frac{1}{A_1} \tag{3.23}$$

Expression 3.23 represents the critical modified reduced velocity of a body whose shape corresponds to the aerodynamic static coefficient A_1 . The information about the dynamics (damping β , mass m, ...) is contained in a single variable: \bar{U} .

Novak presented the post-critical behaviour of a square cylinder on a bifurcation diagram in the plane $(\bar{U}, \bar{\xi})$ and showed that this universal response curve can be defined for any aerodynamic shape, from the knowledge of either the static lift and drag coefficients or by measuring directly the galloping oscillations of the body.

It is interesting to compare the universal curve identified for the present generic bridge section to of that of other simple bluff-body shapes. This comparison is presented in figure 3.31, which shows the universal galloping curve of the generic bridge section studied in this chapter together with the responses of a square prism and a 2:1 rectangular prism (results from Novak [164]).

On the basis of figure 3.31, it can be stated that for the same geometrical and dynamical properties of the structure (B, m, ω, β) , the generic bridge section at 19° is more likely to be unstable than the square cylinder and the 2:1 rectangular prism. Indeed, the (non-dimensional) Hopf airspeed is lower than the corresponding airspeeds of the other two bluff sections. However, the post-critical behaviour of the bridge is less severe than those of the square and the rectangle, i.e. the amplitude of the resulting LCOs are smaller for the bridge section.

3.9 Chapter summary

This chapter describes a complete analysis of the galloping phenomenon of a generic bridge deck. The static equilibrium angle of attack of the bridge is set to 19° in order to lead to galloping oscillations in the range of airspeeds tested during wind tunnel experiments. Both static and dynamic measurements are carried out. The latter concentrating on



Figure 3.31: Universal galloping curves

determining the full bifurcation behaviour of the system. This behaviour consists of a fold bifurcation at an airspeed of around 10 m/s, leading to either LCO or steady responses at higher airspeeds, depending on whether an external excitation is applied or not. A second fold appears to occur at an airspeed of 14.2 m/s.

The quasi-steady theoretical model is presented and adapted to the analysis of a structure with non zero angle of attack ($\alpha_s = 19^\circ$). It is verified that the criterion of applicability of the quasi-steady approach is respected. The predicted critical airspeed is equal to 8.7 m/s, while intermittent LCOs are observed above 10 m/s. Furthermore, when excited from its equilibrium position, the bridge section can undergo stable LCOs above this airspeed. It is concluded that the quasi-steady approach leads to a conservative estimation of the galloping airspeed.

The first order quasi-steady approach is extended up to the third order to obtain a model capable of reproducing the post-critical behaviour of the bridge section. This third order model is based on the measurement of the static aerodynamic coefficients. Nevertheless, the comparison with experimental measurements shows that this model leads to significant overestimation of the LCO amplitudes.

A modification of the model in order to include additional odd nonlinear terms is

suggested. The proposed model consists in a 5^{th} order polynomial of the form

$$C^{a}_{Fy}(\dot{y}, V_{\infty}) = A_{1}\frac{\dot{y}}{V_{\infty}} + A_{2}\frac{\dot{y}|\dot{y}|}{V_{\infty}^{2}} + A_{3}\left(\frac{\dot{y}}{V_{\infty}}\right)^{3} + A_{4}\frac{\dot{y}^{3}|\dot{y}|}{V_{\infty}^{4}} + A_{5}\left(\frac{\dot{y}}{V_{\infty}}\right)^{5}$$

The predictions of this model are compared to those obtained from two other wellknown models proposed by Parkinson and Novak for square and rectangular cylinders. The fitting of these nonlinear models is based on a Harmonic Balance method, using the amplitudes of the LCOs measured during experiments. It is shown that the galloping behaviour closely resembles that of the wind tunnel model for the proposed model and Novak's model. Parkinson's model fails to predict the fold bifurcations observed during experiments.

The shape of the static force coefficient $C_{Fy}(\alpha)$ is not sufficient to capture the inflection points that lead to the nonlinear behaviour observed experimentally. The universal galloping curve of the generic bridge deck at $\alpha_s = 19^\circ$ is presented and compared to the curves of a square cylinder and a 2:1 rectangular cylinder. It is shown that the bridge section is more likely to undergo galloping, but the resulting LCOs have smaller amplitudes.

Chapter 4

Torsional flutter

4.1 Introduction

This chapter deals with the torsional flutter phenomenon, which is an aeroelastic instability leading to Limit Cycle Oscillations of a structure along its pitching degree of freedom. As presented in chapter 1, torsional flutter has been held responsible for the destruction of the Tacoma Narrows bridge in 1940 [77, 105, 106]. The rotating aspect of torsional flutter is the cause of the complexity of the phenomenon, which is driven by unsteady aerodynamics, leading to nonlinear forces and moment. It is worth mentioning that early analyses of the Tacoma disaster wrongly concluded that it was caused by VIV [107] (studied in chapter 2) because the consideration of torsional flutter was limited to aeronautical applications until then.

Two different structures undergoing torsional flutter oscillations are investigated in this chapter: the generic bridge section and a 4:1 rectangular cylinder. In each case the complete aeroelastic behaviour is studied and the characteristics of the torsional flutter phenomenon are analyzed. In the case of the rectangular cylinder, Time-resolved Particle Image Velocimetry (Tr-PIV) measurements are carried-out on the upper surface in order to explain the cause of the phenomenon. After presenting the experimental investigations of both structures, two different methods of prediction of the critical airspeed are presented. The first one is based on a quasi-steady model and the second one deals with phenomenological observations of the large vortical structures observed around the structure.

4.2 Torsional flutter of a bridge deck

This section presents the torsional flutter instability of the generic bridge deck section. The wind tunnel experiments are performed in the aeronautical test section, using the suspension system described in chapter 3.

4.2.1 Experimental set-up

In these experiments, the wind-shields are present on the generic bridge deck, but the acoustic panel is removed. The bridge deck is free to oscillate around its pitch degree of freedom. The pitch restoring force is provided by two extension springs and a lever arm clamped to the pitch axis of the deck. The pitch axis is supported on both extremities by ball bearings. The experimental set-up is shown in figure 4.1. The length of the lever arm is equal to 0.39m.



Figure 4.1: Experimental set-up: torsional set-up in the test section

Aerodynamic forces and moment on the structure are measured using traction/compression sensors and a torque sensor (see figure 3.6). Simultaneous measurements of forces and bridge's motion can be performed. The main drawback of the apparatus is the need to use pneumatic bushes to allow stiction-free force measurement. These bushes induce a

certain amount of flexibility in the structure, resulting in a small degree of freedom in heave. This low amplitude motion is measured and amplitudes below 2 mm are reported. Hence, this heaving motion can be neglected compared to the main dimensions of the bridge section and the pitching amplitudes observed in the section 4.2.4.

Force and moment response signals are recorded for periods of 10 seconds. Motion measurement is carried out using PCB accelerometers with an operating frequency range from 0 to 300 Hz and an acceleration range from -50 to 50 g. Airflow measurements are performed using a one-component hot-wire probe. For all measurements, the acquisition rate is set to 1 kHz.

4.2.2 Wind-off dynamic tests

The equation of motion of the bridge section around its pitching axis can be written as

$$I_{\alpha}\ddot{\alpha} + c_{\alpha}\dot{\alpha} + k_{\alpha}\alpha = M_{ext} \tag{4.1}$$

where I_{α} , c_{α} and k_{α} denote the polar moment of inertia, the damping coefficient and the pitching stiffness respectively. The term M_{ext} on the right-hand side of the equation of motion can be any external moment per unit length applied to the system. The parameters I_{α} , c_{α} and k_{α} , defined per unit length, are identified through vibration tests at zero airspeed. The structure is excited by different initial pitching conditions and the resulting decaying responses are measured by accelerometers. The values of the identified parameters are presented in table 4.1 below.

Table 4.1: Structural parameters of the torsion motion of the bridge

where f_0 and ξ_0 denote the frequency and the damping ratio of the pitching dof respectively.

4.2.3 Wind-on static tests

The aerodynamic coefficients of lift, drag and pitching moment acting on the bridge under static conditions are measured for angles of attack varying between -14° and $+30^{\circ}$. The airflow velocity was set to 20m/s for each angle of attack. The lift and drag coefficients

have already been defined in equations 3.2. The moment coefficient is defined as

$$C_M = \frac{M}{1/2\rho V_{\infty}^2 B^2}$$
(4.2)

where M denotes the aerodynamic moment per unit length, measured in [N.m/m] and defined positive nose up. V_{∞} , ρ and B denote the free-stream velocity, the density of the fluid (air) and the chord of the model respectively.

Figure 4.2 presents the variation of C_L , C_D and C_M with the angle of attack. The slope of the curve of C_L is positive up to an angle of attack of 4°, while from 4° to 24° it is negative. There is an abrupt fall in the value of C_L between 17° and 22°. From 24° to 30°, the lift coefficient increases slightly.



Figure 4.2: Aerodynamic coefficients of the generic bridge deck

The drag coefficient C_D has a maximum value of 0.58 for an angle of attack of 14°. The curve of C_D is relatively flat; to a first approximation it could be said that C_D remains unaffected by the angle of attack. The pitch coefficient C_M increases from -14° to 0° but remains approximately constant from 0° to 30°.

Strouhal number

The Strouhal number is measured at several angles of attack of the deck under static conditions. The deck is fixed in the flow-field and a hotwire probe is placed one chord downstream of the trailing edge. The probe measures the variation of the streamwise component of the velocity. The measured signal is further analyzed in the frequency domain to identify periodic phenomena in the wake of the static deck. The Strouhal number is calculated according to its definition (equation 1.3).

$$St = \frac{f_s D}{V_{\infty}} \tag{4.3}$$

where f_s denotes the vortex shedding frequency and D is the characteristic cross-flow dimension equal to 0.1 m for the model with wind-screens (cfr figure 3.3).



Figure 4.3: Strouhal number of the generic bridge deck vs. angle of attack

Figure 4.3 presents the variation of the Strouhal number for angles of attack varying between -10° and $+10^{\circ}$. The Strouhal number decreases from around 0.16 to around 0.12 in this range, a decrease of nearly 25%. The modification of the model's geometry has a significant impact on the behaviour of the flow around the deck, which in turn affects the vortex shedding process.

4.2.4 Wind-on dynamic tests

Extensive tests are carried out in the initial configuration of the stiffness set-up, characterized by the parameters of table 4.1. The bridge's motion and the aerodynamic forces acting on the deck are measured simultaneously. Additional experiments are performed on the generic bridge section using a second arm with a different length. This allows the investigation of the effect of the modal parameters through the comparison of the responses of the bridge for the two configurations.

Equilibrium deflection

Because static deflections cannot be measured using accelerometers, the equilibrium position of the bridge section, α_s , is lost when measuring pitching oscillations. Nevertheless, it is possible to estimate this angle by equating the restoring moment of the springs to the static aerodynamic moment, from the knowledge of the aerodynamic moment coefficient (equation 4.2):

$$\alpha_s = \frac{1}{2k_\alpha} \rho V_\infty^2 B^2 C_M(\alpha_s) \tag{4.4}$$

The experimental measurements of the pitch angle presented in this section take into account this static deflection, i.e. $\alpha(t) = \alpha_s + \iint \ddot{\alpha}_m(\tau) d\tau$, where $\ddot{\alpha}_m(\tau)$ denotes the measured accelerations.

Experimental procedure

In the previous chapter dealing with the galloping instability of the generic bridge section, two types of excitation were identified: turbulence excitation and external excitation (see section3.5). In the present section, the experimental procedure is limited to turbulence excitation tests, as defined previously, i.e. the bridge is free to oscillate around its equilibrium position and the only external excitation is the turbulence of the oncoming flow-field.

The airspeed is initially increased from 0 m/s to 19.1 m/s, then reduced to 6 m/s. Then it is increased again from 6 m/s to 19.1 m/s in order to highlight the repetitiveness of the observed phenomenon. Figure 4.4 presents the experimental procedure described above and the four corresponding responses measured. It is observed that two types of motion are possible:

- Decaying responses, leading to a small and noisy displacement of the bridge section around its static equilibrium position. Two samples are shown in plots 4.4 (a) and 4.4(d). These small oscillations are due to the turbulence of the free stream and the buffeting response of the bridge section.
- LCOs characterized by large amplitudes oscillations at the natural frequency of the pitching dof of the bridge at zero airspeed (2.1 Hz). Two LCOs are shown in plots (b) and (c).

Note that a LCO is measured for a free-stream velocity of 11.6 m/s (figure 4.4(c)), while decaying response is observed at 11.7 m/s (figure 4.4(a)). This hysteretic effect is characteristic of the torsional flutter phenomenon, as shown in previous work on helicopter blades [55–57], wing structures [67,68], or bluff bodies [109].



Figure 4.4: Schematic experimental procedure (upper plot) and corresponding time responses at different airspeeds

Stability concepts

From a mathematical point of view, this hysteretic dependence of the response type on a system parameter can be explained as the consequence of a fold bifurcation of cycles, followed by a subcritical Hopf bifurcation (see for example [162]). A fold bifurcation of cycles is the sudden appearance of a half-stable limit cycle of non-zero amplitude around a stable focal point, as the value of a system parameter is increased. In the present case the system parameter is the wind tunnel airspeed. As the value of the parameter is further increased, the half-stable limit cycle breaks up into two limit cycles of different amplitude, one stable and one unstable. The unstable limit cycle lies between the stable one and the focal point, as required by the Pointcaré-Bendixson theorem (see for example [165]). As the parameter is further increased, the unstable limit cycle can intersect with the focal point and then disappear, while the focal point itself becomes unstable. This phenomenon is know as a subcritical Hopf bifurcation. The fold and Hopf bifurcation terminology will be used in this work in order to characterize mathematically the aeroelastic response of the bridge deck and the rectangle.

Figure 4.5 demonstrates graphically the fold and subcritical Hopf bifurcations. In this example, a dynamic system has a focal point at 0 for all parameter values. A fold bifurcation of cycles occurs at a parameter value of 3, giving rise to two limit cycles, one stable of high amplitude and one unstable of low amplitude. The amplitude of the unstable limit cycle decreases to zero when the parameter reaches a value of 5. Consequently, there are three regions of stability:

- 1. Parameter < 3: the steady state response of the system is 0.
- 2. 3 < Parameter < 5: the steady state response of the system is either 0 or a LCO with amplitudes between 2 and 3.4.
- 3. Parameter > 5: the steady state response of the system is a LCO with amplitude higher than 3.4.

Notice that if no external excitation is applied to the system while the system parameter is increased from 2 to 7, the response will be 0 until, at 5, it will jump to a LCO. If then the parameter value is decreased back to 2, the system response will remain a LCO down to a parameter value of 3, when it will suddenly jump to 0. This phenomenon corresponds to the turbulence excitation tests carried out on the bridge deck in the wind tunnel. The wind tunnel's free stream turbulence level is very low, therefore the excitation level applied to the system is not sufficient to induce LCOs at subcritical airspeeds.

A system can only reach an unstable limit cycle as time approaches $-\infty$. This means that unstable limit cycles cannot be observed in practice. However, their existence can



Figure 4.5: Fold and Hopf bifurcations

be detected by careful application of different initial conditions for each parameter value of interest. This is the object of the initial condition tests applied to the rectangular cylinder in section 4.3.

Bifurcation diagram

Figure 4.6 shows the bifurcation diagram of the bridge deck: the maximum and minimum amplitudes of the pitching motion are plotted as a function of airspeed.

From 0 m/s to 13.4 m/s (point A to C), small oscillations around the equilibrium angle are observed. This angle is equal to 4.5° at 13.4 m/s. When the airspeed reaches 14.7 m/s (point D), the deck suddenly starts a large oscillating pitch motion between -5° and $+15^{\circ}$ around an equilibrium position of $+5^{\circ}$. This characteristic value of the bifurcation parameter is the Hopf airspeed, V_{hopf} , according to the stability concept of the previous section.

The airspeed is further increased up to 19.1 m/s (point E) and it is observed that the amplitude of oscillation decreases slightly: the peak to peak amplitude is equal to 16° . Note that the static deflection α_s corresponding to 19.1m/s is equal to 7.8° .

The airspeed is then reduced from 19.1 m/s to 6.8 m/s (point E to G) and the peak to peak amplitude reaches 24° . When the airspeed reaches 6.3 m/s the bridge deck suddenly stops its large amplitude motion (point B). This value of the bifurcation parameter is the



Figure 4.6: Bifurcation diagram of the bridge section

fold airspeed, V_{fold} .

The hysteresis loop highlighted above appears clearly through the branch D-G where LCOs are measured while decaying responses were previously measured in the same range of airspeeds (branch A-C).

As stated in the previous section, no initial conditions tests were performed on the bridge deck system. Hence the unstable limit cycle connecting points C and G was not identified experimentally during the wind tunnel tests.

The airspeed is again increased and only small amplitudes are observed up to point H. When airspeed reaches 12.8 m/s (point I), the deck suddenly starts to oscillate as described above at point D. While the amplitudes of the LCO measured during the two occurrences of the phenomenon are very similar, the Hopf airspeed V_{hopf} of is different, i.e. 14.7 m/s the first time and 12.8 m/s the second. Two explanations can be suggested for this observation:

- The Hopf airspeed is equal to 14.7 m/s. At 12.8 m/s, the system must have been excited sufficiently by the natural turbulence of the wind tunnel, above the attraction region of the unstable LCO branch. Hence, a stable LCO is reached.
- The Hopf airspeed is equal to 12.8 m/s. At 14.7 m/s the system is not excited from its equilibrium position.

The second suggestion is not likely because when the Hopf airspeed is exceeded, an

infinitesimal excitation is sufficient to lead to stable LCOs. For that reason the first assumption is retained and $V_{hopf} = 14.7$ m/s.

Aerodynamic forces and motion of the deck

The lift and drag forces and the motion of the bridge deck are measured simultaneously and four points denoted P1 to P4 are selected on the bifurcation diagram 4.7. It should be mentioned that the aerodynamic pitching moment around the pitch axis could not be measured since the pitch degree of freedom was set free.

- Point 1 at 9.7 m/s, below the Hopf airspeed, where the system remains on the stable LCO branch of zero amplitude.
- Point 2 at 12.5 m/s, the system undergoes large amplitudes LCOs.
- Point 3 at 18.7nm/s, the system undergoes LCOs, but the amplitude of oscillation is reduced compared to P2.
- Point 4 at 6.7 m/s, the system undergoes large amplitudes LCOs, below the Hopf airspeed and above the fold airspeed.



Figure 4.7: Bifurcation diagram of the bridge section without static deflection

Note that the static deflection is deliberately not added to the motion measured by accelerometers in figure 4.7. It is observed from that figure that the peak to peak amplitude decreases as the airspeed increases, which had also been observed for the LCOs occurring at points E and G (see figure 4.6).

Figures 4.8 to 4.11 represent three seconds of measurements of the pitch angle, lift coefficient and drag coefficient for the four points defined above. The aerodynamic coefficients are multiplied by 10 to fit on the same plot as the pitch angle, expressed in degrees. The bottom plot of each figure presents the frequency content of the lift coefficient, through Fourier transform in the range 0 to 30Hz.

Point 1

Point 1 corresponds to a decaying response of the bridge section. Small oscillations around the static deflection ($\alpha_s = 2.3^\circ$, not represented on figure 4.7) are measured, corresponding to the vortex shedding and buffeting responses of the bridge. Nevertheless, significant variations of the lift and drag coefficients are observed at this airspeed.



Figure 4.8: Point 1 - Decaying branch at 9.7 m/s

The lower plot of figure 4.8 shows that the only frequency component in the lift coefficient is 13.3 Hz. This frequency differs from the natural frequency in pitch of the

system (2.1 Hz). The excitation responsible for these forces variations is aerodynamic but different from the torsional flutter phenomenon investigated until that point.

Considering the static equilibrium pitch angle at 9.7 m/s, $\alpha_s = 2.3^\circ$, the Strouhal number was measured to be 0.16 (see figure 4.3). According to the definition of the Strouhal number, an estimate of the shedding frequency is $f_s = \frac{StV_{\infty}}{D} = \frac{0.16 \times 9.7}{0.1} = 15.5$ Hz. The difference between 13.3 Hz and 15.5Hz is probably due to the errors either in the measurement of the frequency in the wake or in the estimation of the model's pitch angle. It can be concluded that oscillations of the aerodynamic forces are due to the shedding of Von Karman vortices in the wake of the bridge deck; these oscillatory aerodynamic forces result in very low amplitude oscillations of the bridge. The low amplitude of the oscillations is probably due to the relatively high damping of the structure and to the fact that the natural frequency of the structure is much lower than the vortex shedding frequency.

Point 2

Point 2 lies on the stable LCO at an airspeed of 12.5 m/s, characterized by a peak to peak amplitude of 23° (pitch oscillations between -11.5° and 11.5° in figure 4.7). The pitch angle, lift and drag coefficients are shown in figure 4.9. The pitching frequency is 2.1 Hz, equal to the natural frequency of the pitch degree of freedom of the deck.

The frequency content of the lift coefficient is composed of the pitching frequency at 2.1 Hz and its first and second harmonics (at 4.3 Hz and 6.3 Hz), plus the vortex shedding frequency around 15 Hz.

The simultaneous occurrence of Von Karman vortices and large vortices convecting along the surface of a rectangular cylinder is analyzed further in section 4.3.

It is interesting to note that the lift and drag signals are not totally repeatable, each cycle is slightly different, although the main frequency components are the same.

Point 3

Point 3 sits on the stable LCO where the deck oscillates between -7.5° and 7.5° , as shown in figure 4.10. The corresponding airspeed is 18.7 m/s. The pitching frequency is 2.1 Hz, identical to the pitching frequency at point 2.

The shape of the aerodynamic coefficient signals is similar to the shape obtained at point 2. Nevertheless the strong peaks and troughs appearing at 2.1 Hz have the same magnitude as the variations due to the Karman vortices. This can be explained by the smaller amplitudes of oscillation measured at 18.7 m/s compared to 12.5 m/s. Hence the shedding process of Karman vortices is less affected by the large oscillations.



Figure 4.9: Point 2 - LCO at 12.5 m/s

Point 4

Point 4 corresponds to stable LCOs at 6.7 m/s, where the amplitude of oscillations is maximum: oscillations between -14° and 14° are shown in figure 4.11. The frequency content of the lift coefficient consists of the pitching frequency component and the vortex shedding frequency. The two force coefficients are nearly in phase with the pitching motion. Furthermore the energy level of the peaks at 15.3 Hz (Karman vortices) is much lower than the peak at 2.1 Hz.

It is concluded from the above discussion that two types of instabilities take place: torsional flutter and vortex shedding according to the Strouhal relation, the latter being of much lower amplitude



Figure 4.10: Point 3 - LCO at 18.7 m/s

Period-averaged lift and drag coefficients

It is possible to advance the analysis of the relation between the aerodynamic forces and the motion of the the bridge when it undergoes LCOs by observing the period averaged measurements of C_L and C_D . By averaging over one cycle, the cycle-to-cycle variability of the measured signals is removed, as shown in figures 4.8 to 4.11. Figures 4.8 to 4.11 show that C_L and C_D are in phase, therefore only C_L will be discussed. A bandpass filter is applied to the measured lift coefficient before performing the period averaging. The frequency range of this filter is set to 0 to 7Hz in order to retain the frequency component corresponding to the pitching motion (2.1Hz). The higher frequency components due to the shedding of Karman vortices are not considered here.

The period-averaged lift coefficients are shown for points 2, 3 and 4 in figure 4.12. The pitch angle response over one cycle is shown as a dashed curve, its amplitude normalized



(b) Frequency content

Figure 4.11: Point 4 - LCO at 6.7 m/s

to unity. A global peak is present in the curves of C_L during the upstroke of the bridge deck, before reaching the maximum pitch angle. The first observation concerns the values of the peaks: they are rather similar for points 2 and 4, while clearly inferior for the higher airspeed tested (point 3). The second observation concerns the phase of the peaks, which changes with airspeed. The smallest phase difference between the lift and pitch angle peaks occurs at point 4, the largest occurs at point 3. In addition, secondary peaks are observed in the period averaged responses of points 2 and 3, but not in that of point 4 for which the analysis of the frequency content showed a strong component at the pitching frequency and a lower component around the shedding frequency (see figure 4.11(b)).

The phase shift of the lift force, ϕ_L , is defined relative to the pitching displacement $\alpha(t) = A \sin(2\pi f t)$, according to $C_L(t) = C_1 \sin(2\pi f t + \phi_L)$, where f denotes the pitching frequency and the constants A and C_1 denote the amplitudes of the measured signals.

The phase shift is presented in table 4.2 for each of the three points 2, 3 and 4, together



Figure 4.12: Time averaged lift coefficient for points 2, 3 and 4

with the corresponding airspeed, peak to peak amplitude of the LCOs and maximum value of C_L . Note that the columns of this table are sorted in order of increasing airspeed.

	Point 4	Point 2	Point 3
V_{∞}	$6.7 \mathrm{m/s}$	$12.5 \mathrm{~m/s}$	18.7m/s
$\Delta \alpha$	28°	23°	15°
C_L^{max}	1.5	1.65	1.0
ϕ_L	24.4°	80.2°	128.9°

Table 4.2: Phase delays of lift coefficient for points 2, 3 and 4

Table 4.2 shows that the phase shift increases with airspeed while the amplitude of oscillation reduces with airspeed. By contrast, the maximum C_L does not depend monotonously on the airspeed, therefore it cannot be the cause of the reduction in response amplitude with increasing airspeed. Hence it can be concluded that the delay between the aerodynamic force and the pitch angle response is the main factor that reduces the amplitude of oscillation of the bridge section.

The discussion above is based on the assumption that the aerodynamic moment coefficient (not measured in these experiments) will follow the same tendency as the lift coefficient. The main characteristic of torsional flutter is demonstrated through the analysis of these measurements: the aerodynamic moment is not in phase with the motion of the structure. This delay is due to the separation and re-attachment process of the flow-field around the structure undergoing large amplitude oscillations.

Effect of the modal parameters

T

It is of interest to study the effect of different modal parameters on the torsional flutter phenomenon. Hence, another lever arm is used in the pitching apparatus. Its length is equal to 0.5 m, leading to an increase of the natural frequency of the pitching motion. In addition, an increase of the structural damping is measured during wind-off dynamic tests performed in this configuration. The characteristics of this second system, denoted S2, are shown together with the initial structural parameters (S1) in table 4.3.

Set-up	f_0 [Hz]	ξ_0 [%]	I_{α} [kgm]	$c_{\alpha} \; [\rm kgm/s]$	$k_{\alpha} [\mathrm{N/rad}]$
S1	2.1	1.6	0.148	0.063	26.17
S2	2.7	1.8	0.150	0.091	43.02

Table 4.3: Structural parameters for both set-up

Figure 4.13 shows the bifurcation diagrams corresponding to the two systems: S1 and S2. The shapes of the bifurcation diagrams are qualitatively equivalent. Nevertheless several interesting remarks can be made:

- The amplitudes of S2 are higher than the ones measured in the case of S1.
- The static deflection α_s is naturally lower for S2 than S1. This is due to the value of k_{α} used in equation 4.4 corresponding to each system (see table 4.3).
- The value of the Hopf airspeed observed for S1 (14.7 m/s) is denoted on figure 4.13 by the symbol H1. For the second set-up, S2, the Hopf airspeed is equal to 14.9 m/s, i.e. comparable with S1. It is denoted by H2 in the figure.
- The fold airspeeds are identical for the two set-ups and is equal to 6.3m/s (points F and F').

It is concluded that the effect of the damping coefficient is weak on the bifurcation airspeeds (Hopf and fold), while an important difference of amplitude is observed. This is counter-intuitive, since the increase of the torsional stiffness should lead to the decrease of the resulting oscillations.



Figure 4.13: Bifurcation diagrams for both set-ups: star markers for S1 and circles for S2

Figure 4.14 shows the variation of the frequency of the LCO motion of the deck with airspeed. Both stiffness set-ups are presented. It can be observed that the frequencies match the natural frequencies of the pitch degree of freedom for each stiffness set-up and are rather constant with airspeed. This is in accordance with the theory that states that torsional flutter is a damping-driven phenomenon.



Figure 4.14: LCO frequencies for both set-ups
4.2.5 Concluding remarks

This section presents the experimental investigation of a generic bridge section with a pitch degree of freedom. The analysis of the motion of the deck reveals the existence of both decaying and LCO responses. The bifurcation diagram features a hysteretic loop, characterized by a sub-critical Hopf bifurcation followed by a fold bifurcation.

The effect of the torsional stiffness and the structural damping on the torsional flutter is studied. It is observed that the torsional flutter oscillations occur at the natural frequency of the pitch degree of freedom and are independent of the airspeed. In addition, the fold airspeed is not affected by the variations of damping and stiffness studied here.

The simultaneous measurements of the aerodynamic forces and acceleration response of the structure allowed the observation of several interesting phenomena. As the aerodynamic pitching moment was not measured, the discussion presented in this work is based on the lift and drag forces only.

When starting from rest, the structure oscillates with very small amplitudes and a frequency imposed by the vortex shedding process. For higher airspeeds, the deck jumps to the stable large amplitude LCO and the lift coefficient possesses two main frequency components:

- A frequency component corresponding to the vortex shedding process, as for the small amplitude motion.
- A lower frequency component equal to the bridge deck's pitch oscillation frequency.

In addition, important phase delays between the lift force and the pitch angle of the bridge deck are reported. The ejection of major vortices when high amplitudes are reached can be interpreted as peaks and troughs in the lift and drag coefficient signals. These vortices are investigated in the next section, dealing with the torsional flutter of a rectangular cylinder.

4.3 Torsional flutter of a rectangular cylinder

A deeper investigation of the torsional flutter phenomenon is carried-out in this section. Particular attention is given to the tight interaction between the unsteady flow patterns in the flow-field and the response of the structure. The investigation focuses on a rigid rectangular cylinder with side ratio B/D equal to 4, free to oscillate around its pitching axis. This specific bluff-body is chosen for three reasons:

1. It is well documented in significant research papers from the end of the 70's [108, 109], to the recent works by Matsumoto [78]. These works present experimental

investigations of the behaviour of rectangular cylinders with different aspect ratios B/D. They represent a good starting point to investigate the torsional flutter phenomenon and to compare with the new observations and analysis presented in this thesis.

- 2. Its simple geometry facilitates the design and the construction of the experimental set-up (standard beam dimensions).
- 3. The numerical simulations of chapter 5 are easier to perform, especially concerning the meshing of the body and the resulting density of vortical particles, which is a critical point in vortex methods. The calibration of the numerical tool is enhanced by the comparison with data of other experiments [109].

It is worth mentioning that the aerodynamic characteristics of a simple bluff-body shape such a rectangle cylinder are still of interest to the research community. Indeed, a global consortium named Benchmark on the Aerodynamics of a Rectangular 5:1 Cylinder (BARC), was launched in July 2008 by several of the most active researchers in the domain of bluff-body aerodynamics. The objective of this consortium is to connect experimental results from wind tunnel and water tunnel laboratories to numerical tools though a generic benchmarking model⁽¹⁾.

The choice of a 4:1 rectangular cylinder for the present work is dictated by two factors: first the area of interest of this thesis is the aeroelastic behaviour of bluff-bodies, while the BARC project is limited to aerodynamics. Furthermore, the aeroelastic behaviour of 4:1 rectangular cylinders is under investigation by important members of the BARC project [78]. Secondly, the chord to thickness ratio B/D of the generic bridge section varies between to 3.2 and 4.5, depending on wether the height of the wind-screens is taken into account in the definition of D (cfr figure 3.3). The sketch of figure 4.15 presents the two sections centered on their common pitching axis. Noting that the wind-shields have a permeability factor of 42% and no acoustic panel is present on the deck section in these experiments, it can be stated that the ratio B/D of the two sections is approximately equal.

The complete aeroelastic behaviour of the rectangular cylinder with pitch degree of freedom is identified during the wind tunnel experiments. Similarly to the generic bridge deck experiments, structural accelerations but also the longitudinal component of the velocity in the wake of the model at the mid-span point are measured and analyzed. Flow-field visualization by means of the Time resolved Particle Image Velocimetry (Tr-PIV) technique is used to demonstrate some interesting aspects of the unsteady flow-field,

⁽¹⁾Details about the structure and objectives of the project or data available can be found at http://www.aniv-iawe.org/barc/



Figure 4.15: Comparison of the B/D ratios for the generic bridge and the 4:1 rectangle. The common pitching axis is shown as a circle

such as the manifestation of flow separation and the shedding of vortices. As the damping ratio is more important in the present experimental set-up than in previous wind tunnel works [78, 108], the vortex-excited responses and the torsional flutter responses can be decoupled, at least as far as the onset velocities of these phenomena is concerned.

Furthermore, a quantitative analysis of the aeroelastic responses of the system is carried out. It is based on the CPOD technique presented in chapter 2. An artifice is proposed for building a set of experimental data containing the solid velocities inside the model and the unsteady velocity fields measured using PIV. This technique allows the quantitative comparison of the responses of a system for different values of the airspeed. Interesting information about the torsional flutter instability can be extracted from the CPOD analysis of the coupled fluid and structural responses of the system at different airspeeds. Large vortical structures, denoted Motion Induced Vortices (MIV), are identified as the fundamental cause of the torsional flutter phenomenon.

As stated above, results of numerical simulations are described in chapter 5 in order to complete the study of the torsional flutter of bluff bodies. The experimental results of the present chapter are also used for the validation of the numerical aeroelastic tool.

4.3.1 Experimental setup

A support apparatus has been developed in the framework of this research, with the aim to study the aeroelastic instability of quasi-two-dimensional structures free to oscillate around a pitching axis.

This apparatus is distinctive in the sense that it is of cantilever type (see figure 4.16), allowing an optimal use of the PIV measurement system. The supporting frame is designed such that the complete structure is rigid in all directions except for the pitch degree of freedom, which is allowed by two ball bearings supporting the pitching axis. The small dimensions of the model are dictated by the capabilities of the PIV system installed in the laboratory (maximum visualization window of 12 cm by 12 cm). Note that the



Figure 4.16: Experimental set-up in the test section

resulting design leads to a very low value of the blockage factor (< 4%). The experimental apparatus is installed perpendicularly to the oncoming airflow and the model is centered inside the aeronautical test section in the vertical direction. Hence, the flow-field around the model is not perturbed by the floor and roof of the wind tunnel.

Model

The model consists of a rectangular cylinder with chord B = 0.08 m, depth D = 0.02 m and length L = 1 m. It is made from a rectangular aluminum tube with a thickness of 2 mm, resulting in very high stiffness values for the model's flexible modes compared to the low stiffness of the rigid body pitching degree of freedom. The position of the pitching axis is set at the geometric center of the section of the model, as shown in figure 4.17. All the experiments are performed with a wind-off equilibrium pitch angle α_s equal to zero.



Figure 4.17: Cross section of the wind tunnel model showing its dimensions and the position of the pitching axis

The flow around the model is approximately two-dimensional since the model's spanwise dimension is 12 times greater than the chord. Furthermore, the gaps between the tips of the model and, one one side the end plate and on the other side the wall of the wind tunnel are 5mm each. As a consequence, three-dimensional flow effects at the tips are suppressed.



Figure 4.18: Pitching system

The pitching motion of the model is measured using two accelerometers attached to the spring adaptor arm (see figure 4.18). The length of the arm is equal to 14 cm. The frequency range of the accelerometers is 0-300 Hz and the acceleration measurement range is ± 50 g. From these measurements, the pitching response, $\alpha(t)$, of the model around its wind-off equilibrium pitch angle is reconstructed by numerically integrating the acceleration signals twice.

Static loading experiments are performed in order to characterize the linearity of the complete pitching system, composed of the adaptor arm connected to two extension springs (as shown in figure 4.18). Figure 4.19 shows the variation of the restoring moment with the pitch angle. It is observed that the system remains linear in the range between 0° and 22° . Structural nonlinearity appears for values of α above 22° and is of softening nature.

Wind-off dynamic measurements are performed to identify the wind-off natural frequency, f_0 , and damping ratio, ξ_0 , of the system. These parameters are used in order to create a wind-off mathematical model of the system of the form

$$I_{\alpha}\ddot{\alpha} + c_{\alpha}\dot{\alpha} + k_{\alpha}\alpha = 0$$

where I_{α} is the moment of inertia of the rectangle around its pitch axis, c_{α} the damping coefficients and K_{α} the spring stiffness. Table 5.3 presents the identified values for the wind-off natural frequency, damping ratio, moment of inertia, damping coefficient and



Figure 4.19: Restoring moment

spring stiffness.

f_0 [Hz]	ξ_0 [%]	I_{α} [kgm]	$c_{\alpha} \; [\rm kgm/s]$	$k_{\alpha} [\mathrm{N/rad}]$
8.65	2.6	0.00152	0.0045	4.47

Table 4.4: Structural parameter of the linear system (wind-off)

Hot wire anemometry

A hot wire probe is used to measure the longitudinal component of the velocity, u(t), at one position in the wake of the model. The probe is located at mid-span, at a distance of 0.04 m from the trailing edge of the model (see figure 4.20). It is supported by a vertical square beam which is located 0.14 m downstream of the model, thus minimizing the influence of its presence on the response of the model. The accelerometer and hot wire signals are acquired simultaneously at a sampling rate of 1.0kHz.

The wind tunnel's free-stream turbulence has significant frequency components between 0 and 2 Hz while the modal frequencies of the different configurations of the model range between 5.5 Hz and 10 Hz. For this reason, a bandpass filter between 2 and 120Hz is applied to all the measurements recorded during the experiments. This frequency range excludes irrelevant low frequency phenomena and includes all the important fundamental frequencies and higher harmonics.



Figure 4.20: Sketch of the experimental measurement systems

A preliminary analysis of the hot wire measurements is dedicated to the identification of the Strouhal number of the static model. The Strouhal number (see equation 4.3) is calculated with D = 0.02 m. The model is securely clamped at the selected angle of attack (0° and 5°) and the fluctuations of the longitudinal velocity u(t) in the wake of the model are measured for different airspeeds. The shedding frequency is extracted through the analysis of the Fourier Transform of u(t) and the Strouhal number is calculated according to equation 4.3. The identified Strouhal numbers are 0.152 and 0.149 for 0° and 5° respectively. The former result compares well with the experimental works by [113] and [166], where values ranging from 0.13 to 0.15 are reported for zero angle of attack.

Tr-PIV set-up

The unsteady velocities in the flow-field are measured using the technique of Tr-PIV, in a window covering the entire upper surface of the model. The acquisition frequency of the Tr-PIV measurements is set to 1.0 kHz, as the accelerometers and the hot-wire probe.

Figure 4.20 shows a sketch of the experimental set-up with the position of the PIV window at mid-span on the upper surface of the model. The measurement window is illuminated using a double-pulse laser (527 nm, 2×10 mJ). For each pair of images, a time of 30 ns is chosen between the acquisition of the first image and the second one. Digital image sequences are acquired using a charge-coupled-device camera (Phantom V9.1, max. 1600×1200 pixels) mounted on a traversing system outside the test section.



Figure 4.21: Experimental set-up - PIV

The size of the measurement window, dictated by the energy available in the laser sheet, is equal to 10 cm by 10 cm and allows a good visualization of the flow-field on the upper surface of the model. The laser sheet is shown in figure 4.21. The post-processing of the recorded images (1000×600 pixels) is performed using an adaptive cross-correlation algorithm with an initial grid of 64×64 pixels and a sub-correlation, resulting in a spatial resolution of 32×32 pixels (corresponding to a square mesh of 2.5×2.5 mm). An additional treatment of the post-processed PIV measurements was implemented in order to handle the pitching motion of the model in the PIV window.

4.3.2 Experimental procedure

Two types of test are performed:

- Turbulence excitation tests: The wind tunnel airspeed is varied in steady increments and the model's response is observed and measured at each stabilized airspeed. The only source of excitation in these tests is the wind tunnel's free-stream turbulence.
- Initial condition tests: At selected airspeeds, different initial conditions are applied to the model. These conditions consist of initial pitch angle displacements. Initial angles α_0 between 1° and 10° can be applied, in increments of 1°.

At low airspeeds, the system response is a low amplitude response to the wind tunnel's free stream turbulence. This type of response will be referred to as stable. The top plot

of figure 4.22 shows an example of a stable response, where the signal recorded from one of the accelerometers is plotted against time. It can be seen that the form of the signal is stochastic and its acceleration amplitude is low (below 0.05 g).

At higher airspeeds, the system response becomes a Limit Cycle Oscillation (LCO), i.e. a symmetric oscillation in pitch with constant frequency and amplitude. An example of a LCO response is shown in the bottom plot of figure 4.22 where the signal of one of the accelerometers is plotted against time. In this case, the signal is deterministic and the amplitude is much higher (around 5 g).



Figure 4.22: Stable (top) and LCO (bottom) acceleration responses (in g)

4.3.3 Bifurcation

The turbulence excitation tests show that, as the airspeed is increased from 0 m/s, the system's response is stable up to an airspeed, $V_{\infty} = 14.6$ m/s. At this critical airspeed, the response jumps abruptly to a high amplitude LCO. The wind tunnel airspeed is then gradually decreased but the system still undergoes LCOs, down to an airspeed of 6.7 m/s. Therefore, there is clearly a hysteretic dependence of the response type on the airspeed, as in the case of the bridge deck.

Notice that no tests are carried out beyond the critical airspeed for safety reasons: the model undergoes pitching oscillations with a 25° amplitude. The deflections of the extension springs become large and increasing further the airspeed would lead to significant nonlinear structural effects in the pitching system, as stated in section 4.3.1. The bifurcation diagram for the rectangular cylinder, showing the variation of the LCO amplitude with airspeed is presented in figure 4.23. The airspeed ranges between 6 m/s and 15 m/s. The positive amplitudes are shown only because of the symmetry of the observed motion around the zero degree equilibrium position. Notice that the bifurcation diagram features the same subcritical Hopf and fold combination of bifurcations observed in the bridge deck experiments (see figure 4.6).

During the turbulence excitation tests, the airspeed is first increased step by step from zero to 14.6 m/s. This airspeed can be seen as the approximate Hopf airspeed and is denoted by a star marker in figure 4.23. The dashed arrow represents the sudden change of stability experienced by the system at this airspeed. Then the airspeed is reduced by small increments from 14.6 m/s to 6.7 m/s; the system undergoes LCOs at each stabilized airspeed. The amplitudes of these LCOs are shown as open circles in figure 4.23. When the airspeed reaches 6.6 m/s the response becomes a long decaying motion. This airspeed is the approximate fold speed. Therefore, the turbulence excitation test leads to the identification of the Hopf and fold airspeeds as 14.6 m/s and 6.6 m/s respectively. Furthermore, the amplitudes of the stable LCOs are measured for each intermediate airspeed.



Figure 4.23: Bifurcation diagram

The black dots and the x's in figure 4.23 correspond to the initial condition tests. Each dot or x corresponds to an initial pitch angle of the model at a chosen airspeed. Values between 1° and 10° are imposed during these experiments. A cross corresponds to an initial condition that leads to a stable response. The black dots represent the initial conditions that lead to a stable LCO. Multiple initial conditions are applied in order to define the domains of attraction of the system in the finest possible detail. The following conclusions are drawn:

- 1. At $V_{\infty} = 6.5$ m/s all initial conditions up to 10° lead to stable responses. Therefore, the fold has not occurred yet.
- 2. At 6.7 m/s an initial condition of 2° leads to a stable response while initial conditions of 3° or higher lead to LCOs. Therefore, the unstable limit cycle lies between the pitch angles of 2° and 3°. Furthermore, the fold bifurcation must occur between 6.5 m/s and 6.7 m/s. Figure 4.24 shows the pitching responses of the system at two different airspeeds close to the fold airspeed. In the upper plot, corresponding to 6.7 m/s, the oscillation amplitude is 7.5° and is constant in time; the system lies on a stable LCO. The lower plot of the same figure corresponds to the response of the system at 6.6 m/s, i.e. at the fold airspeed identified above. The model is given an initial condition of 5° but the resulting oscillation has a slowly decreasing amplitude, leading to a stable condition after 40 s.
- 3. At $V_{\infty} = 6.9$ m/s the $\alpha_0 = 1^{\circ}$ initial pitch angle leads to a stable response while initial conditions of 2° and higher lead to LCOs. Therefore, the unstable limit cycle must lie between 1° and 2°.
- 4. At airspeeds between 6.9 m/s and 14.6m/s even an initial condition of 1° will lead to LCOs. As the turbulence excitation tests demonstrate that the response of the system can be stable at these airspeeds, it follows that there is an unstable limit cycle, which lies between 0° and 1°.

Note that the actual experimental apparatus does not allow an initial angle setting resolution lower than 1°. For this reason, a horizontal dashed line has been sketched in figure 4.23 to represent the hypothetical position of the unstable limit cycle.

The similarity between the bifurcation diagram of the rectangular cylinder (figure 4.23) and the bridge section (figure 4.6) is obvious: the subcritical characteristic of the torsional flutter is demonstrated on both structures.

Another interesting feature of the bifurcation plot of figure 4.23 is the fact that there is a jump in LCO amplitude at $V_{\infty} = 9.4$ m/s. The cause of this jump could be a second fold occurring around this airspeed. As it is not possible to set initial conditions of 15°, the occurrence of this second fold cannot be verified.



Figure 4.24: Rectangular cylinder: Time signals

Frequency-Velocity Plots

Figures 4.25 and 4.26 show the evolution of the frequency content of the pitching angle response $\alpha(t)$ and wake velocity u(t) with airspeed. Figure 4.25 plots the frequency content of these two quantities for the stable responses only. Figure 4.26 plots the frequency content for the LCO responses. Both figures present the Power Spectral Density (PSD) of the signals measured for different airspeeds and calculated using Welch's method. White areas denote high positive values of the PSD while dark areas correspond to small values. Frequencies up to 120 Hz are included in the plots.

In the absence of LCOs (figure 4.25), the hot wire measures the variations of the longitudinal velocity due to the shedding of vortices following the Strouhal relation. Indeed, the peaks of the PSDs match the dashed line corresponding to equation 4.3. The plot of the PSD of $\alpha(t)$ (left) contains a strong frequency components between 8 and 9Hz as well as a weak component corresponding to the vortex shedding. Therefore, the stable responses of the rectangle contain two components:

- Response at the rectangle's natural frequency due to the free stream turbulence excitation.
- Response at the Strouhal frequency due to the vortex shedding.

When the system undergoes LCOs (figure 4.26), a strong peak is measured around 8Hz in the PSDs of $\alpha(t)$ and u(t). Furthermore, harmonics at integer multiples of this

frequency appear in both PSDs. This spread of the energy of the signal to its harmonics is characteristic of the response of nonlinear systems; in the present case the nonlinearity lies in the aerodynamic force caused by the high amplitude oscillations.



Figure 4.25: PSDs of $\alpha(t)$ and u(t), without LCOs

Figure 4.27 shows a zoom of the PSDs of $\alpha(t)$ and u(t) between 4 and 12Hz for the LCO case. It is observed that the frequencies decrease as the airspeed increases. The highest peak in the PSD of $\alpha(t)$ passes from 8.3 Hz at 7.0 m/s to 7.5 Hz at 15 m/s, which corresponds to a decrease of 10%.

The frequency content of the responses demonstrate the shedding of two types of vortices in the wake of the model: Karman Vortices (KV) and Motion Induced Vortices (MIV). The former are shed periodically following the Strouhal relation (equation 4.3) when the response is stable around $\alpha_0 = 0$. Their frequency content is concentrated on a single shedding frequency.

On the other hand, MIVs are shed when the model undergoes LCOs. The self-excited oscillations are due to this type of vortex, which results in a nonlinear aerodynamic pitching moment. The energy of the MIV is distributed over the pitching frequency and its harmonics. The next section deals with the response of the system at the vicinity of the fold bifurcation, where both MIV and KV can be observed at a single airspeed.



Figure 4.26: PSDs of $\alpha(t)$ and u(t), with LCOs



Figure 4.27: Zoom on the PSDs of $\alpha(t)$ and u(t), with LCOs

Frequency-Time Plot

The response of the system at 6.6 m/s is interesting because it lies in the neighborhood of the fold bifurcation as presented above in figure 4.23. It is of interest to analyze the horizontal component of the measured wake velocity, u(t), for different types of response of the model at this airspeed.

The time evolution of the pitch response of the system is shown in the upper plot of figure 4.28. Three samples are highlighted in this figure, according to the type of motion

observed: in the first sample, the system undergoes oscillations with a nearly constant amplitude of 5° . The second sample contains the decaying portion of the motion. The third sample corresponds to the part of the response where the model responds to the free stream turbulence around its equilibrium position.



Figure 4.28: Decaying response from LCO at 6.6 m/s

The lower plot in figure 4.28 shows the Short Time Fourier Transform (STFT) of the longitudinal component of the velocity measured in the wake of the model. The STFT is obtained by dividing the hot wire measurement into 50 windows and computing the Fourier transform of each window. Each window has a length of 1000 samples without overlap.

The graphical representation of the STFT shows the time evolution of the frequency content of the longitudinal velocity u(t). Figures 4.29 to 4.31 show one second of measurements of u(t) and $\alpha(t)$ in plain and dashed curves respectively, for each of the three samples presented above.

Figure 4.29 corresponds to sample 1 and demonstrates that the system undergoes nearly constant amplitude oscillations, i.e. LCO. The wake velocity and pitch angle signals have the same fundamental frequency (8.3 Hz) but are slightly out of phase, u(t) leading



Figure 4.29: Pitching response at 6.6 m/s - sample 1

 $\alpha(t)$. The wake velocity signal contains higher frequency components, which are integer multiples of the fundamental frequency, as seen in the relevant section of the STFT plot of figure 4.28. This frequency content is typical of MIV shedding at the system's natural frequency. However, the MIVs are not strong enough to maintain the motion indefinitely; the amplitude of the motion starts to decay markedly after about 40 s.



Figure 4.30: Pitching response at 6.6 m/s - sample 2

Figure 4.30 corresponds to the decaying phase of the response of the system (sample 2). In this case, the hot wire signal is composed of two main frequency components: the pitching frequency at 8.3 Hz and the vortex shedding frequency at 44.5 Hz. Peaks are observed in the STFT at these frequencies, in the dashed zone denoted 2 in figure 4.28. Each peak corresponds to a type of vortex: MIV at 8.3 Hz and KV at 44.5 Hz. The wake velocity and pitch angle signals appear to be nearly in phase.



Figure 4.31: Pitching response at 6.6 m/s - sample 3

Figure 4.31 shows the time signals for the third sample: the model is stable around the equilibrium position at zero degree. This figure shows a unique frequency component at 44.5 Hz, corresponding to Karman Vortices. Note that the time signal is not repetitive, which is characteristic of responses to turbulence. The shedding frequency (44.5 Hz) is 11% lower than the 50.2 Hz value predicted by the Strouhal relation. This difference may be due to inaccuracies in the estimation of the Strouhal number detailed in section 4.3.1.

4.3.4 Tr-PIV measurements

Tr-PIV measurements are performed in order to investigate the relation between the unsteady characteristics of the flow-field (vortex convection, separation and re-attachment of the flow-field) around the model and the resulting aeroelastic response. Figure 4.32 shows a PIV snapshot of the velocity field on the upper surface of the model at an airspeed of 7.4 m/s. The grayscale backgrounds correspond to the images recorded by the high speed camera, where the seeding particles, lit by the laser sheet, appear as white dots. The white line appearing in both pictures represents the upper surface of the model and is the result of the reflection of the high intensity laser light on the aluminum surface. This reflection allows the accurate measurement of the instantaneous position of the model and its synchronization with the accelerometer signals. The arrows denote the instantaneous local flow velocities calculated after post-processing the images.



Figure 4.32: Sample PIV snapshot

The unsteady flow-field is investigated when the model is either static at 13.9 m/s or undergoes LCOs at two airspeeds: 7.4 m/s and 13.4 m/s. Referring to the bifurcation diagram presented in figure 4.23, small and large amplitudes LCOs are observed at these airspeeds respectively.

Static model at 5°

Primary PIV measurements are performed on the upper surface of the model statically retained with an angle of attack equal to 5° . The airspeed is set to 13.9 m/s and 500 snapshots of the flow-field are measured. The mean flow, presented in figure 4.33, shows a large separated shear layer without re-attachment on the surface of the model.



Figure 4.33: Averaged flow-field around the static model at 5° and 13.9 m/s

From the time resolved velocity fields measured in this configuration, it is observed that the separated shear layer contains large vortical structures that are periodically created in this region and convected downstream. Several trajectories of the cores of these vortices are shown as segment lines in figure 4.33. The average trajectory of the vortices passes through the points of the flow-field where the mean flow changes its orientation. The trajectories start at the quarter-chord of the model and stop before reaching the trailing edge of the model, where the size of the vortex is larger and its intensity decreases due to the convection in the separated flow-field. The limited size of the PIV window does not allow to track the vortex cores further downstream.

The analysis of the frequency content of the PIV measurements shows that the periodicity of the creation and convection process discussed above follows the Strouhal relation 4.3. Hence the vortices observed on the upper surface of the static model correspond to Karman Vortices (KV), as pointed out in the discussion about the frequency content in page 139.

The mean convection velocity of the KVs is calculated on the basis of the identified trajectories. Its value is equal to 60% of the free-stream velocity, which is in agreement with the estimates presented by Shiraishi and Matsumoto [167]. These authors observed that the convection airspeed is generally equal to 60% of V_{∞} for rectangular cylinders with different aspect ratios B/D, at zero angle of attack. On this basis it is possible to derive an interesting expression for the Strouhal number for this type of bluff-body. The

convection velocity of a Karman vortex, V_c , is equal to

$$V_c = 0.6V_\infty = \frac{B}{T_c}$$

where T_c denotes the time needed for the vortex to convect from the leading edge to the trailing edge. If it is assumed that there is no time delay between the shedding of two consecutive vortices (i.e. a new vortex starts to get formed near the leading edge as the old one clears the trailing edge), then the shedding period is equal to the convection time. The shedding frequency becomes $f_s = 1/T_c$ and, from the definition of the Strouhal number,

$$St = \frac{f_s D}{V_{\infty}} = \frac{D}{T_c V_{\infty}} = \frac{0.6 V_{\infty}}{B} \frac{D}{V_{\infty}} = 0.6 \frac{D}{B}$$

When applied to the present 4:1 rectangular cylinder, this expression yields: $St = 0.6\frac{1}{4} = 0.15$. This result is in agreement with the hot wire measurements performed at 5°, for which the Strouhal number measurement is 0.149. Nevertheless it should be pointed out that this simplified presentation of the vortex shedding phenomenon is only valid for large Reynolds numbers and it does not take into account any aerodynamic details (sharpness of the edges, rugosity ...), which can have an important effect on the flow. Furthermore, the discussion above is based on rectangular cylinders at zero angle of attack. Recalling that the value of the Strouhal number at zero degree is equal to 0.152, it is concluded that the unsteady flow-field is roughly the same over the surface of the rectangle at 0° and 5°.

Despite the attractiveness of this empirical formula, it must be used with care. It can be too simple, or even dangerous, to reduce the complex Navier-Stokes equations, governing unsteady viscous aerodynamic phenomena, to simplified rules of thumb.

Figure 4.34 shows the fluctuation intensity of the horizontal velocity on the upper surface and around the trailing edge of the model. This quantity measures the influence of the KV on the separated shear layer [137]. Figure 4.34(a) shows the occurrence of two sharp peaks, one at the edge of the shear layer near the leading edge and one in the vicinity of the trailing edge. The leading edge peak is due to the unstable behaviour of the shear layer (oscillating randomly) but also because it is the location where the KVs are created. The trailing edge peak is due to the shedding of the Karman vortices and, possibly, also due to the interaction between the vortices shed from the upper and lower surfaces.



Figure 4.34: Horizontal velocity fluctuation intensity (expressed in m/s) for the static model at 5° and 13.9 m/s

Oscillating model

The unsteady flow-field is investigated when the model undergoes LCOs at two airspeeds: 7.4 m/s and 13.4 m/s. Referring to the bifurcation diagram presented in figure 4.23, small and large amplitude LCOs are observed at these airspeeds. On the fluid side, MIVs are expected to occur around the model.

Airspeed = 7.4 m/s

Figure 4.35 shows the pitch response $\alpha(t)$ during one cycle of the LCO observed at $V_{\infty} = 7.4$ m/s, where 15 time instances are selected. They correspond to the 15 snapshots of the velocity field presented in figure 4.37.



Figure 4.35: Time reference for velocity snapshots at 7.4 m/s

The following observations can be made:

- At $\alpha(t) = 0$ (snapshot 1), there is a small separated flow region near the leading edge. The extent of this region is roughly equal to one third of the chord length.
- The situation is similar in snapshots 2 and 3, with an increase in the size of the separated region, up to the mid-chord position. These first three snapshots correspond to the creation of the MIV.
- Snapshot 4 shows a clear vortical pattern inside the separated region: this MIV is characterized by a clockwise orientation. The flow-field remains attached from the mid-chord to the trailing edge.
- In snapshots 5 to 9, the MIV convects downstream and lifts off the surface. It convects downstream with a speed equal to 20% of the free stream velocity (7.4 m/s).
- In snapshot 10 the flow is completely separated from the surface of the model. Referring to figure 4.35, this time instance corresponds to 35% of the period of the motion. Separation occurs just after the maximum pitch angle is reached.
- In snapshots 10 to 12 the flow remains completely separated. Weak secondary vortices form inside the separated flow region.
- Snapshot 13 shows the beginning of the re-attachment process of the flow-field at the trailing edge. The re-attachment process starts at 50% of the period of oscillation, i.e. when the pitch angle is equal to zero. In snapshot 14 the flow is partially re-attached. In both cases, the flow is nearly stagnant in the separated region.
- Finally, the flow is completely attached to the surface of the model at time instance 15, which occurs just before the minimum pitch angle is reached, at 70% of the cycle.

Airspeed = 13.4 m/s

The LCO amplitude observed at $V_{\infty} = 13.4$ m/s is 22°. The flow-field is plotted in figure 4.38 at the time instances shown in figure 4.36. The following observations can be made:

• Snapshots 1 and 2 are similar to the first five snapshots of the 7.4 m/s case (figure 4.37). They correspond to the creation of the MIV at the leading edge.



Figure 4.36: Time reference for velocity snapshots at 13.4 m/s

- Snapshots 3 to 5 show the downstream convection of the MIV. As in the 7.4 m/s case, the convection velocity is equal to 20% of the free stream velocity.
- Snapshot 6 marks the beginning of the separation process. It is interesting to note that separation occurs just before reaching the maximum pitching amplitude (20% of the cycle). The MIV is still present in the PIV window in snapshot 6, but the flow is separated over the whole surface of the model.
- The flow remains completely separated in snapshots 7 to 9.
- The reattachment process begins in snapshots 10 to 12, i.e. between 35% and 50% of the cycle. Small secondary vortices are observed in the separated region of the flow. These secondary vortices are strongly unstable, due to the unsteadiness of the motion of the separation boundary. They are quite distinct from the MIV present in the flow in the first 20% of the cycle.
- Snapshots 13 and 14 do not feature any significant vortices and the size of the separated flow region is significantly decreased, especially in snapshot 14.
- Finally, the flow-field is completely attached to the surface of the model in snapshot 15, just before the minimum pitch angle is reached, at 65% of the cycle.

The flow separation and reattachment mechanisms are the same for the two airspeeds illustrated in figures 4.37 and 4.38. Three different states of the flow-field can be defined:

- Attached flow: the flow is attached to the upper surface of the model.
- MIV generation and convection: the MIV is created at the leading edge of the model and convects downstream. Initially, the flow-field is attached to the surface on the rear of the rectangle, from the pitch axis to the trailing edge.

• Flow separation: this state is characterized by reversed flow or stagnation on the upper surface of the model. It is split into two parts: separation and re-attachment. The separation phase occurs just after the MIV has cleared the trailing edge. Then a large recirculation region is observed in the flow-field. The re-attachment process starts when the flow stagnates near the surface. Between flow reversal and stagnation, secondary vortices can be generated and convect downstream.



Figure 4.37: Instantaneous velocity snapshots at 7.4 m/s $\,$



Figure 4.38: Instantaneous velocity snapshots at 13.4 m/s

4.3.5 Quantitative analysis of the stall mechanism

This section presents a quantitative analysis of the unsteady flow-fields presented above. It is based on the Common-base Proper Orthogonal Decomposition (CPOD) approach, presented in chapter 2. This technique consists in a mathematical decomposition of the unsteady velocity field, resulting in a modal description of the data. Each mode features a spatial pattern and a time coordinate. The complete theoretical background of CPOD was presented previously in section 2.4 and only the specific implementation is described below.

The previous section showed that the flow mechanisms observed at 7.4 m/s and 13.4 m/s are similar. The analysis of CPOD outputs confirms this affirmation and allows to draw interesting conclusions about the mechanisms of flow separation, MIV convection and flow re-attachment around the oscillating rectangle.

The CPOD analysis is based on the concatenation of the time resolved velocity fields for 7.4 m/s and 13.4 m/s. For each airspeed, three periods of the pitching motion are used in the decomposition. Since the pitching frequency decreases with airspeed (see figure 4.27), the number of PIV snapshots retained for the CPOD analysis varies: 368 snapshots for 7.4 m/s and 391 snapshots for 13.4 m/s.

Rectangle motion

Since the model undergoes LCOs, some grid points in the PIV visualisation window find themselves instantaneously inside the rectangle, where the flow velocity is zero. This phenomenon introduces a strong discontinuity in the values of the flow velocity at these grid points, rendering the application of CPOD difficult. Nevertheless a physical meaning can be given to these points by introducing the concept of a solid velocity corresponding to the rigid body motion of the rectangle. There is a different velocity vector associated to every grid point inside the rectangle, given by

$$u_{i}^{s}(t_{k}) = C\dot{\alpha}(t_{k})(y_{i}^{k} - y_{f}) \qquad v_{i}^{s}(t_{k}) = -C\dot{\alpha}(t_{k})(x_{i}^{k} - x_{f})$$
(4.5)

where the superscript s stands for the solid velocity and the subscript i denotes any of the grid points of the PIV visualization window that are inside the model at the k^{th} time step. The position of the pitching axis of the rectangle is denoted by x_f and y_f . The coefficient C in equations 4.5 is a multiplying factor used in order to scale the solid velocities. Due to the low frequency of the pitching oscillation and the small dimensions of the model, the maximum solid velocity is much smaller than the mean flow airspeed. Considering the LCOs at 7.4 m/s, the pitching amplitude is 10° and the frequency is 8.1Hz. Hence the maximum solid velocity, before scaling, is equal to 0.35 m/s, i.e. less than 5% of the

free-stream velocity. The value of C is chosen as C = 20 in order to scale the solid velocity to the same order of magnitude as the fluid velocity.

The solid velocity concept is demonstrated schematically in the shapshots of figure 4.39 where the pitching angle is increasing (upstroke) and the instantaneous pitch angle of the model is equal to 9.4°. The left hand snapshot shows the situation without scaling, i.e. C = 1, while the right hand snapshot corresponds to C = 20. This scaling factor is the same for all the sets of data used in the CPOD analysis.



Figure 4.39: Solid velocities inside the model - not scaled (left) - scaled (right)

After applying solid velocities, the grid points inside the rectangle can be physically considered in the orthogonal decomposition. The resulting global matrix is composed of fluid and solid velocities, hence it is possible to draw some aeroelastic conclusions from the CPOD analysis.

CPOD results

Figure 4.40 shows three estimates of the first four spatial mode shapes, obtained from the POD of the flow-field at 7.4 m/s, the POD at 13.4 m/s and the CPOD of the velocity fields at both airspeeds. This figure shows that the first three mode shapes obtained by POD at 7.4 m/s and 13.4 m/s and by CPOD are very similar. There is a slight difference between the mode shapes of mode 4 identified at the two airspeeds. This difference is discussed in more detailed below. The similarity between the mode shapes at these two different airspeeds reinforces the observation made earlier that the flow separation and reattachment mechanisms are essentially the same. Consequently, the CPOD mode shapes can be used to describe and compare both datasets.

The first four CPOD modes contain 93.1% of the energy of the flow velocity signals and are considered sufficient to describe the torsional flutter phenomenon. The corresponding generalised coordinates are plotted in figures 4.41(b) to 4.41(e) for both airspeeds (solid line 7.4 m/s, dashed line 13.4 m/s) during three cycles of the motion. The corresponding



Figure 4.40: Comparison between POD modes and CPOD modes

pitch angle responses are plotted in the top of figure 4.41(a). On the basis of figures 4.40 and 4.41, it is possible to discuss the contribution of each mode to the unsteady velocity fields observed through PIV.

Mode 1

The first mode depicts mainly the state of the system when the flow is essentially irrotational, i.e. no vorticity is present in the observation window. While it is the most uniform of the modes, the horizontal velocity is faster near the top of the visualisation window and the vertical velocity is upwards near the leading edge and downwards near the trailing edge. Notice that the rectangle is immobile in this mode.

The local mean velocity is not subtracted from the measured PIV data, hence the first mode is equivalent to the mean flow, shown in figure 4.42. The resemblance of the mean flow to the first CPOD mode in figure 4.40(c) is obvious.

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Figure 4.42: Mean flow-field

The generalized coordinate of the first mode is constant and positive when the flowfield is attached and when the MIV is being generated. At 7.4 m/s, $q_1(t)$ takes its minimum value as the pitch angle decreases from its maximum, but this minimum the trough is not very low, highlighting the fact that the first mode participates in the flow separation. At 13.4 m/s, $q_1(t)$ reaches a much lower minimum (nearly zero) slightly earlier in the cycle, just as the pitch angle reaches its maximum. This means that the mode does not participate in the flow-field when separation begins (snapshots 6 to 9 in figure 4.38). When the flow starts to re-attach, at 50% and 35% of the cycle for 7.4 m/s and 13.4 m/s respectively, $q_1(t)$ recovers to its high nearly constant positive value, until the next vortex convection begins.

Mode 2



Figure 4.41: CPOD - Generalized coordinates - plain for 7.4 m/s and dashed for 13.4 m/s

The second mode depicts a counter-clockwise vortex centered over the body, combined with a nose-up motion of the rectangle. This pattern signifies that, when $q_2(t)$ is positive, the mode participates to the generation of the MIV, occurring at the beginning of the upstroke motion, i.e. when the pitching velocity is a maximum (see snaphots 1 to 5 of figure 4.37 and 1 to 4 of figure 4.38). On the other hand, when $q_2(t)$ is negative, the mode induces large recirculation close to the surface of the rectangle. The peak negative value of the generalized coordinate is reached at 35% of the period for 7.4 m/s. This instant of the cycle corresponds to the beginning of the separated state of the flow-field (snaphot 9 in figure 4.37). The situation is similar in the case of 13.4 m/s, with the occurrence of the negative peak at 20% of the cycle. Note that the width of this peak is more important at 13.4 m/s than at 7.4 m/s. Furthermore, a small local maximum is observed in the curve of $q_2(t)$ at 13.4 m/s (dashed curve in figure 4.41(c)).

Mode 3

The third mode depicts a nose-down motion of the rectangle with a counter-clockwise vortex near the leading edge and a large clockwise recirculation region near the trailing edge. The mode participates in flow reversal when $q_3(t)$ is positive and in MIV convection when it is negative. In the latter case, the MIV is present in the forward half of the rectangle while the flow-field is attached over the rear (see snapshots 4 to 7 of figure 4.37 and 3 to 4 of figure 4.38). The large velocity values observed in the mode shape play an important role in flow re-attachment over the rear of the rectangle. This point is reinforced by the large negative values of $q_3(t)$ at the end of the upstroke.

Mode 4

Unlike the first three modes, the POD modes corresponding to 7.4 m/s and 13.4 m/s are slightly different: in the case of 7.4 m/s, the fourth POD mode depicts two counterrotating vortices (see figure 4.40(j)), while only one appears clearly in the POD mode corresponding to 13.4 m/s (see figure 4.40(k)). Furthermore, the movement of the solid in the POD mode for 7.4 m/s is negligible, unlike the 13.4 m/s case. Because of the single basis feature of the CPOD method, the fourth CPOD mode contains the characteristics of both sets of data, with a clear resemblance with the POD calculated at 13.4 m/s. For this reason, the generalized coordinates corresponding to 7.4 m/s and 13.4 m/s will also show differences: in the case of 7.4 m/s, $q_4(t)$ is roughly constant and close to zero up to 50% of the cycle, where a maximum value is reached. The same maximum is reached at 50% of the cycle for 13.4 m/s, but in this case, a minimum is also observed in the curve of $q_4(t)$. According to these observations, it can be concluded that the fourth mode participates to the re-attachment process at both airspeeds but also in the vortex convection at 13.4 m/s.

Modal participation

Figure 4.43 presents a summary of the participation of each mode in the three different states of the flow-field defined in section 4.3.4. The timings of these states are shown in the upper part of figures 4.43(a) and 4.43(b) for airspeeds of 7.4 m/s and 13.4 m/s respectively.

The lower parts of these figures show four grayscale bars, one for each mode. The black and grey colors, correspond to positive and negative values of the generalized coordinates respectively. The white areas correspond to the instances when the effect of the mode is negligible, i.e. the generalized coordinate is positive or negative but its absolute value is low.

The following remarks can be made:

- Modes 1 and 2 participate in MIV generation at both airspeeds. Modes 1-3 participate in MIV convection at both airspeeds. At 13.4 m/s, the fourth mode also participates in MIV convection.
- Flow reversal is a combination of modes 1, 2 and 3 at 7.4 m/s. At 13.4 m/s, mode 1 does not participate.
- The re-attachement phase includes all the modes at 13.4 m/s while only the first and the fourth modes participate at 7.4 m/s.
- The attached flow is composed of modes 1 and 2, at both airspeeds.

Generally speaking, it is observed from figure 4.43 that the participation of the modes is more complex at 13.4 m/s than 7.4 m/s. Modal contributions are more diverse in the former case, with up to four modes participating in a single flow phase. Conversely, at 7.4 m/s all flow phases can be described by the combination of Mode 1 and one more mode. The two airspeed cases studied here lie on either side of the amplitude jump occurring at 9.4m/s, as seen in figure 4.23. This fact may be reflected in the differences in the degree of modal participation observed in the CPOD decompositions of the torsional flutter motions observed at these two airspeeds.

4.3.6 Role of the MIV

The results presented here indicate that the fundamental cause of the torsional flutter phenomenon is the MIV. This vortex is generated at the leading edge but increases in strength as it travels downstream. It becomes strongest as it clears the trailing edge, at which point it induces high reversed flow velocities on the rear half of the rectangle. As the



Figure 4.43: Participation of the CPOD modes over one period

pitching axis lies on the half-chord, the resulting low pressure induces a strong nose-down moment around this axis. This MIV-induced moment is so strong that it can force the rectangle to rotate to a negative pitch angle. Then, a MIV is created on the lower surface, while the flow is generally attached on the upper surface. As the lower surface MIV clears the trailing edge, it generates a strong nose-up moment and the motion continues.

This description of the torsional flutter phenomenon requires that the MIV is strong enough when it moves over the rear part of the rectangle to generate a strong nosedown moment. This moment is only applied over a short period of time (around 15% of the cycle's duration), corresponding to the 'reversal' phase of the flow in figures 4.43(a) and 4.43(b), so it acts as an impulse that adds energy to the motion. Two such impulses are applied over a cycle, one due to the MIV on the upper surface and one due to the MIV on the lower surface. Therefore, the MIV is the mechanism by which the structure absorbs energy from the free stream.



Figure 4.44: Amplitude vs Period of the LCOs

It is interesting to note that, as the airspeed increases, the MIV convects faster. One would expect that the frequency of the motion should increase. However, as shown earlier, the frequency of the motion slowly decreases with airspeed (see figure 4.27). Indeed, the nose-down moment is applied earlier at higher airspeeds but it is also stronger, so that the motion's amplitude is higher. The MIV is stronger, the reversed flow velocities higher, and the extent of the separated flow greater. As a consequence, the flow takes longer to re-attach. Figure 4.43 shows that the vortex convection stage is shorter at higher airspeeds but the re-attachment stage is longer. The vortex convection stage lasts for 35% of the cycle at 7.4 m/s but only 20% at 13.4 m/s. On the other hand, the flow re-attachment lasts for 20% of the cycle at 7.4 m/s and 30% at 13.4 m/s. The attached flow state is also slightly prolonged by 5% of the cycle at 13.4 m/s.

On balance, the period of the motion increases with airspeed instead of decreasing. The relationship between the period and the amplitude of the LCOs is shown in figure 4.44: it is clear that the period increases with amplitude, which is an image of the strength of the MIV. A linear relation can be established between the amplitude of the motion and the period of oscillation, as shown by the straight line in figure 4.44, which is a least squares linear curve-fit of the data.

4.3.7 Concluding remarks

This section presents the experimental investigation of the torsional flutter of a 4:1 rectangular cylinder. The complete aeroelastic behaviour is investigated with a strong emphasis of the cause of the phenomenon, which is analyzed through flow visualization: Tr-PIV measurements are performed on the model, under static and oscillating conditions.

These measurements highlight the complex behaviour of the unsteady flow over the upper surface and the creation and convection of large vortical structures termed Motion Induced Vortices (MIV). Because of the large pitching oscillations of the model, no Karman Vortices are observed during the flutter oscillations. Three main states of the flow-field are identified: vortex convection, flow separation and attached flow. The convection velocity of the MIV is equal to 20% of the free-stream velocity. The relative timings of the successive states over one period are established for both airspeeds.

A quantitative analysis of the PIV measurements, based on the CPOD method is also presented. The first four modes, containing 93.1% of the energy of the data are extracted from the unsteady flow-field. An interpretation of each mode, based on its mathematical properties, is proposed and its participation in the different physical states of the flow-field is discussed.

The generation and convection of the MIV, from the leading edge to the trailing edge of the rectangle, is identified as the fundamental cause of the torsional flutter phenomenon. A discussion of its role at two different airspeeds is presented, highlighting the self-excited characteristic of the phenomenon.

4.4 Estimates of the critical airspeed

Up to this point the dynamic behaviour of a bridge section and a rectangular cylinder undergoing torsional flutter has been studied. This section presents two types of criterion giving an estimate of the critical airspeed for torsional flutter. The first type is devoted to the quasi-steady approach of the pitching system. It is shown that its application has no physical meaning and leads to inaccurate results. The second type presents a criterion based on phenomenological observations concerning the timing of the MIV described in section 4.3.6.

4.4.1 Quasi-steady critical airspeed

Quasi-steady theory can be used to model the aerodynamic forces applied on the oscillating body, providing an estimate of the critical airspeed of the torsional flutter instability. After presenting the main equations of the theory, different approaches are applied to both structures (bridge and rectangle) and the results are compared to the experimental measurements presented in sections 4.2 and 4.3.

According to the quasi-steady theory, the equation of motion 4.1 can be expressed as

$$I_{\alpha}\ddot{\alpha} + c_{\alpha}\dot{\alpha} + k_{\alpha}\alpha = \frac{1}{2}\rho V^2 B^2 C_M(\alpha, \dot{\alpha})$$
(4.6)

where $C_M(\alpha, \dot{\alpha})$ denotes the aerodynamic moment coefficient relative to the pitching axis of the body for the oscillating model. This instantaneous coefficient depends on the position and the pitching velocity of the structure at instant t only, i.e. no memory effect is considered by the quasi-steady approach [109].

For small variations of α around the equilibrium position, it is possible to approximate $C_M(\alpha, \dot{\alpha})$ from the knowledge of the static aerodynamic moment coefficient C_M :

$$C_M(\alpha, \dot{\alpha}) = \frac{\partial C_M}{\partial \alpha_s} \alpha_{eff}(\alpha, \dot{\alpha})$$
(4.7)

where $\alpha_{eff}(\alpha, \dot{\alpha})$ denotes the effective instantaneous angle of attack of the oscillating body.

The difficulty of the quasi-steady approach of pitching structures is that the effective angle α_{eff} depends on the position λ , where it is evaluated. This non-dimensional quantity is measured relative to the geometric center of the body and varies positively towards the leading edge and negatively towards the trailing edge (see figure 4.45).



Figure 4.45: Position of the aerodynamic center

Restraining the approximation of $C_M(\alpha, \dot{\alpha})$ to the first order, it can be expressed by

$$C_M(\alpha, \dot{\alpha}) = \frac{\partial C_M}{\partial \alpha_s} \left(\alpha - \lambda \frac{B \dot{\alpha}}{V_\infty} \right)$$
(4.8)

Note that the airspeed V appearing in equation 4.6 also depends on λ . Nevertheless, the term V^2 is commonly approximate by $V^2 = V_{\infty}^2$ to the first order [3,81].
Different possibilities for the values of λ can be envisaged, according to the type of application:

- For an airfoil with attached flow (small angle of attack), the aerodynamic center is used to build the aeroelastic model of an airfoil free to oscillate around its pitching and heaving dof's [156]. Its position is denoted by the non-dimensional quantity r. According to the thin airfoil theory [168], it is assumed that r is independent of α and its value is equal to 1/4 (using the sign convention above).
- For rectangular cylinders, Nakamura proposed to used the half chord for λ [109]. Hence the effective angle of attack is evaluated at the leading edge of the structure. According to the sign convention $\lambda = 1/2$ and it is considered independent of α .
- For bridge applications, Cremona [169] proposed to used the position of the aerodynamic center, similarly to the airfoil approach, with an additional assumption which is detailed below.

Remark that for quasi-steady aerodynamics based on thin airfoil theory [156], there is an additional rear aerodynamic center, which lies at $\lambda = -1/4$, and which is not taken into account by either Nakamura or Cremona.

The critical torsional flutter airspeed can be estimated using any of the choices for λ described above. The resulting expressions will be applied here to two bridge sections and the rectangular cylinder in order to compare the quasi-steady predictions to the experimental results.

Aerodynamic center

The aerodynamic center is a static concept, which defines a position where the static moment coefficient, denoted C_{M_F} , is independent of the angle of attack. This position is denoted by $r(\alpha_s)$ with the same sign convention as λ . Note that the notation α_s is chosen, in accordance with the rest of the thesis, in order to insist on the static feature of the proposed quasi-steady model.

The moment coefficient around the aerodynamic center is calculated using

$$C_M = C_{M_F} + r(\alpha_s)C_z \to C_{M_F} = C_M - r(\alpha_s)C_z \tag{4.9}$$

In this equation C_z is the static lift coefficient defined normally to the chord of the body:

 $C_z(\alpha_s) = C_D \sin \alpha_s + C_L \cos \alpha_s$. Hence, from the definition of the aerodynamic center:

$$\frac{\partial C_{M_F}}{\partial \alpha_s} = \frac{\partial C_M}{\partial \alpha_s} - \frac{\partial r(\alpha_s)}{\partial \alpha_s} C_z(\alpha_s) - r(\alpha_s) \frac{\partial C_z(\alpha_s)}{\partial \alpha_s} = 0$$
(4.10)

Inspired by the thin airfoil theory, Cremona and Fouriat proposed to retain the assumption of the independence of the position of the aerodynamic center with angle of attack, and to apply it in the case of bridge deck sections [169]. Hence $\frac{\partial r(\alpha_s)}{\partial \alpha_s} = 0$ and equation 4.10 yields,

$$r^*(\alpha_s) = \frac{\frac{\partial C_M}{\partial \alpha_s}}{\frac{\partial C_z}{\partial \alpha_s}} \tag{4.11}$$

where the star superscript is used to mark the difference between the exact expression of $r(\alpha_s)$ from equation 4.10 and the approximate solution from equation 4.11.

Conversely, if the position of the aerodynamic centre is allowed to vary with static angle of attack, the resolution of the differential equation 4.10 leads to an analytical expression for $r(\alpha_s)$ assuming that the position of the aerodynamic center varies with the angle of attack, i.e. $\frac{\partial r(\alpha_s)}{\partial \alpha_s} \neq 0$. In this equation, C_M is known and C_z is deduced from the knowledge of C_L and C_D ($C_z(\alpha_s) = C_D \sin \alpha_s + C_L \cos \alpha_s$). The quantities C_M , C_L and C_D are static aerodynamic coefficients, available from the experiments for the bridge section (section 4.2.3) and from the literature for the rectangular cylinder [109]. Hence a polynomial expression can be fitted to each curve $C_M(\alpha_s)$ and $C_z(\alpha_s)$:

$$C_M = \sum_{k=0}^N A_k \alpha_s^k \qquad \qquad \frac{\partial C_M}{\partial \alpha_s} = \sum_{k=0}^N A_k k \alpha_s^{k-1} \qquad (4.12)$$

$$=\sum_{k=0}^{N} B_k \alpha_s^k \qquad \qquad \frac{\partial C_z}{\partial \alpha_s} = \sum_{k=0}^{N} B_k k \alpha_s^{k-1} \qquad (4.13)$$

The coefficients A_k and B_k can be evaluated in a least squares sense from the measured aerodynamic coefficients. They are presented in Appendix F for the two structures studied in this work.

Using expressions 4.12 and 4.13 in equation 4.10 yields,

 C_z

$$\sum_{k=0}^{N} \left[A_k k \alpha_s^{k-1} - \frac{\partial r(\alpha_s)}{\partial \alpha_s} B_k \alpha_s^k - r(\alpha_s) B_k k \alpha_s^{k-1} \right] = 0$$
(4.14)

Expression 4.14 is a differential equation of the first order. The analytical solution, derived

in Appendix E, is

$$r(\alpha_s) = \frac{r_0 + \sum_{k=1}^{N} A_k \alpha_s^k}{\sum_{k=0}^{N} B_k \alpha_s^k}$$
(4.15)

where r_0 is a constant, defined as $r(\alpha_s = 0) = \frac{r_0}{B_0}$. It is necessary to choose a value of r when the angle of attack α_s is equal to zero. It is proposed to select the thin airfoil assumption, hence $r_0 = B_0/4$.

Four expressions are available for the value of λ in equation 4.8: $\lambda = 1/2$, $\lambda = 1/4$, $\lambda = r^*(\alpha_s)$ and $\lambda = r(\alpha_s)$. From equations 4.6 and 4.8, the first order equation of motion is

$$I_{\alpha}\ddot{\alpha} + c_{\alpha}\dot{\alpha} + k_{\alpha}\alpha = \frac{1}{2}\rho V_{\infty}^{2}B^{2}\frac{\partial C_{M}}{\partial\alpha_{s}}\left(\alpha - \lambda\frac{B\dot{\alpha}}{V_{\infty}}\right)$$
(4.16)

The torsional flutter instability occurs when the total damping vanishes, i.e.

$$c_{\alpha} + \frac{1}{2}\rho V_{\infty} B^3 \frac{\partial C_M}{\partial \alpha_s} \lambda = 0$$
(4.17)

leading to the critical airspeed

$$V_{crit} = \frac{-2c_{\alpha}}{\rho B^3 \frac{\partial C_M}{\partial \alpha_s} \lambda}$$
(4.18)

Equation 4.18 corresponds to the critical airspeed of torsional flutter when the system starts from rest, i.e. no initial displacement is applied. It is based on the static aerodynamic forces and moments, through $\frac{\partial C_M}{\partial \alpha_s}$ and λ , which depend on the static equilibrium angle α_s . This expression is equivalent to Den Hartog's criterion, used for the galloping phenomenon and presented in chapter 3. The main difference comes from the term λ in equation 4.18, which has a strong influence on the resulting estimation, as shown below.

The predictions of the quasi-steady theory, calculated through the four different choices for λ are presented in the next sections.

Application to the generic bridge section

In the case of the generic bridge section, the equilibrium static angle α_s is expressed in section 4.2.3 through equation 4.4: a different static angle α_s corresponds to each value of V_{∞} . For example the static deflection is equal to 3.4° at 11.7m/s (point H in figure 4.6).

Figure 4.46 plots the flutter speed estimates from the four different choices for the value of λ . The vertical axis of each plot corresponds to the flutter estimate and the horizontal axis is the static angle of attack α_s . This figure corresponds to the case of the generic bridge deck.

It is observed from figure 4.46 that the situation is rather similar for λ equal to 1/2



Figure 4.46: Critical airspeed from quasi-steady theory - Generic bridge section

and 1/4: no instability is predicted by equation 4.18 for negative angles of attack. When α_s is positive, a U-shaped curve is observed in the top right corner of figures 4.46(a) and 4.46(b). The minimum value of these curves is reached for $\alpha_s = 6.2^{\circ}$ and the critical airspeeds are 29.3m/s and 58.7m/s for $\lambda = 1/2$ and $\lambda = 1/4$, respectively. Note that the ratio of two between these airspeeds is simply due to the corresponding constant values chosen for λ . This highlights the strong and arbitrary dependence of λ on the predicted airspeed.

Figure 4.46(c) is similar to figures 4.46(a) and 4.46(b) for positives values of α_s . However, for negatives α_s , the choice $\lambda = r(\alpha_s)$ leads to instability: for $\alpha_s = -5^\circ$, the prediction states that the system is unstable a zero airspeed, which has no physical meaning.

As stated above, the static angle of the bridge varies with airspeed. The positive value of the static moment coefficient (figure 4.2) is responsible for the positive static angles measured during experiments. Hence, only the positive values of α_s are of interest

in figure 4.46. It is shown in section 4.2.4 that the Hopf airspeed is identified around 13.5 m/s, starting from a static equilibrium position of 4°. The corresponding predictions for the critical airspeeds from the choices of λ equal to 1/2, 1/4 and $r(\alpha_s)$ are 29.3m/s, 58.7m/s and 73.9m/s respectively. Hence it can be concluded that these choices for the value of λ fail to yield a good estimate of the critical airspeed of torsional flutter.

The situation is different when considering the choice of Cremona, i.e. selecting the position of the aerodynamic center with the additional assumption that the latter does not vary with the angle of attack: $\lambda = r^*(\alpha_s)$. The resulting critical airspeed is plotted versus α_s in figure 4.46(d). A vertical curve is observed for positive static angles of attack, around 4°. From this observation, it could be argued that the use of $\lambda = r^*(\alpha_s)$ fails to estimate correctly the critical airspeed.



Figure 4.47: Critical airspeed of the bridge section using $\lambda = r^*(\alpha_s)$

Figure 4.47 presents the critical airspeed as a function of the free-stream velocity. The free-stream velocity corresponding to the value of α_s of figure 4.46(d) is computed from equation 4.4. The equation of the dashed line in this figure is simply $V_{crit} = V_{\infty}$. The intersection between this line and the plain curve corresponds to the critical airspeed of 13.3m/s, which is close to the experimental Hopf airspeed (13.5m/s).

From this latter result, it could be argued that the quasi-steady approach proposed by Cremona gives a good estimate of the critical airspeed of the torsional flutter phenomenon. Nevertheless, a deeper analysis shows that this result is fortuitous and could not be applied to the rectangle cylinder.

Application to the rectangular cylinder

Because of the symmetry of the rectangular cylinder, the static equilibrium angle is zero and the discussion of the validity of the quasi-steady estimation of the critical airspeed is slightly different from above. It is possible to propose an alternative interpretation remembering that two different types of tests have been performed during the experiments in the wind tunnel: turbulence excitation tests and initial conditions tests (see section 4.3.2). Hence the static angle α_s appearing in expression 4.18 may have two values:

- zero in the case of turbulence excitation tests.
- the value of the initial angle applied by the operator during initial conditions tests.

Nevertheless, the quasi-steady theory is only valid around the equilibrium position. Hence the second type of test (initial condition test) must be discarded from this discussion. Only the turbulence excitation tests are discussed herein.



Figure 4.48: Critical airspeed from quasi-steady theory - Rectangular cylinder

Figures 4.48(a) and 4.48(b) show that the predicted critical airspeeds are equal to 21.3m/s and 42.6m/s respectively. Figure 4.48(c), corresponding to the exact solution for the aerodynamic center, $\lambda = r(\alpha_s)$, states that the equilibrium position $\alpha_s = 0$ is unstable at zero airspeed, which is not physically possible. Finally, figure 4.48(d) shows that the choice $\lambda = r^*(\alpha_s)$ does not predict instability for an airspeed lower than 100m/s.

Recalling that the Hopf airspeed identified experimentally during turbulence excitation tests is equal to 14.6m/s, the four predictions of the critical airspeed are wrong, as far as turbulence tests are concerned. Note that the turbulence excitation tests correspond to the assumption that no initial displacement is applied to the structure, i.e. starting from rest, as stipulated in Den Hartog's criterion, applicable for the onset of the galloping phenomenon.

Discussion

In the previous sections it is demonstrated that the quasi-steady theory does not represent a reliable method to estimate the critical airspeed of torsional flutter. It is shown that the choices $\lambda = 1/2$, $\lambda = 1/4$ and $\lambda = r(\alpha_s)$ lead to wrong predictions for the two structures tested. Nevertheless, it is shown that the method of Cremona yields good results for the generic bridge section and erroneous predictions for the rectangle.

Figure 4.49 shows the curves of $r^*(\alpha_s)$ for the two structures of interest. In the case of the generic bridge, an asymptote is observed at the static angle for which the torsional flutter is predicted ($\alpha_s = 4^\circ$). This singularity is obviously due to the definition of $r^*(\alpha_s)$ and the fact that the slope of C_z at this angle is equal to zero (see Appendix F). Hence the estimated critical airspeed, which is inversely proportional to $r^*(\alpha_s)$, is equal to zero, as shown in figure 4.46(d). Because of the non-zero static deflection, it was possible to propose a physical meaning to that zero critical airspeed (figure 4.47).

The curve of $r^*(\alpha_s)$ for the rectangular cylinder also shows asymptotes, but because of its symmetric shape, no static deflection is measured.

It can be concluded from the present discussion that the aerodynamic center has no physical meaning in the case of separated flow-fields. Its choice as the reference position for the calculation of the effective angle of attack is not supported by any rational argument. The complexity of the separated flow-field around a bluff structure does not allow this choice.

Finally, another criterion, based on the quasi-steady theory is discussed. It concerns a simple statement about the sign of the derivative of the moment coefficient. According to [170], the structure is torsionally stable if $\frac{\partial C_M}{\partial \alpha_s} > 0$. This simplistic criterion is in fact based on equation 4.18, indicating that the critical airspeed calculated by this equation is physical when positive. As stated by Nakamura: such an explanation is too simple



Figure 4.49: Position of the aerodynamic center: $r^*(\alpha_s)$

to be accepted, although in some experiments the criterion has been shown to be fairly reasonable [109]. Regarding the moment coefficient of the two structures discussed in this chapter (see figure F.1 in Appendix F), it could be argued that the criterion is satisfied. Indeed, for the static angles where torsional flutter occurs, the slope of C_M becomes negative. Nevertheless, because of the sub-critical characteristic of torsional flutter, if sufficiently excited, the system can jump to a stable LCO, even if $\frac{\partial C_M}{\partial \alpha_s} > 0$, as was shown in the case of the generic bridge deck in section 4.2. Hence, this criterion can be considered as a condition sufficient but not necessary. This restriction was also discussed by Novak [161] concerning Den Hartog's criterion, which deals with the galloping of bluff structures (see chapter 3)

This final comment adds to the arguments presented in this thesis against the use of quasi-steady theory for the estimation of the critical onset airspeed of torsional flutter. The author believes that fluid memory (neglected in the quasi-steady approach) is a key factor in the modelling of the aerodynamic forces around an oscillating bluff-body [109].

4.4.2 Empirical criteria

Several authors attempted to develop simplified stability criteria for the onset of torsional flutter based on phenomenological observations [106, 167]. The discussion presented by Larsen [106] is based on the timings of the MIVs of the upper and lower surfaces of the

bluff-body, when undergoing pitching oscillations. It is stated that no torsional flutter instability occurs when the distance between the upper surface MIV and the lower surface MIV is higher than the half chord of the body (B/2).

We propose to apply this simplified model to the experimental observations of the rectangular cylinder. It was shown in section 4.3 that a MIV is generated on the upper surface, at the beginning of each pitching period, hence the above mentioned condition is equivalent to

$$T_c > T/2$$

where T_c denotes the time needed for the MIV to convect from the leading edge to the mid-chord of the model. Using the non-dimensional reduced airspeed $U = V_{\infty}/fB$, where f denotes the pitching frequency (f = 1/T), in the condition above yields,

$$T_c > \frac{BU}{2V_{\infty}} \tag{4.19}$$

Recall that the convection airspeed of the MIV, V_c , is equal to 20% of the free-stream velocity, i.e. $V_c = 0.2V_{\infty}$, hence

$$V_c T_c = \frac{B}{2} \to 0.2 V_\infty T_c = \frac{B}{2} \to T_c = \frac{B}{0.4 V_\infty}$$

This expression of T_c is replaced in the stability criterion 4.19, leading to

$$\frac{B}{0.4V_{\infty}} > \frac{BU}{2V_{\infty}} \to U < 5 \tag{4.20}$$

The resulting critical airspeed corresponds to the equality

$$U = \frac{V_c}{fB} = 5 \tag{4.21}$$

Note that in the work of Larsen, dealing with the failure of the Tacoma Narrows bridge [106], the convecting velocity was identified between 25-27% of the oncoming free-stream velocity, hence the resulting critical airspeed (equation 4.21) is expressed as

$$U = \frac{V_c}{fB} = 3.6 - 4. \tag{4.22}$$

In their works, Shiraishi and Matsumoto [167] carried out the same type of phenomenological observations resulting in the following expression

$$U' = \frac{V_c}{fD} = \frac{2}{2N - 1} 1.67 \frac{B}{D}$$

where N = 1, 2, ... and the prime symbol is used to highlight the different definition of the reduced airspeed of Shiraishi and Matsumoto. Adapting this definition to the one used in this thesis, one finds:

$$U = \frac{V_c}{fB} = \frac{2}{2N - 1} 1.67 \tag{4.23}$$

This expression becomes U = 3.34 when N takes the value of 1. The three expressions of the critical reduced airspeed U, 4.21, 4.22 and 4.23 are equivalent and the constants on the right hand side are similar.

It is observed from the development of the the critical airspeed U estimates mentioned above, that the structural damping, always present in any engineering system, is not considered. In fact the simplified model presented here assumes that the damping is zero [106, 167].

All the results available from the experimental investigations of sections 4.2 and 4.3, together with the results of the above mentioned authors [77, 106, 167] are tabulated in table 4.5. The table allows the comparison of the experimental observations with the prediction of the empirical formulas.

Models		В	D	f_0	ξ_0	Sc	V_{Hopf}	V _{fold}	$\frac{V_{Hopf}}{fB}$	$\frac{V_{fold}}{fB}$
		[m]	[m]	[Hz]	[%]	[-]	[m/s]	[m/s]	[-]	[-]
(a1)	Bridge S1 (§4.2.3)	0.317	0.1	2.1	1.6	243	13.5	6.3	20	9
(a2)	Bridge S2 (§4.2.3)	0.317	0.1	2.1	1.8	273	14.9	6.3	17	7.5
(b)	Rectangle $(\S4.3.3)$	0.08	0.02	8.6	2.7	2631	14.6	6.7	21	10
(c1)	Real Tacoma [106]	11.9	2.4	0.2	1.0	547	19.0		8.	0
(c2)	Exp. Tacoma [106]	NA	NA	NA	7.0	4.0	NA	NA	12	.4
(d1)	Exp. Tacoma [77]	0.297	0.061	0.995	0.06	77	0.68		2.3	
(d2)	Exp. Tacoma [77]	0.297	0.061	2.1	2.7	750	2.7		4.4	
(e)	Exp. rectangle [78]	0.3	0.075	4.9	0.2	7.9	2.	6	7.	0

Table 4.5: Empirical critical airspeed, comparison between models

The first three lines of table 4.5 present both Hopf and fold airspeeds, due to the sub-critical behaviour observed experimentally in sections 4.2.3 and 4.3.3.

The Scruton number, Sc, appearing in the sixth column of the table is defined as $Sc = 2I_{\alpha}\delta_0/\rho D^4$, where δ_0 denotes the decremental damping, $\delta_0 = 2\pi\xi_0$. This nondimensional number characterizes the amount of damping in the system with respect to the inertia of the latter. It was introduced in chapter 1, where it is usually employed in order to quantify the susceptibility of a structure to undergo VIVs. Nevertheless, it is useful in the case of torsional flutter too, as shown in the work by Matsumoto et al. [77], where the effect of aerodynamic interference between heaving and torsional vibrations is investigated. In this work, a new explanation of the collapse of the Tacoma Narrows bridge is proposed. Through wind tunnel experiments, each single dof (heaving and torsion) was first tested separately, then a two dofs tests were carried out. These tests have been performed for different values of the Scruton number and concluded that the heaving and torsional dofs interfere, depending on the value of the Scruton number. The models (d1) and (d2) in table 4.5 correspond to single dof results in torsion for two values of Sc (77 and 750), where torsional flutter oscillations were observed. It is clear from these results that the critical value of equation 4.21 strongly depends on the Scruton number and hence on the structural damping of the system.

The same observations were made by Larsen when studying the Tacoma section in water tunnel experiments: the large structural damping, 7%, reported in line (c2) of table 4.5, leads to a Scruton number of $4.0^{(1)}$ and the deck undergoes torsional flutter for a reduced critical airspeed equal to 12.4, while his simplified model stated a maximum value of 4 (equation 4.22).



Figure 4.50: Critical reduced airspeed vs Scruton number

Generally speaking, it is observed from the last two columns of table 4.5 that the corresponding reduced airspeeds $\frac{V}{fB}$ vary strongly between the eight cases presented in this table. This fact is reinforced by figure 4.50, which shows the values of the critical ratio V/fB as a function of the Scruton number. It is obvious from this figure that

⁽¹⁾Note that the Scruton number is calculated with the water density, $\rho = 1000 kg/m^3$ in this case, hence it results in a small Sc.

the constant value ranging between 3.34 and 5 in equations 4.21, 4.22 and 4.23 does not compare well with the experimental results.

This section demonstrates the difficulty to estimate a critical airspeed for the torsional flutter phenomenon. It is in agreement with the statements of section 4.3.4 concerning the estimation of the Strouhal number based on phenomenological observation of the KV. Regarding the large dispersion of the experimental data in figure 4.50, it seems difficult, if not impossible, to summarize the torsional flutter phenomenon into a simple formula, even in attempting to include the Scruton number. Hence, it is concluded that this type of formula must be used with care, even if the shape of the bluff-body is simple.

4.5 Chapter summary

This chapter presents an extensive investigation of the torsional flutter phenomenon. Two different structures are studied:

- The generic bridge deck section;
- A 4:1 rectangular cylinder.

Despite the different geometrical shapes, it is shown that the main characteristics of torsional flutter for both structures are comparable. Hence, these two experimental investigations can be considered as complementary.

The generic bridge deck section presented in the previous chapter for the analysis of the galloping instability is now free to oscillate around its pitching degree of freedom. The dynamic of the aeroelastic system is investigated, showing a hysteretic loop in the bifurcation diagram. Furthermore, it is shown that the aerodynamic forces are delayed compared to the pitching motion of the deck. This nonlinear characteristic is found responsible for the LCOs observed during experiments. Because of the large size of the wind tunnel model, it is not possible to carry out flow visualization. Consequently, another experimental set-up was designed and tested.

This second structure consists in a 4:1 rectangular cylinder. The complete aeroelastic behaviour of the rectangle is identified, showing the same hysteretic loop as the bridge section. Furthermore, additional initial condition tests are performed in order to identify the unstable part of the limit cycle branch. The design of this experimental set-up was realized with the objective to perform flow measurements using Tr-PIV technique. The small size of the model enables the measurement of the velocity field over the entire upper surface of the rectangle. This valuable set of data is analyzed over one cycle, for two different airspeeds where LCOs are observed. A quantitative analysis based on the CPOD technique presented in chapter 2 is carried-out. It is concluded that the Motion Induced

Vortices (MIV) convecting from the leading edge to the trailing edge of the rectangle are responsible for the torsional flutter oscillations. A discussion of the role of the MIVs at two different airspeeds is presented, highlighting the self-excited characteristic of the phenomenon.

Finally, this chapter presents different methods to estimate the onset airspeed of torsional flutter. Quasi-steady and empirical simplified models are presented and compared to the experimental results. In addition, other results from the Tacoma Narrows bridge are integrated into the discussion. It is concluded that both the quasi-steady and the empirical approaches fail to give a reliable estimate of the onset airspeed of the torsional flutter phenomenon. Despite the attractiveness of these simplified approaches, they must be used with care, because the aerodynamics leading to torsional flutter oscillations is highly sensitive to many details (geometrical or structural).

Chapter 5

Numerical simulations

5.1 Introduction

This final chapter presents a numerical tool capable of simulating the aeroelastic behaviour of 2D bluff bodies. The aerodynamic part of the model is based on a Discrete Vortex Method (DVM), introduced in the early 80's by Leonard [171]. It is demonstrated that DVM is well suited for modelling the flow around bluff bodies, which are characterized by flow separation and re-attachment processes, as discussed all throughout this thesis.

Aeroelastic responses can be simulated by coupling an aerodynamic model to a structural model, so that one acts as a feed-back input to the other through the modification of the boundary conditions of the fluid domain. The results of simulations described in this chapter are compared quantitatively and qualitatively to the experimental flow visualisation and aeroelastic systems presented in the previous chapters. The following aerodynamic and aeroelastic systems are simulated:

- A 4:1 rectangular cylinder, free to vibrate along its pitch dof (cfr section 4.3)
- A circular cylinder, oscillating along its heave dof (cfr chapter 2)
- A static bridge section (cfr chapter 3)

Discrete Vortex Methods have been used in the past to simulate unsteady separated flow-fields by a number of researchers [172–175]. Basically, a vortex method is a Lagrangian approach which consists of tracking vortical particles shed from the surface of a body immersed in a flow domain. These methods represent an alternative to grid based schemes, such as finite difference/volume/element approaches. Vortex methods satisfy exactly the boundary conditions for external flows, i.e. internal boundaries only, undisturbed at infinite distance from the body. Some very complete overviews of vortex methods can be found in Leonard [171] or Spalart [173]. An interesting and critical review of these methods is proposed in a recent paper by Voustinas [176].

A DVM is selected in the scope of this thesis because it has the following advantages:

- No meshing of the flow domain is needed, which is an important advantage when dealing with moving bodies: only the body is discretized by a limited number of panels similarly to the classic steady panel method [17].
- No pre-specification of separation points is required, which represents a strong advantage over other implementations of the method (see for example [172] or [174]).

The main drawback of the DVM is the important computational cost induced by the calculation of the interaction of vortices on each other, which is typically proportional to the square of the number of vortices. This may become problematic when the number of vortices representing the wake is important. Researchers developed algorithms to reduce the number of wake vortices or to accelerate the way to compute their mutual interactions [177].

The present implementation of the DVM is based on the works of Morgenthal [87], Walther [178], Lin [179] and Andrianne [180, 181]. The theoretical background and implementation issues are introduced, then the numerical applications mentioned above are presented with an emphasis on the case of the torsional flutter of the rectangle, since the simulation tool was developed with this particular application in mind [181].

5.2 Discrete Vortex Method

The theoretical background and the main characteristics of the method are briefly presented below, paying particular attention on the specific implementation issues. The numerical parameters are presented and the choice of their values is discussed for each application.

5.2.1 Theoretical background

Under the assumption of 2D, incompressible flow at constant kinematic viscosity ν , the Navier-Stokes equation can be expressed as the vorticity transport equation:

$$\frac{\partial\omega}{\partial t} + (\mathbf{u} \cdot \nabla)\,\omega = \nu \nabla^2 \omega \tag{5.1}$$

where **u** denotes the velocity vector and $\boldsymbol{\omega}$ is the vorticity vector. In 2D, the vorticity vector $\boldsymbol{\omega}$ becomes a scalar quantity: $\boldsymbol{\omega} = \nabla \times \mathbf{u} = \omega \mathbf{e}_z$, where \mathbf{e}_z is normal to the plane of the flow-field.

The vortex method solves equation 5.1 in a Lagrangian framework: at each time step, vortical particles are released in the flow-field. In order to perform the convection process, the knowledge of the local velocity at the position of each particle is necessary. Considering the incompressibility condition $\nabla \cdot \mathbf{u} = 0$, the velocity vector \mathbf{u} can be expressed through the stream function Ψ according to:

$$\mathbf{u} = \nabla \times \Psi \mathbf{e}_z + \mathbf{U}_\infty$$

where \mathbf{U}_{∞} is the free-stream velocity. The Poisson equation $\nabla^2 \Psi = -\omega$ relates the vorticity to the stream function. The solution of this equation allows the expression of the stream function $\Psi(\mathbf{x})$ as a function of the vorticity:

$$\Psi(\mathbf{x}) = \Psi_{\infty} - \frac{1}{2\pi} \iint_{D} \log |\mathbf{x} - \mathbf{x}'| \,\omega(\mathbf{x}') d\mathbf{x}'$$

where \mathbf{x} denotes the spatial coordinates in the plane of the flow-field and D denotes the flow-field domain. The velocity vector follows from the definition of the stream function:

$$\mathbf{u}(\mathbf{x}) = \mathbf{U}_{\infty} - \frac{1}{2\pi} \iint_{D} \frac{(\mathbf{x} - \mathbf{x}') \times \omega(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2} d\mathbf{x}'$$
(5.2)

Equation 5.2 is commonly referred to the Biot-Savart relation.

The calculation of the vorticity and velocity is carried out by enforcing the impermeability kinematic boundary condition on the surface of the body: $\mathbf{u}(\mathbf{x}) \cdot \mathbf{n} = 0$, on the surface A_b , where \mathbf{n} denotes a unit vector normal to the surface. The impermeability condition can be expressed in terms of vorticity only, using equation 5.2:

$$\frac{1}{2\pi} \iint_{A_b} \frac{(\mathbf{x} - \mathbf{x}') \times \omega(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2} d\mathbf{x}' = \mathbf{I}(\mathbf{x}_{A_b}) + \mathbf{U}_{\infty}$$
(5.3)

where \mathbf{x}_{A_b} are the coordinates of the surface and $\mathbf{I}(\mathbf{x}_{A_b})$ denotes the velocity induced by the wake and the motion of the body. By defining the surface vortex sheet γ , which is also a scalar quantity: $\frac{\partial \gamma}{\partial n} = \omega$, equation 5.3 becomes:

$$\frac{1}{2\pi} \int_{B} \frac{(\mathbf{x} - \mathbf{x}') \times \gamma(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^2} dB = \mathbf{I}(\mathbf{x}_{A_b}) + \mathbf{U}_{\infty}$$
(5.4)

Equation 5.4 contains the unknown γ and its solution is unique up to a constant. The solution is made unique by enforcing an additional condition, known as Kelvin's theorem. This condition states that the total amount of vorticity in the domain (fluid and body)

is constant:

$$\frac{d}{dt} \iint_D \omega d\mathbf{x} = 0 \tag{5.5}$$

Equations 5.4 and 5.5 form the basis of the vortex method implemented here. Diffusion is modelled using the random walk method proposed by [182].

The aerodynamic forces applied on the body are computed from the integration of the pressure along the surface. Several authors proposed to compute the surface pressure from the vorticity flux through the surface [183,184]. This approach is based on the expression

$$\frac{\partial p}{\partial s} = \nu \frac{\partial \omega}{\partial n}$$

so that, using the definition of the surface vorticity γ , one finds

$$\frac{\partial p}{\partial s} = -\rho \frac{\partial \gamma}{\partial t} \tag{5.6}$$

This last expression can be used to compute the unsteady pressure distribution as a function of the surface vorticity.

5.2.2 Discretization of the flow-field

The key feature of DVM is that vortical particles are present in the region of the flow-field where vorticity arises, i.e. in the boundary layer and in the wake of the body. At large Reynolds numbers, the rest of the flow-field is potential, as discussed in chapter 1.

Because of the singular nature of these vortical particles, also denoted point vortices [173], numerical instabilities can arise if two or more come too close (very large, unphysical induced velocities). The solution frequently adopted in DVM consists in using blob vortices that are smoothed vortex particles instead of point vortices. Mathematically, the velocity induced at a point (x_j, y_j) by a blob vortex located at (x_i, y_i) is expressed by

$$\begin{pmatrix} u_{j,i} \\ v_{j,i} \end{pmatrix} = \frac{\Gamma_i}{2\pi d^2} \begin{pmatrix} y_i - y_j \\ x_j - x_i \end{pmatrix} \left[1 - \exp(-\frac{d^2}{\sigma_c^2}) \right]$$
(5.7)

where $d^2 = (x_j - x_i)^2 + (y_j - y_i)^2$ and Γ_i is the surface integral of the vorticity of the i^{th} vortex blobs, defined by $\Gamma_i = \int_{S_i} \omega dS_i$. The parameter σ_c appearing in this equation denotes the cut-off distance, which is an important parameter of vortex methods.

The continuous vorticity field $\omega(\mathbf{x}, t)$ is represented by the sum of all the vortex blobs present in the flow-field. If N_k vortical particles are present at the k^{th} time step, the vorticity field is expressed by:

$$\omega(\mathbf{x}, t_k) = \sum_{i=1}^{N_k} \delta(\mathbf{x} - \mathbf{x}_i(t_k)) \Gamma_i$$
(5.8)

where δ denotes the kronecker delta.

5.2.3 Implementation

As presented above, the DVM is a time stepping method that could be roughly described as an unsteady panel method: at every time step, the system is defined by its kinematics, i.e. the displacement and velocity of the body along its degrees of freedom. Similarly to the experimental systems, the body itself is rigid and the motion is only possible through its pitching (α) and/or heaving (h) dofs.

The flow chart in figure 5.1 presents the different steps of the DVM implemented in this thesis. They appear inside the dashed rectangle entitled DVM in the figure. The step number 10, corresponding to the structural model, is presented in section 5.3. The drawing on the left of each step shows the corresponding conceptual situation around a rectangular body.

Step 1. Discretization

The surface of the body is represented by N_p panels, as shown in figure 5.2 for a rectangular cylinder. In this figure, \mathbf{x}_i and \mathbf{x}_{i+1} denote the end-points of panel i and \mathbf{x}_c^i is the corresponding collocation point defined in the middle of each panel. The normal vector to the i^{th} panel, \mathbf{n}_i , is defined positive outwards. The body is discretized using a limited number of panels, typically 20 to 300.

Step 2. Solving for the vorticity distribution at t_0

The vorticity distribution varies piecewise linearly from panel to panel all around the body. It is defined on the boundaries of each panel γ_i , with $i = 1, \ldots, N_p$ and is continuous. The Kutta condition is not enforced in any form.

The vorticity distribution γ_i is calculated by enforcing the no-penetration condition at the center of each panel (N_p equations). An $N_p + 1^{th}$ equation is added to enforce Kelvin's theorem (equation 5.5). During the initial time step, the wake does not exist yet, hence Kelvin's theorem is expressed as

$$\sum_{i=1}^{N_p} \gamma_i ds_i = \Gamma_0 \tag{5.9}$$



Figure 5.1: Flow chart of the DVM



Figure 5.2: Discretization of the body

where Γ_0 is the initial amount of circulation in the control volume. The resulting system of equation is linear and over-determinated:

$$[COI][\gamma] = [RHS] \tag{5.10}$$

where the matrix of Coefficients of Influence [COI] depends only on the geometry of the body. The Right Hand Side term [RHS] is a column vector with $N_p + 1$ elements containing the normal velocities at the N_p panels. The last element is of this vector contains the contribution from Kelvin's theorem, expressed by equation 5.9.

Step 3. Motion of the body

The motion of the body is defined by the pitch angle α and the heave position h, along with the corresponding velocities. Since the shape of the body is unchanged and the number of panels is not high, imposing the motion of the body is an easy and cheap numerical task. At each time step, the kinematics of the system is known from the structural model or imposed motion. The algorithm consists in translations and rotations of the panel coordinates \mathbf{x}_i and the collocation points \mathbf{x}_c^i in the 2D plane of the flow. The simplicity of the motion description algorithm represents one of the main advantages of DVM over meshed flow methods that require grid adaptation, a computationally expensive task.

Step 4. Shedding process

The surface vorticity distribution is broken down into blob vortices, one for each panel. The circulation of each vortex blob is equal to $\Gamma_i = \gamma_i ds_i$. The blobs are released in a direction normal to the surface at a chosen distance δ_r^i from the body, as shown in figure 5.3, where one panel is represented. In this figure, \mathbf{n}_i is the unit vector normal to the panel and \mathbf{x}_r^i is the location of the vortex release point. The value of the releasing distance is calculated according to $\delta_r^i = \sqrt{4/3\nu dt}$, as proposed by Porthouse [185].



Figure 5.3: Definition of the releasing distance δ_r^i

Step 5. Convection

After being released, the blob vortices are convected and diffused. The simulation of these two phenomena is carried out simultaneously. Convection is performed by computing the local velocity at each vortex location using equation 5.2. The motion of each vortex is due to the combined effects of the free-stream velocity, the motion of the body and the other blob vortices forming the wake. The new position, after convection, is computed using an Adam-Bashforth scheme:

$$\mathbf{x}_{w,c}^{k+1} = \mathbf{x}_w^k + \frac{dt}{2} \left(3\mathbf{u}_w^k - \mathbf{u}_w^{k-1} \right)$$

where \mathbf{x}_{w}^{k} and \mathbf{u}_{w}^{k} stand respectively for the position and the velocity of each blob vortex at the k^{th} time step and dt is the time step. The subscript c stands for the fact that the new position calculated is due to convection only.

Step 6. Diffusion

Diffusion is performed using the Random Walk method proposed by Chorin [182]. After the convection step, the position of each blob vortex is modified according to

$$\mathbf{x}_w^k = \mathbf{x}_{w,c}^k + \eta$$

where η is a 2 × 1 vector whose elements are real random variables with zero mean and standard deviation equal to $\sqrt{2dt/Re}$, where Re is the Reynolds number of the flow. Despite the non-deterministic feature of this method, it is often selected in vortex methods because of its simplicity and the good approximation it represents of the diffusion phenomenon [186]. After the convection/diffusion step the new position of each blob vortex is given by

$$\mathbf{x}_{w}^{k} = \mathbf{x}_{w}^{k-1} + \frac{dt}{2} \left(3\mathbf{u}_{w}^{k} - \mathbf{u}_{w}^{k-1} \right) + \eta$$

Step 7. House keeping

An important aspect of vortex methods is the treatment of the blob vortices after the convection/diffusion step. Two types of vortex require a special treatment: vortices entering the boundary of the body and vortices located far from the body.

Blob vortices crossing the body's surface are absorbed by the body. The sum of the circulation of these vortices, denoted by Γ_{in}^{k-1} , is taken into account during the next computation of the vorticity distribution.

The effect of a blob vortex located at a great distance (typically 5 to 10 chords) from the body is weak. Indeed, the velocity induced by this vortex on the body is inversely proportional to the distance, as shown above in equation 5.7. Hence, this vortex is not taken into account when establishing the no-penetration condition on the body surface. Nevertheless, its circulation, Γ_{far}^{k-1} , must be taken into account in order to satisfy Kelvin's theorem.

Despite the use of blob vortices to avoid numerical instability, large concentrations of blob vortices must be avoided, as they increase unnecessarily the computational cost of the simulation. A merging algorithm can be used in order to limit the number of vortices in the wake.

All the vortices located inside a certain region and exceeding a chosen density are merged into an equivalent vortex. The approach retained here was proposed by Spalart [173] and used by Morgenthal [177]. It consists in merging a pair of vortices into one, which is located at the centroid of the two original vortices and is characterized by the sum of their circulation, hence obeying Kelvin's theorem. Spalart showed that this method can be applied without important effects on the flow-field if

$$\left|\frac{\Gamma_1\Gamma_2}{\Gamma_1+\Gamma_2}\right|\left|\mathbf{x}_1-\mathbf{x}_2\right|<\epsilon$$

where ϵ is a small number and Γ_i and \mathbf{x}_i denote the circulation and the position of the pair of vortices.

The operation is repeated until a suitable vortex density is reached. In practice, this method is carried out by defining a circle with a chosen radius. The merging algorithm is repeated until only one equivalent vortex lies in this circle. A value of $\frac{1}{2}\overline{ds}$, where \overline{ds} is the mean panel length, is a suitable choice for the radius as it has a negligible effect on the shape of the wake. Although the merging of vortices has a certain computational cost,

the subsequent reduction in the number of vortices speeds up the simulation considerably.

Step 8. Solving for the vorticity distribution

The linear system of equations 5.10 is solved again but Kelvin's theorem 5.9 is modified into $$_{\rm N}$$

$$\sum_{i=1}^{N_p} \gamma_i^k ds_i + \Gamma_{wake}^{k-1} + \Gamma_{far}^{k-1} + 2A_b \Omega = \Gamma_0 + \Gamma_{in}^{k-1}$$
(5.11)

where k denotes the k^{th} time step, A_b is the area of the body section and $\Omega = \dot{\alpha}_k - \dot{\alpha}_{k-1}$ is the variation of angular velocity of the body between two time steps. The three global counters introduced above are:

- Γ_{wake}^{k-1} : sum of the circulation of all the vortices present in the wake.
- Γ_{far}^{k-1} : sum of the circulation of all the vortices at a chosen far-field distance from the body.
- Γ_{in}^{k-1} : sum of the circulation of all the vortices entering the body after the convection/diffusion process.

These counters, corresponding to the $k - 1^{th}$ time step, are considered in the calculation of the surface vorticity distribution at the k^{th} time step. The position of Γ_{in}^{k-1} in the right hand side of equation 5.11 means that the circulation of all the absorbed vortices is re-created on the surface of the body.

Step 9. Aerodynamic force calculation

The aerodynamic forces are computed from the integration of the instantaneous pressure distribution around the body. The calculation of the pressure is based on expression 5.6, which states that the pressure is proportional to the flux of vorticity through the surface. In order to use equation 5.6 on a discretized body it is necessary to establish the amount of circulation entering each panel at each time step. A local counter, $\Gamma_{in_i}^{k-1}$ with $i = 1, ...N_p$, is used to sum up the circulation of every vortex entering the i^{th} panel at the $k - 1^{th}$ time step. These N_p local counters are related to the global counter Γ_{in}^{k-1} , introduced in equation 5.11, by $\Gamma_{in}^{k-1} = \sum_{i=1}^{N_p} \Gamma_{in_i}^{k-1}$. The discretized expression for the pressure on panel i at the k^{th} time step is

$$p_i^k = p_1^k - \frac{\rho}{dt} \sum_{j=1}^i \left(\gamma_j^k ds_j - \Gamma_{in_j}^{k-1}\right)$$

where p_1 denotes a reference pressure at which the integration starts around the body. Since the pressure closes exactly around the surface of the body, because of the vorticity conservation, p_1 cancels out when calculating the aerodynamic forces through the closed integration over the body. It is clear that the calculation of the absolute pressure distribution over the body is a complex task using DVM, because it is based on the vorticity equation 5.1. On the other hand, this task is much easier in solutions of the full Navier-Stokes or Euler equations, where the pressure is a primary unknown.

At this point, the aerodynamic model has been described and the last step (number 10 in figure 5.1) deals with the calculation of the structural response of the aeroelastic system to the aerodynamic forces. This calculation is described in section 5.3. The aerodynamic solver can also be used without this step: static or imposed motion of a body in a flow-field. The next section presents the numerical parameters appearing in the aerodynamic model.

5.2.4 Numerical parameters

The most important parameters in a vortex method are the time step dt, the radius of the blob vortex σ_c and the number of panels N_p . Note that the releasing distance of the blob vortices δ_r and the radius of the merging circle are also numerical parameters but of less importance. Hence, their values are computed according to the relations presented in the foregoing.

Time step

The time step dt is calculated by $dt = k_1 \frac{B}{V_{\infty}}$, where B denotes the chord of the body. k_1 is a constant that takes values between 0 and 1, depending on the frequency content of the investigated flow-field and the dynamic characteristics of the flow-field and the body.

Cut-off distance

This parameter defines the radius of the blob vortex below which the induced velocity profile is smoothened (see section 5.2.2). The blob radius σ_c is chosen to be of the same order of magnitude as the mean panel length, so that $\sigma_c = k_2 \overline{ds}$, where k_2 is another constant that takes values between 0 and 1.

Number of panels

The number of panels, N_p , used to represent the body is also an important parameter. It is closely related to the cut-off distance presented above. On the one hand, the number of

panels should be large enough to allow a good spatial resolution of the pressure on the body and of the flow-field around it. On the other hand, if N_p is too large, numerical problems can arise and the computational cost can increase dramatically. Furthermore, when N_p is very large, \overline{ds} and hence σ_c are so small that the blob vortices become essentially classical vortices, inducing nearly infinite airspeeds near their centers. In the present numerical applications, the number of panels varies between 30 for the rectangular and circular cylinders, up to 268 panels for the generic bridge section.

5.3 Structural model

The aerodynamic model presented in the first section of the chapter can be coupled to a structural model in order to obtain an aeroelastic tool. This coupling corresponds to step 10 in the flowchart presented in figure 5.1. The three dofs of the body in the plane of the flow-field can be considered: pitching and heaving, as previously shown, plus the in-flow dof, denoted by x. Nevertheless, since only single dof aeroelastic systems involving α or h are studied here, the equation of motion is

$$I_{\alpha}\ddot{\alpha} + c_{\alpha}\dot{\alpha} + k_{\alpha}\alpha = M_a(t) \tag{5.12}$$

or,

$$m\ddot{y} + c\dot{y} + ky = F_y(t) \tag{5.13}$$

where $M_a(t)$ and $F_y(t)$ are the aerodynamic moment and vertical force, respectively, calculated using the DVM. The parameters I_{α} , c_{α} and k_{α} (m, c and k) stand for the structural inertia, damping and stiffness of the pitch (heave) degree of freedom. Equations 5.12 and 5.13 assume that no external forces or moments are applied to the system, apart from the aerodynamic forces.

The aerodynamic and structural models are loosely coupled, i.e. a new pitch angle $\alpha(t_k)$ (heave displacement $h(t_k)$) is computed at each time step from the knowledge of the aerodynamic moment $M_a(t_{k-1})$ (force $F_y(t_{k-1})$), resulting from the DVM computation using the previous kinematic state of the system $[\alpha(t_{k-1}), \dot{\alpha}(t_{k-1})]$ ($[h(t_{k-1}), \dot{h}(t_{k-1})]$). This calculation is performed through the numerical solution of equation 5.12 or 5.13 using a Newmark time integration scheme [73].

5.4 Application 1: Flow over a rectangle

The first numerical application deals with the torsional flutter oscillations of a 4:1 rectangular cylinder. Extensive experimental investigations have been presented in section 4.3. The comparison between the experimental and numerical results is carried out in terms of both flow characteristics and aeroelastic responses.

Two types of simulation are carried out: imposed motion of the rectangle and aeroelastic motion. For the former, the position of the rectangle at each time step is imposed and the fluid has no effect on the motion of the solid. For the latter, only initial conditions are imposed, the ensuing solid and fluid motions are simulated.

The imposed motion simulations are carried out for cases where the experimental system undergoes Limit Cycle Oscillations; the experimentally measured pitch response is the motion imposed on the simulations. Aeroelastic motion simulations are carried out for different airspeeds and initial conditions. Particular attention is paid to the effect of the structural damping on the simulation results.

5.4.1 Sensitivity analysis

As presented in section 5.2.4, the values of the numerical parameters must be set. The analysis of the effects of k_1 , k_2 and N_p is carried out through the simulation of the Strouhal number of the static rectangular cylinder. This choice is justified by the fact that the Strouhal number represents a global characteristic of the unsteady flow around the body and in its wake. The Strouhal number was measured experimentally in chapter 4. Its value is equal to 0.152 in the range of Reynolds numbers investigated during the wind tunnel experiments ($Re = 3 \times 10^4 - 10^5$).

Simulations are performed at airspeeds ranging from 2 m/s to 15 m/s with the body static at zero of angle of attack. The longitudinal component of the velocity u(t) is computed in the wake, one chord downstream of the trailing edge of the rectangular cylinder. For each airspeed, the fundamental frequency f_s of u(t), corresponding to the vortex shedding process, is extracted and the Strouhal number calculated according to its definition, $St = f_s D/V_{\infty}$.

Table 5.1 presents the Strouhal number identified for three numbers of panels ($N_p = 20, 45 \text{ and } 68$), two values of k_1 (0.1 and 0.2) and three values of k_2 (0.1, 0.5 and 1). It is observed that none of the parameters has a consistent effect on St. Values between St = 0.124 and St = 0.155 are obtained, leading to a maximum error of 18% with respect to the experimental measurements. The values $N_p = 45, k_1 = 0.1$ and $k_2 = 1$ yield a very good Strouhal estimate (2% error) and result in an adequate spatial resolution.

5.4.2 Imposed motion

As shown through experiments, the system undergoes large amplitude LCOs at airspeeds higher than 6.6 m/s (see figure 4.23). The unsteady flow over the upper surface of the

		$N_p = 20$	$N_p = 45$	$N_p = 68$
$k_{2} = 0.1$	$k_1 = 0.1$	0.151	0.143	0.144
$\kappa_2 = 0.1$	$k_1 = 0.2$	0.152	0.131	0.124
$k_{2} = 0.5$	$k_1 = 0.1$	0.129	0.142	0.134
$\kappa_2 = 0.5$	$k_1 = 0.2$	0.125	0.128	0.131
$k_{2} = 1$	$k_1 = 0.1$	0.143	0.155	0.134
$\kappa_2 - 1$	$k_1 = 0.2$	0.136	0.135	0.130

Table 5.1: Effect of the parameters on St

rectangle is visualized at two airspeeds, 7.4 m/s and 13.4 m/s, using Tr-PIV. The pitch response amplitude, A, and frequency, f are presented in table 5.2 for both these airspeeds.

Table 5.2: Amplitude and frequency of the experimental LCOs

DVM simulations are performed by imposing the corresponding pitch motion of the body at both airspeeds and the velocity field is computed at the same locations as in the PIV measurements. The experimental responses are approximated as simple sinusoids of the form $\alpha(t) = A \sin 2\pi f t$. Three cycles of oscillation are selected for comparison with experiments (cfr figures 4.35 and 4.36)

Because of the time stepping feature of this Lagrangian approach, it is necessary to perform a preliminary computation to ensure that the flow behaviour is well established, i.e. to remove starting effects. The rectangle is held motionless at its initial condition, while the flow is simulated until the starting vortex moves outside of the computation domain. Figure 5.4 shows two snapshots of the flow-field at two different time instances of this preliminary simulation: after 30 and 150 time steps, corresponding to 0.032 s and 0.16 s. The domain where the velocities are computed spans from one chord upstream to 5 chords downstream of the body. It is observed from the top snapshot of figure 5.4 that the computed flow-field remains undisturbed where no vortices are present, approximately two chords downstream of the trailing edge. Conversely, after 150 time steps, the wake is completely developed within the domain. Subsequently, the rectangle is set in motion at



Figure 5.4: Snapshot of the flow-field: 30 time steps (top) and 150 time steps (bottom)

the prescribed frequency and amplitude and the simulation continues for a further 1000 time steps. The flowflow-field field simulated over the last three cycles of the oscillation is selected for further analysis.

Figure 5.5 shows three pairs of instantaneous snapshots of the velocity field when the airspeed is set to 7.4 m/s. PIV measurements and simulation results are displayed on the left-hand and right-hand plots, respectively. The flow-fields are plotted at three different time instants, corresponding to instantaneous pitch angles of $\alpha = 8.8^{\circ}$, 5.2° and -5.8° . In accordance with the different flow states defined in chapter 4, it can be stated that:

- The first pair of snapshots, (a-b), corresponds to the convection of the Motion Induced Vortex (MIV). The simulation result reproduces correctly the MIV, spanning from the leading edge to the mid-chord position. The shape of the flow-field is well reproduced by the DVM, but the thickness of the simulated MIV is slightly lower than that of the PIV measurement.
- The second pair of snapshots, (c-d), corresponds to the flow reversal phase, reached at roughly 40% of the cycle. It is observed that the flow-field is completely separated from the surface of the rectangle for both PIV measurements and DVM simulations.



Figure 5.5: Snapshots of the velocity fields: Experiment (left) and simulation (right)

The simulated separated flow region is slightly smaller than the experimental one.

• The third pair of snapshots, (e-f), corresponds to the downstroke part of the motion, at 60% of the cycle, where the re-attachement of the flow begins (the separated flow region reaches its maximum size at 50% of the cycle). It is observed that the simulated and measured velocity fields are similar, with an additional secondary vortex in the rear part of the rectangle for the former.

The snapshots of figure 5.5 demonstrate that the physics of the unsteady flow-field is well captured by the DVM simulation. However, comparisons of flow snapshots are qualitative rather than quantitative. They are also laborious, as there are many snapshots in a cycle. A more quantitative evaluation of the quality of the DVM simulation results can be obtained using a modal decomposition of the unsteady flow-fields. Here, the numerical results of the simulations are analyzed and compared to the experimental velocity fields using the Common-base Proper Orthogonal Decomposition method (CPOD), presented in chapter 4.



Figure 5.6: Three periods for each airspeed considered in the CPOD analysis

Figure 5.6 shows the pitch response over three cycles, first at 7.4 m/s and then at 13.4 m/s. Following the CPOD methodology, the corresponding experimental and numerical data sets are placed in a row vector in order to create the global covariance matrices \mathbf{U} and \mathbf{V} , as presented in chapter 2.

Figure 5.7 shows the first four CPOD mode shapes, as calculated from the experimental measurements (left column) and the DVM simulations (right column). It is obvious that there are clear similarities between the two sets of mode shapes. The shapes of modes 1 and 2 are nearly identical. The shapes of mode 3 are very similar but the experimental shape features a slightly stronger reversed flow component over the rear half of the rectangle. There are more significant differences in mode 4, the experimental shape featuring two recirculation areas, while the simulated shape only one.

Figure 5.8 presents the associated generalized coordinates associated to the first four CPOD modes. Referring to figure 5.6, the first 0.37 seconds correspond to the three cycles of oscillations at 7.4 m/s. The rest of the time signals is associated to the LCOs at 13.4 m/s.

The figure shows that the responses of the generalized coordinates computed from the DVM method agree with the corresponding experimental results. Nevertheless, small differences concerning the values of maxima and minima are observed. In addition, these differences are stronger at 13.4 m/s. At this higher airspeed there is a clear phase shift between the two sets of generalized coordinates for all modes. It should be mentioned that the pitch response of the experimental system is not sinusoidal, while the motion imposed on the simulation is. As a consequence, the simulated flow-field could never be identical to the experimental one.

Nevertheless, the present results demonstrate the capability of the DVM to reproduce correctly the main characteristics of the flow-field around the oscillating rectangle. This is of great importance since torsional flutter is caused by the interaction between the strongly nonlinear behaviour of the flow-field and the structural response of the body.

5.4.3 Aeroelastic behaviour

Simulations of the aeroelastic behaviour of the system are carried out at airspeeds between 5 m/s and 15 m/s and different initial conditions. Unsteady aerodynamics is characterized by a memory effect: the aerodynamic loading on the structure depends on the history of the flow-field around it. Hence it is required that the wake of the body is fully developed before applying the initial condition to the system. This fact has already been pointed out in the previous section where the state of the flow-field after 30 and 150 time step was shown in figure 5.4. In the present aeroelastic simulations, the preliminary computation consists in 1000 time steps with the body held motionless at the selected initial pitch angle. After this preliminary computation, the body is 'released' and its motion is dictated by the fluid-structure interaction calculation, marking the beginning of the aeroelastic simulation. Note that the situation is similar to the experimental procedure: the model is set at a known pitch angle, then the wind tunnel is turned on and when the velocity of the airflow is stabilized at the desired airspeed, the model is released and its response is measured.

Structural model

As presented in section 5.3, the structural model of the system can be coupled to the aerodynamic solver in order to obtain an aeroelastic tool. It was shown through static



Figure 5.7: CPOD modes - PIV (left) and DVM (right)



Figure 5.8: CPOD generalized coordinates: PIV measurements (dashed), DVM simulations (plain)

experiments that the model behaves linearly in the pitch angle range of interest. Hence, the equation of motion can be expressed by equation 5.12 and the structural parameters, identified experimentally through wind-off vibration tests (see section 4.3.1) are repeated in table 5.3 for convenience. They correspond to a linear damping ratio of $\xi_0 = 2.6\%$ and

I_{α} [kgm]	$c_{\alpha} \; [\rm kgm/s]$	$k_{\alpha} [\mathrm{N/rad}]$
0.00152	0.0045	4.47

Table 5.3: Structural parameters of the linear system (wind-off)

a natural frequency of $f_0 = 8.65$ Hz.

Bifurcation diagram

The bifurcation diagram, showing the variation of the LCO amplitudes with airspeed is presented in figure 5.9. The experimental data from chapter 4 and the corresponding simulation results are shown as circles and dots respectively. The value of the structural damping ξ_0 is set to 2.6% in accordance with table 5.3. The following observations can made regarding figure 5.9:

- The simulated aeroelastic system is characterized by a sub-critical Hopf bifurcation, as observed experimentally.
- The amplitudes of the simulated LCO are slightly higher but in good agreement with the corresponding experimental measurements.
- At 10 m/s, the system's fixed point becomes unstable, i.e. the Hopf airspeed of the simulated results is equal to 10 m/s. It is lower than the Hopf airspeed identified experimentally, which was equal to 14.6 m/s. This difference may be explained by the presence of low levels of turbulence in the wind tunnel. Turbulence can have a stabilizing effect on the torsional flutter phenomenon [187].
- At 9 m/s, the simulated system undergoes a fold bifurcation, i.e. the unstable limit cycle branch generated at the Hopf point changes direction and becomes stable. At this airspeed, if a high enough initial condition is imposed to the body, it will reach a stable LCO; otherwise, the response will decay to equilibrium. The experimental system also undergoes a fold bifurcation but at a much lower airspeed of 6.6 m/s.

LCO amplitude is not the only result of interest; the frequency content of the system response is also very important. The variation of the fundamental frequency of both the simulated and experimental responses with airspeed is shown in figure 5.10. The two



Figure 5.9: Torsional Flutter (Exp. and DVM) - Bifurcation diagram

sets of results agree in the sense that the fundamental frequency decreases with airspeed. This tendency is more pronounced in the experimental results: the frequency drops by 4% between 7 m/s and 15 m/s. The simulated results feature a slightly smaller frequency decrease of 2.7%.



Figure 5.10: Torsional Flutter (Exp. and DVM) - Softening effect

Effect of structural damping

Additional simulations are performed for different values of the structural damping parameter in order to determine whether the bifurcation airspeeds can be simulated more accurately. The results of the simulations carried out with $\xi_0 = 2.0\%$, $\xi_0 = 2.6\%$ and $\xi_0 = 3.0\%$ are shown in figure 5.11.

The baseline result is taken to be the bifurcation plot for $\xi_0 = 2.6\%$, which was already

$\xi_0 \ [\%]$	2.0	2.6	3.0
U_{Hopf} [m/s]	9.0	10.0	12.0
$U_{fold} [\mathrm{m/s}]$	8.25	9.0	10.0

Table 5.4: Critical velocities - effect of the structural damping

presented in figure 5.9. It is observed that a reduction of the damping ratio to 2.0% results in the doubling of the amplitude of the LCOs, while an increase of the damping to 3.0% brings about a 20% decrease in amplitude. There is also a strong effect on the bifurcation airspeeds, as summarised in table 5.4. In general, as the damping is decreased, both the Hopf and fold airspeeds decrease, but at a decaying rate. The difference between the two airspeeds decreases from 2 m/s at $\xi_0 = 3.0\%$ to 0.75 m/s at $\xi_0 = 2.0\%$, allowing to speculate that at even lower damping rates the Hopf bifurcation becomes supercritical and the fold disappears completely. It can be concluded that, while the structural damping has an important effect on the simulation results, its variation does not bring about a more accurate prediction of the two bifurcation airspeeds.



Figure 5.11: Torsional Flutter (Exp. and DVM) - Effect of the structural damping

The aeroelastic results presented so far show that the simulation can predict the occurrence, amplitude and frequency of LCOs with accuracy. The major differences between the experimental and simulated results concern the airspeeds at which the Hopf and fold bifurcations occur. These airspeeds are very sensitive to the amount of structural damping in the system.
Bifurcation airspeeds

It was stated in chapter 4 that the torsional flutter mechanism is a Motion Induced Vortex (MIV) that is generated and shed periodically on the upper and lower surfaces of the rectangle. As the MIV clears the trailing edge, it generates a large area of recirculation over the rear of the rectangle, resulting in a strong restoring impulse. This impulse transfers energy from the free-stream to the structure twice per cycle, thus preserving the motion.

The resulting aerodynamic moment applied to the structure is nonlinear and leads to LCOs. Figure 5.12 shows a phase plot of the simulated aerodynamic moment coefficient $C_M = \frac{M(\alpha, \dot{\alpha})}{1/2\rho B^2 V_{\infty}^2}$ against α and $\dot{\alpha}$ at four airspeeds where LCOs are observed: 9 m/s, 11 m/s, 13 m/s and 15 m/s. If the aerodynamic moment was linear, the shape of the C_M curve would be elliptical. The figure shows that C_M is not elliptical, even at the lowest airspeed. Furthermore, as the airspeed increases, C_M becomes less and less elliptical. This phenomenon is a strong indication of the increasing degree of nonlinearity at increasing airspeeds.



Figure 5.12: Moment coefficient $C_m(\alpha, \dot{\alpha})$

The MIV can be either generated spontaneously or caused by an initial pitch displacement. The fold airspeed is the lowest airspeed at which a MIV can be generated by an initial displacement. At lower airspeeds, either the MIV is not generated at all or it is not strong enough to preserve the motion and the system is characterized by a decaying response.



Figure 5.13: Effect of the initial condition at 12 m/s

The Hopf airspeed represents a distinct instability mechanism. At airspeeds equal to or higher than the Hopf, an infinitesimal initial displacement gives rise to a weak oscillation. The amplitude of this oscillation grows in time, until MIVs start to get generated and the amplitude is stabilised. Figure 5.13 shows the simulated pitch response at an airspeed of 12 m/s for two different initial conditions: $\alpha_0 = 0^\circ$ and 30° . The top plot of this figure corresponding to $\alpha_0 = 0^\circ$ shows that, at this airspeed, the system's fixed point is an unstable focal point causing an oscillation with slowly increasing amplitude. It takes nearly 8 seconds for the oscillation amplitude to be stabilized. The bottom plot was obtained at the same airspeed but with an initial pitch angle of 30° . In this case, MIVs are produced immediately and the oscillation amplitude is stabilized in less than one second.

Going back to the top plot of figure 5.13, the response during the first 4 seconds is reminiscent of a classical linear instability with exponentially increasing amplitude. Therefore, the complete bifurcation behaviour of the system can be seen as the result of two instabilities:

- 1. A strongly nonlinear instability related to the generation and shedding of a MIV and leading to LCOs.
- 2. A nearly linear instability related to a change in the stability of the system's equilibrium point and leading to oscillations with exponentially increasing amplitude.

The MIV is the dominant mechanism since, at airspeeds when the two instabilities coexist, the steady state response is a LCO. The MIV instability, as it is strongly nonlinear, can only be simulated numerically. The linear instability can be modeled using an equation of motion of the form

$$I_{\alpha}\ddot{\alpha} + c_{\alpha}\dot{\alpha} + k_{\alpha}\alpha = \frac{1}{2}\rho V_{\infty}^2 B^2 \left(a_1\alpha + a_2\frac{B\dot{\alpha}}{V_{\infty}}\right)$$
(5.14)

where a_1 and a_2 are coefficients to be determined. This equation undergoes a degenerate Hopf bifurcation when the total damping, c_{tot} , vanishes

$$c_{tot} = c_{\alpha} - \frac{1}{2}\rho V_{\infty}B^3a_2 = 0$$

The value of the total damping present in the simulation responses at low response amplitudes can be identified using a linear damping assumption. Only low-amplitude, transient parts of the response signals are used in this identification. Figure 5.14 shows the variation of the total damping ratio, $\xi_{tot} = c_{tot}/(2\omega I)$, with airspeed. The value of the structural damping ratio at zero airspeed, $\xi_0 = 2.6\%$, is denoted by a star on the vertical axis of this figure. It is observed that the total damping is positive at airspeeds equal to or lower than 9 m/s. It becomes negative at an airspeed between 9 m/s and 10 m/s and stays negative for all higher airspeeds. The airspeed at which the total damping ratio becomes zero corresponds to the fold airspeed observed in the time responses of the DVM simulations. At airspeeds higher than the Hopf airspeed, the total damping is negative, which is consistent with the initial exponential increase in the response amplitude seen in the upper plot of figure 5.13.



Figure 5.14: Variation of the total damping ξ_{tot} with airspeed

Several authors have attempted to describe the torsional flutter phenomenon using linear equations of the form 5.14 and to determine the values of the a_1 and a_2 coefficients from quasi-steady considerations [109, 169]. The simulations presented in this section, in agreement with the experimental results of section 4.3, show that the MIV instability mechanism can cause large amplitude LCOs at airspeeds significantly lower than the Hopf condition, be it subcritical or degenerate. Therefore, linear quasi-steady models of torsional flutter only represent one aspect of the phenomenon and are incomplete.

In conclusion, it is shown that the present numerical tool gives good estimates of the characteristics of the aeroelastic system. The tool allows a deeper analysis of the unsteady aerodynamics and highlights the role of the MIV in the onset of the torsional flutter phenomenon.

5.5 Application 2: Flow over an oscillating cylinder

Tr-PIV measurements of the flow behind an oscillating circular cylinder were presented in chapter 2 of this thesis. In this section, the DVM method is used in order to simulate the same system. Comparisons between the experiments and the simulation results are presented, aiming to demonstrate the capability of the DVM tool to simulate accurately and efficiently the flow around an oscillating circular cylinder.

In chapter 2, it was shown that the cylinder undergoes VIV for airspeeds varying between 13.5 m/s and 18.0 m/s. Test 4 corresponds to the configuration where the heave oscillation amplitude reaches nearly 2 mm at 14.8 m/s. Tr-PIV measurements of the wake are available for this test, at 0.4 m from the cylinder's midpoint (cfr position 2 in figure 2.3).

DVM simulations are carried out for the configuration of Test 4. The motion of the cylinder is imposed and the velocity field in the wake of the cylinder is computed. The results presented below are obtained with the following numerical parameters: $N_p = 36$, $k_1 = 0.1$ and $k_2 = 0.5$. According to the airspeed of Test 4, the time step dt is equal to 2.41×10^{-4} s, rounded to 2.5×10^{-4} s. By doing so, four numerical time steps correspond to one Tr-PIV measurement ($dt_{PIV} = 0.001$ s), which facilitates the comparison between numerical and experimental results.

Figure 5.15 shows one cycle of the vertical motion of the cylinder, extracted from the images acquired through PIV measurements during the wind tunnel experiments. The heave position varies between -1 mm and 1mm at a frequency of 70.5 Hz. Eight time instances evenly distributed on the cycle, are selected and shown in this figure. Figures 5.16 and 5.17 present the corresponding snapshots, where the left and right columns show the PIV measurements and the DVM results, respectively. In each snapshot, the Von Karman vortices are highlighted using black dots in order to facilitate the comparison. It is observed that two vortices are present in the visualization window at all time instances, a phenomenon that is well reproduced by the DVM simulations. Nevertheless, slight

differences in the position of these vortices can be observed, especially in the first four time instances (figure 5.16). In the second half of the cycle (snapshots 5 to 8), the DVM results are in better agreement with the experiments (figure 5.17). Generally speaking, it is difficult to simulate accurately the unsteady aerodynamics in the wake of a bluff body, even in the presence of large vortical and energetic patterns, such as the Von Karman vortices. The difficulty is further increased by the flow regime characteristic of this experiment: 'transitional in the shear layer' (see section 2.3.1), where the ejection of Karman vortices is transitional and combined with small turbulent eddies.



Figure 5.15: Selection of the 8 time instances in one period of oscillation of the cylinder

The Strouhal number is identified from the frequency content of the velocity component in the wake of the cylinder. The identification is carried out on a point on the symmetry axis of the cylinder, two diameters downstream. The Fourier transform of the vertical component of the instantaneous velocity vector is shown in figure 5.18, where two peaks are clearly identifiable at 70 Hz and 82 Hz. These frequencies correspond to the heaving motion and the vortex shedding process respectively. The resulting Strouhal number is equal to $St = 82 \times 0.036/14.8 = 0.1995$, which corresponds to the value of 0.2 from figure 1.9 of chapter 2.

This section demonstrates that the DVM tool is capable of simulating correctly the frequency content in the wake of the cylinder, despite small differences in the positions of the Von Karman vortices. Note that, despite the apparent geometric simplicity of the circular cylinder, the simulation of the unsteady flow-field around it represents a complex numerical task [188]. This is principally due to the strong effect of the Reynolds number on the behaviour of the boundary layer and its separation from the surface of the cylinder [36].



Figure 5.16: Comparison between PIV (left) and DVM simulations (right)



Figure 5.17: Comparison between PIV (left) and DVM simulations (right) (cont'd)



Figure 5.18: Fourier transform of the vertical velocity component in the wake

5.6 Application 3: Flow around a generic bridge deck

The last numerical application presented in this thesis concerns the generic bridge section studied in chapters 3 and 4. The sharp edges of the deck are likely to lead to flow separation at the leading-edge, while the equivalent ratio $B/D \approx 4$ leads to re-attachment of the flow on the upper surface of the deck. Unfortunately no experimental visualization of the flow-field around the deck was possible, because of the important size of the chord of the model (0.317 m), compared to the limited size of the PIV window (0.1 m × 0.1 m). Hence, flow comparisons between the experiments and simulations are not possible.

Calculations of the Strouhal number and the aerodynamic forces and moment are carried out around the static bridge as a function of the angle of attack. These results are compared to the experimental measurements presented in chapters 3 and 4. In addition, on the basis of the good results obtained using the DVM tool in the previous sections, it is proposed to carry out flow simulations around the generic bridge deck in order to observe the main characteristics of the flow-field. In particular, the occurrence of flow separation and re-attachment is related to the predicted aerodynamic forces and Strouhal number.

5.6.1 Geometric modifications

Before using the DVM tool, it is necessary to discuss different geometric modifications that must be carried out in order to represent as well as possible the aerodynamic shape of the bridge. Two different types of appendages, shown in figure 5.19, induce a certain amount of three-dimensionality in the geometry and in the aerodynamics of the deck:

- Wind-shields on the top of the deck, with an equivalent porosity of 42%. The diameter of the wires composing these shields is smaller than 1 mm on the model (see figure 3.2).
- Stiffeners on the bottom part of the deck, spaced at 46 mm over the span dimension of the bridge section. The diameter of these rods is 3.8 mm.



Figure 5.19: Generic bridge deck: appendages

The present DVM implementation is limited to single body modelling, i.e. it is not possible to model the wind-shields as additional small circles. Even with the possibility to handle multiple bodies, as in the work of Morgenthal [87], the size of the panels used to model the wires of the wind shields would lead to numerical problems. Furthermore, the house keeping task would become unnecessarily complex and computationally expensive.

For these reasons, the two-dimensional representation of the deck section must be adapted in order to represent as well as possible the aerodynamic shape of the bridge. The modified geometry of the bridge section is shown in figure 5.20. First, the stiffeners below the deck are removed, because their effect on the global behaviour of the flow is low: small 3D vortices are shed from the inclined rods on the leeward face with a limited effect, while the rearward face lies in a re-criculation/zero airspeed region of the flow domain. The drag force of these rods is assumed to be negligible compared to drag of the main part of the deck.

The wind-shields are modelled as impermeable wind-shields of equivalent height h_e (see figure 5.20). This quantity takes values between 0% and 100% of the real wind-shields, as shown in the next sections.

Note that the acoustic panel, located near the trailing edge of the deck, as shown in figure 3.7, is modelled exactly in 2D, since it is completely impermeable to air. The additional appendages on the upper part of the deck correspond to security rails. They are kept unchanged in all the simulations presented below.



Figure 5.20: Sketches of the bridge section - Equivalent impermeable wind-shields

5.6.2 Strouhal number - sensitivity analysis

Following the procedure applied to the rectangular simulations, the Strouhal number is used in order to set the values for k_1 and k_2 appearing in expressions $dt = k_1 \frac{B}{V_{\infty}}$ and $\sigma_c = k_2 \overline{ds}$ (see section 5.2.4). The configuration of the bridge deck of chapter 4 is selected here because the variation of the Strouhal number with angle of attack was measured experimentally for this configuration (for which the acoustic panel is absent) and shown in figure 4.3.

Four 2D models of the bridge are presented in figure 5.21. In this figure, h_e takes the values of 0% (the very small appendages present in sketch 5.21(a) correspond to the foundations of the wind shields), 33%, 66% and 100%.



Figure 5.21: Sketches of the bridge section (flow from left to right)

Numerical simulations are carried-out for each configuration of figure 5.21 with the following numerical parameters: $k_1 = 0.1$, $k_2 = 1$ and $N_p = 262$. The resulting discretiza-

tion of the bridge section is shown in figure 5.22. It is observed that an important number of panels is necessary compared to the rectangular and circular cylinders presented in the previous sections. This is of course due to the small size of the details on the upper part of the deck. Note that N_p could be reduced by decreasing the panel density in the lower part of the deck (trapezoidal section). Nevertheless, it was decided to keep the panel density constant in order to obtain a good spatial resolution of the flow-field over the entire deck. In addition, because of the variation of angle of attack α , certain faces of the section that are initially in an attached region of the flow may change into separated flow region when α varies.



Figure 5.22: Grid on the bridge section - $N_p = 262$

Figure 5.23 shows the variation of St with α for the four values of h_e presented in figure 5.21. As expected, the effect of h_e is important: the impermeable equivalent wind shields change considerably the nature of the flow on the upper part of the deck. The appropriate height seems to lie between 66% and 100%. The 66% value leads good results for positive angles of attack while the 100% value is better for negative α . It is not desired to carry out an optimization process in order to identify the best value of h_e , but only to highlight the important effect of these aerodynamic appendages. Generally speaking, it can be stated that the DVM tool yields to good estimates of the Strouhal number, for all angles of attack tested.

Additional computations of the flow-field around the bridge section and in its wake are carried-out to investigate the effect of h_e . Figures 5.24 and 5.25 show the instantaneous and the mean velocity field for $h_e = 0\%$ and 100%. The static angle of attack is set to 0° and the airspeed is equal to 15 m/s. It is observed from figure 5.24 that the effect of h_e on the flow-field is strong on the upper part of the deck: the instantaneous flow is roughly attached for $h_e = 0\%$ (see figure 5.24(a)), except in the close vicinity of the surface of the deck where small protuberances lead to flow separation. A small local recirculation bubble is observed at the leading edge, due to the sharp edge of the deck. On the other hand, the flow is completely separated in the case of $h_e = 100\%$ (figure 5.24(b)), where the impermeable equivalent wind-shields lead to a flow-field similar to the one around a



Figure 5.23: Generic bridge deck - DVM vs. Experiments - St

simple U shape. Note the presence of a large clockwise vortex at the center of the deck. The bottom windward part of the flow-field is similar for $h_e = 0\%$ and 100%: a strong recirculation region is observed with low velocities.

Figure 5.25 shows the corresponding mean flows and confirms the remarks made above. The large recirculation on the entire upper surface of the deck appear as a light gray and even white zone in figure 5.25(b). In the case of $h_e = 0\%$, the vertical gradient confirms the global attachment of the flow on the surface of the deck. The recirculation in the bottom part of the two snapshots has the same dimensions, confirming that h_e has no effect on this large separated region of the flow-field.

Note that these additional simulations are numerically expensive. Because of the Lagrangian nature of the vortex method, the knowledge of the velocity field on a fixed grid is not necessary. The identification of the Strouhal number requires to compute the velocity at only one point in the wake. The force calculations presented in the next section does not require the calculation of the full wake either. The computations of the flow-field on the grid of figure 5.24 increases the CPU time by a factor of 100!

This remark highlights an important difference between the DVM and the grid-based methods: the velocity (and pressure) fields are not known from the imposition of the non-penetration condition. They can be considered as secondary unknowns that can be computed a posteriori. This difference represents an advantage of the DVM if only the forces and/or Strouhal number are of interest. This is the case in aeroelastic applications, such as the rectangle presented in section 5.4.



Figure 5.24: Flow-field around the bridge deck - Instantaneous velocity field



(a) $h_e = 0\%$



Figure 5.25: Flow-field around the bridge deck - Mean velocity field

5.6.3 Aerodynamic forces and moment

In this section, the aerodynamic forces and moment are computed using the DVM tool and compared to the experimental results of chapter 4. The configuration of the deck includes the wind-shields and the acoustic panel. From the observations of the previous section, two values for h_e are chosen: 66% and 100%. The corresponding bridge models are shown in figure 5.26.



Figure 5.26: Sketchs of the bridge section - with acoustic panel

Similarly to the experimental tests, the airspeed is set to 15 m/s and the angle of attack varies between -15° and 30° . The aerodynamic coefficients, C_L , C_D and C_M , obtained from the simulations are compared to the experimental values (cfr figure 4.2).

The length of the simulation is set to 2000 time steps, which corresponds to 4 seconds with the numerical parameters selected above ($\Delta t = 0.002$ s). Because of the time stepping feature of the DVM mentioned previously, it is necessary to eliminate the starting effects: the mean coefficients are computed on the basis of the values of $C_L(t_k)$, $C_D(t_k)$ and $C_M(t_k)$ for time steps 300 to 2000 (corresponding to a total duration of 3.4s).

The comparison between experiments and numerical results is presented in figures 5.27, 5.28 and 5.29. It is observed from these figures, that the calculated the lift and moment coefficients are in very good agreement with the experimental results. Similarly, the drag coefficient is well estimated for $-5^{\circ} \leq \alpha \leq 10^{\circ}$, but outside this range an important overestimation of C_D is observed (see figure 5.29). The drag coefficient corresponding to $h_e = 0\%$ is also plotted in figure 5.29 in order to highlight that the effect of h_e is limited: in the absence of an equivalent wind-shield the overestimation of the drag at high angles of attack persists.



Figure 5.27: Generic bridge deck - DVM vs. Experiments - C_L



Figure 5.28: Generic bridge deck - DVM vs. Experiments - ${\cal C}_M$



Figure 5.29: Generic bridge deck - DVM vs. Experiments - C_D

The explanation of this overestimation can be found in the size of the wake of the bridge deck at high angles of attack. Figure 5.30 presents additional simulations of the mean flow around the static deck at 15° for two values of h_e : 0% and 100%. It is observed from figures 5.30(a) and 5.30(b) that the width of the wake is much larger than in the case of $\alpha_s = 0^\circ$. Note that this comparison in not totally consistent because of the presence of the acoustic panel in figure 5.30 while no acoustic panel appears in figure 5.25. Nevertheless, because of the large static angle, $\alpha_s = 15^\circ$, this panel clearly does not affect the size of the separated flow region in figure 5.30.

The increase of the width of the separated flow region at 15° is logical: flow separation over the bottom part of the deck occurs at the edge denoted by the letter A in figure 5.30. On the upper surface the separation takes place at the leading edge. Hence the distance between the separation points is more important for $\alpha_s = 15^\circ$ than $\alpha_s = 0^\circ$ and the resulting wake is also larger.

Nevertheless, it can be argued that the width of the wakes presented in figures 5.30(a) and 5.30(b) is too important. The wind-shields may have a stabilizing effect on the flow and hence cause local re-attachment on the upper surface of the deck. Consequently, the flow may be locally separated at the leading edge, attached further downstream and separated again behind the acoustic panel.

This situation is sketched in figure 5.31, where the dashed lines represent the separated shear layers under the assumption described above. The plain line corresponds to the shear layer observed in figure 5.30. In this figure, the width of the wake is clearly reduced, possibly causing a reduction in drag. This aerodynamic effect of the wind-shields was



Figure 5.30: Flow-field around the bridge deck - mean flow at 15°

investigate in a paper by Taylor and Vezza [27], which aimed at altering the characteristics of the flow-field on the upper surface of a footbridge.

Another explanation can be attributed to an inadequate dissipation of the vortical structures, caused by the use of the random walk method [173]. This intrinsic limitation of the vortex method is also reported by Taylor [189] and Morgenthal [87].

This discussion shows the difficulty in simulating accurately the drag forces of a bluff body because small geometrical details have a strong influence on the behaviour of the flow and hence on the resulting aerodynamics forces. The lift force and the pitching moment are less affected because the leeward faces on the bottom part of the bridge deck (see edges ABCDE in figure 5.31) do not feature such small details. However, these faces



Figure 5.31: Assumed flow-field around the bridge deck

are responsible for the most important part of the aerodynamic forces, the rest of the bridge being in separated flow regions.

5.7 Chapter summary

A numerical tool is developed by coupling a 2D unsteady aerodynamic solver based on the Discrete Vortex Method (DVM) to a linear structural model. The tool can be used either as an aerodynamic model, where the motion of the body is imposed (including static conditions), or as an aeroelastic tool, where the free response of the system is simulated.

Three different numerical applications are demonstrated: (i) aeroelastic modelisation of the torsional flutter of a rectangle, (ii) flow simulations in the wake of an oscillating cylinder and (iii) aerodynamic forces and moment calculations on a static generic bridge deck.

The validation of the aerodynamic solver is performed through qualitative and quantitative comparisons of the simulated flow-field to the experimental measurements presented in previous chapters of the thesis. Quantitative comparison is carried out by means of the CPOD decomposition. The comparison confirms that the physics of the unsteady aerodynamic phenomena is well captured by the DVM simulation. For each of the three applications listed above, the capabilities of the numerical tool are also demonstrated through comparison with flow measurements leading to the identification of the Strouhal number and/or the calculation of the aerodynamic forces and moments.

The key advantage of the DVM implemented in the scope of this thesis is that the discretization of the system is limited to the meshing of the surface of the body only (the fluid remains grid-free). Hence the numerical tool can easily deal with other bluff

body shapes. Note that the application to the generic bridge deck showed that the tool can fail to simulate accurately the flow-field around small geometric details. It may be necessary to refine the discretization in such cases. This aeroelastic model represents a useful general tool for the study of the stability of bluff bodies free to oscillate in a fluid flow, or simply of the unsteady aerodynamic characteristics of the flow over the body.

Chapter 6

Conclusions

The objective of this thesis is to provide a better understanding of the aerodynamics and aeroelasticity of bluff bodies. In particular, the different studies presented here concern the field of civil engineering, where the flexibility and the aerodynamic shapes of the structures lead to a specific problematic termed **bluff body aeroelasticity**.

Two complementary approaches are followed in this work:

On the one hand, extensive experimental aeroelastic studies are carried out in the wind tunnel. The main concern is the investigation of the post-critical behaviour of the unstable structures, hence *free vibration tests* are preferred. It is demonstrated, through forces and flow measurements, that nonlinear aerodynamic excitations are responsible for the Limit Cycle Oscillations (LCOs) observed experimentally.

On the other hand, an engineering approach is undertaken with the objective to predict the possible occurrence of aeroelastic instabilities. This aspect of the thesis is motivated by the desire to bring fundamental findings into practical insight. This task is based on the quasi-steady theory and on empirical criteria, but also on a numerical simulation tool developed herein.

The two approaches presented above are applied through chapters 2 to 5, dealing respectively with VIV, galloping and torsional flutter instabilities. The main achievements of this thesis are presented qualitatively below, while more quantitative results are detailed in each chapter summary.

Chapter 2 presents a successful adaptation and usage of the POD and CPOD techniques to the analysis of aeroelastic systems [190]. In the case of the VIV of a circular cylinder, the (C)POD analysis allows to extract the main characteristics of the unsteady flow-field. The most energetic vortical structures are identified as 'laminar modes', while

the others are classified as 'turbulent modes'. The latter are responsible for the random variations in the wake due to small turbulent eddies, typical of the Reynolds range explored in these experiments $(2 \times 10^4 - 5 \times 10^5)$. Furthermore, the possibility of creating input-output models, based on the (C)POD analysis is established.

Another original analysis technique is developed in the scope of (C)POD. It concerns the identification of the structural damping of linear dynamic systems [127]. This work is naturally integrated to the objectives of this thesis, since most of the aeroelastic instabilities concern the cancellation of the total damping of the fluid/elastic system. Accurate estimates of the structural damping were obtained on simulated and experimental systems.

In chapter 3, a substantial experimental analysis of the galloping phenomenon is presented in the case of a generic bridge section [191]. It is shown that the bifurcation diagram contains a subcritical Hopf and two folds. It is demonstrated that the first order and the third order quasi-steady approaches are conservative in the estimation of the fold airspeed and hence completly miss the subcritical Hopf bifurcation. For that reason, a new polynomial expression is proposed in order to model the behaviour observed experimentally. This model succeeds in representing the response of the bridge, which can be compared to other basic shapes (square and 2:1 rectangular cylinders) using the concept of the universal galloping curves.

Chapter 4 presents extensive investigations of the torsional flutter phenomenon dealing with two complementary applications: a generic bridge deck and a 4:1 rectangle cylinder. Each aeroelastic system undergoes a subcritical Hopf bifurcation, followed by a fold bifurcation. In the case of the bridge section, simultaneous force measurements are performed when the deck undergoes LCOs at different amplitudes and airspeeds. It is observed that the delay between the aerodynamic forces and the pitch angle is associated with the reduction of the amplitude of oscillation of the bridge section. In the case of the rectangular cylinder, extensive Tr-PIV measurements are carried out. The Motion Induced Vortex (MIV) is identified as the source of excitation responsible for the torsional flutter oscillations. The study of the behaviour of the MIVs as a function of the oncoming airspeed is carried out with the CPOD technique [192]. An interpretation of each CPOD mode, based on its mathematical properties, is proposed and its participation in the different physical states of the flow-field is discussed. Finally, it is demonstrated that the simplified approaches fail to predict the critical airspeed of torsional flutter. This failure is due to the high complexity of the excitation mechanism (MIV) due to the pitch motion of the structure.

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In the light of the limitations of the simplified approaches to predict correctly the onset airspeed of the aeroelastic instabilities, a numerical aeroelastic tool is developed in **Chapter 5.** A 2D aerodynamic solver based on the Discrete Vortex Method (DVM) is implemented and coupled to a linear structural model. The resulting numerical tool is very computationally efficient: the longest computational run lasted 3 hours on a standard computer. The tool is demonstrated successfully on the three experimental configurations studied in chapters 2, 3 and 4. Good comparisons are obtained between the Tr-PIV measurements in the wake of the oscillating cylinder and the numerical results performed at the same location. Furthermore, the DVM tool is capable of simulating accurately the aerodynamic forces applied on the static deck section and the variation of the Strouhal number with angle of attack. Finally, the aeroelastic simulations of torsional flutter of the 4:1 rectangle cylinder are carried out [181]. An extensive comparison between the numerical results and the Tr-PIV measurements of the flow-field around the oscillating rectangle cylinder is carried out using the CPOD technique. The quantitative comparisons confirm that the physics of the unsteady flow is correctly captured by the DVM simulations. It can be concluded that this DVM model constitutes a valuable tool for the study of the aeroelastic behaviour of bluff bodies.

Directions for Further Work

The experimental and numerical results presented herein shed some light on several selected aeroelastic phenomena. As stated in the introduction, the aerodynamics of bluff bod-bodies depend strongly on small geometric details (security rails or wind-shields on bridges, ice or even rain on electrical conductors, etc). Hence, each civil engineering application necessitates a dedicated study in order to characterize correctly its aerodynamics and aeroelastic behaviour.

Nevertheless, it is believed that the existing experimental set-ups and the analysis tools developed in this thesis could be extended to the investigation of:

- The effect of the turbulence content of the oncoming flow on aeroelastic stability. Experimentally, a controlled free-stream turbulence intensity [193] could be considered as a parameter in the CPOD analysis based on the Tr-PIV measurements. This parametric analysis could lead to interesting conclusions on the effect of turbulence on the aeroelastic stability. In addition, the effect of turbulence could be integrated in the DVM tool, as shown recently by Rasmussen et al. [194] for the calculation of the aerodynamic admittance of a bridge section.
- The quantification of the <u>three-dimensionality of the flow-field</u>, still considered as a second order effect by most studies dealing with bluff body aeroelasticity, including this thesis.

Appendix A Wind tunnel facility

The wind tunnel of University of Liège was built in 1999 on the site of Sart-Tilman thanks to Walloon region and European Commission fundings. It is a low speed multidisciplinary wind tunnel that can be operate in closed loop or in open loop, under atmospheric pressure conditions. Furthermore, experiments can be carried-out in two test section, depending on the size of the model:

- Aeronautical test section (TS1)
- Wind engineering test section (TS2)

The main characteristics of both sections are specified in table A.1.

Characteristics	TS1	TS2
Dimensions (Width x Height x Length)	2 m x 1.5 m x 5 m	2.5 m x 1.8 m x 5 m
Airspeed Range (Closed/Open loop)	2-65 m/s / 2-40 m/s	2-40 m/s / 2-30 m/s
Test section turntable diameter	$1.5 \mathrm{m}$	2.0 m
Thermal stability	$1^{\circ}\mathrm{C}$	$1^{\circ}\mathrm{C}$
Streamwise static pressure gradient	0.3~% per meter	0.5~% per meter
Mean angle divergence	< 0.2~%	$< 0.2 \ \%$
Speed non-uniformity	$< 0.5 \ \%$	< 0.8 %
Turbulence level	$< 0.15 \ \%$	$< 0.23 \ \%$

Table A.1: Characteristics of the wind tunnel of ULg



Figure A.1: Wind Tunnel of ULg: schematic view



Figure A.2: Wind Tunnel of ULg: aeronautical test section (TS1)



Figure A.3: Wind Tunnel of ULg: wind engineering test section (TS2)

Appendix B

PIV set-up

The PIV system used for these experiments consists of the following components:

- 1. A Litron LDY301-PIV Q-switched laser system. It is a dual power, dual cavity laser with a wavelength of 527 nm, switching at 1000 Hz. The two laser beams contain 2×10 mJ of energy.
- 2. Optical modules for producing a laser sheet.
- 3. A Phantom V9.1 camera with a maximum resolution of 1600×1200 pixels at a frequency of 1 kHz and 6GB of internal memory buffer.
- 4. A timer box for synchronizing the laser with the camera.
- 5. A seeding generator with 3 bar back pressure suitable for PIV particle generation.
- 6. Dantec Dynamics Studio PIV data acquisition and analysis software.
- 7. A high specification Personal Computer.

The entire PIV set-up, apart from the camera and the computer, is installed on top of the Aeronautical test section of the wind tunnel. Figure B.1 shows four photographs of the components mentioned above.



(a) Laser of its translation system

(b) Laser



(c) High speed camera

(d) Power unit



Appendix C

Damping identification of linear dynamic systems using CPOD

This appendix presents the mathematical developments relative to the use of CPOD to identify the damping of linear dynamic systems. It is developed from the Proper Orthogonal Decomposition (POD) of the free response of the system and extended to the Common-base POD (CPOD).

From POD to modal analysis

Consider the equation of motion of a typical linear dynamic system without external force:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{C}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{0} \tag{C.1}$$

where $\mathbf{y}(t)$ are the displacements of the N degrees of freedom of the system, **M** is the mass matrix, **C** is the damping matrix and **K** is the stiffness matrix.

As stated in chapter 2, the objective is to relate the co-variance matrix **D**

$$\mathbf{D} = \frac{1}{M} \sum_{k=0}^{M} \mathbf{y}(k\Delta t) \mathbf{y}^{T}(k\Delta t)$$
(C.2)

to the modal characteristics of the system, that is the damping matrix C.

The equation of motion D.3 can be written in first order state space form, i.e.

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \tag{C.3}$$

where **A** is a $2N \times 2N$ matrix

$$\mathbf{A} = \begin{pmatrix} -\mathbf{M}^{-1}\mathbf{C} & -\mathbf{M}^{-1}\mathbf{C} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}$$
(C.4)

and $\mathbf{x} = [\dot{\mathbf{y}}^T \mathbf{y}^T]^T$.

From the classical solution of the state space equation C.3, $\mathbf{x}(t)$ is given by

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) \tag{C.5}$$

where $e^{\mathbf{A}}$ denotes the matrix exponential of \mathbf{A} . Assume that the system responses are a set of discrete data obtained at times $0, \Delta t, 2\Delta t, \dots, M\Delta t$, where Δt is a small time increment.

$$\mathbf{x}(k\Delta t) \approx e^{\mathbf{A}k\Delta t}\mathbf{x}(0) \tag{C.6}$$

Based on this definition, $\mathbf{x}(k\Delta t)$ contains the response of the system in terms of displacements and velocities. The POD method is based on the displacement responses only, hence it is necessary to select the second half of $\mathbf{x}(k\Delta t)$, to obtain the response displacements $\mathbf{y}(k\Delta t)$.

This is achieved by projecting expression C.6 onto a selection matrix \mathbf{Q} , defined by $\mathbf{Q} = [\mathbf{O}_N \mathbf{I}_N]$, where \mathbf{O}_N is the $N \times N$ zero matrix and \mathbf{I}_N is the $N \times N$ identity matrix.

$$\mathbf{y}(k\Delta t) = \mathbf{Q}\mathbf{x}(k\Delta t) = \mathbf{Q}e^{\mathbf{A}k\Delta t}\mathbf{x}(0)$$
(C.7)

Using equation C.7, the expression C.2 of \mathbf{D} becomes

$$\mathbf{D} = \frac{1}{M} \sum_{k=0}^{M} \mathbf{Q} e^{\mathbf{A}k\Delta t} \mathbf{x}(0) \mathbf{x}^{T}(0) e^{\mathbf{A}^{T}k\Delta t} \mathbf{Q}^{T}$$
(C.8)

The analysis can be taken a step further using the definition of the matrix exponential: $e^{\mathbf{A}} = \mathbf{V}e^{\mathbf{L}}\mathbf{V}^{-1}$, where \mathbf{V} is the eigenvector matrix of \mathbf{A} and \mathbf{L} is the eigenvalue matrix of \mathbf{A} .

Notice that the sizes of matrices \mathbf{A} , \mathbf{V} and \mathbf{L} are identical and equal to $2N \times 2N$. \mathbf{V} and \mathbf{L} contain respectively the *m* complex eigenvectors and complex eigenvalues of the system and their corresponding complex conjugates. Expression C.8 for the co-variance matrix becomes

$$\mathbf{D} = \mathbf{Q}\mathbf{V}\left(\frac{1}{M}\sum_{k=0}^{M} e^{\mathbf{L}k\Delta t}\mathbf{Z}_{0}e^{\mathbf{L}^{*}k\Delta t}\right)\left(\mathbf{Q}\mathbf{V}\right)^{T}$$
(C.9)

where $\mathbf{Z}_0 = \mathbf{V}^{-1} \mathbf{x}(0) \mathbf{x}^T(0) (\mathbf{V}^{-1})^T$ and the asterisk denotes the complex conjugate. When the matrix is complex, the superscript ^T means transposition of the matrix plus conjugation of its elements.

According to the POD technique, the eigenvalue decomposition of the co-variance matrix gives the POD modes $\Phi_k(x)$ and Λ , which is a square matrix with the eigenvalues λ_k on its diagonal.

$$\mathbf{D}\boldsymbol{\Phi} = \boldsymbol{\Lambda}\boldsymbol{\Phi} \tag{C.10}$$

The Spectral Theorem states that a matrix can be expressed as a linear combination of its eigenvectors. Using the notation of its eigenvalue decomposition, equation C.10, the co-variance matrix \mathbf{D} can be expressed by

$$\mathbf{D} = \mathbf{\Phi} \Lambda \mathbf{\Phi}^{-1} \tag{C.11}$$

Since **D** is symmetric by construction, its eigenvectors are orthonormal and $\Phi^{-1} = \Phi^T$. Combining equations C.9 and C.11 yields

$$\mathbf{QV}\left(\frac{1}{M}\sum_{k=0}^{M}e^{\mathbf{L}k\Delta t}\mathbf{Z}_{0}e^{\mathbf{L}^{*}k\Delta t}\right)\left(\mathbf{QV}\right)^{T}=\mathbf{\Phi}\Lambda\mathbf{\Phi}^{T}$$
(C.12)

Equality C.12 expresses that the POD modes Φ are proportional to the eigenvectors \mathbf{V} of the system after projection onto the selection matrix \mathbf{Q} . However the matrix \mathbf{QV} contains the eigenvectors and their complex conjugates, hence the dimensions of \mathbf{QV} are $N \times 2N$, while the dimensions of Φ are $N \times N$. Nevertheless Φ and \mathbf{QV} are in fact equivalent.

Up to this point, no assumption has been made on the mass distribution of the system. From their definition, Φ are orthonormal, i.e. perpendicular to each other, and normalized to unity. By associating the mode shapes to the POD modes, it is assumed that the mode shapes are also orthonormal, which is only true when the mass matrix **M** is proportional to the identity matrix.

This fact was reported before by several authors [140,141] and represents the intrinsic limitation of the POD method. One solution among others is to multiply the co-variance matrix by the mass matrix **M**, in order to obtain the true mode shapes of the system. In this case, the a priori knowledge of the mass matrix is necessary. In the present approach, only the response of the system is known and no mass matrix estimation is considered, hence this solution is discarded. Another interesting technique, named SOD, was proposed by Chelidze and Zhou [195]. It allows the estimation of mode shapes without any knowledge about the mass matrix.

The identification is taken a step further by developing a relation between the eigenvalues of the system and the POD eigenvalues, i.e. the diagonal matrices \mathbf{L} and $\boldsymbol{\Lambda}$ respectively.

As stated before, the matrix **L** contains the complex eigenvalues and their conjugates. Hence, it is convenient to reduc the size of this matrix from $2N \times 2N$ to $N \times N$, without losing any information about the dynamics of the system. If $\mathbf{x}(0)$ is expressed in terms of $\mathbf{y}(0)$, i.e. $\mathbf{x}(0) = \mathbf{Q}^{T}\mathbf{y}(0)$, \mathbf{Z}_{0} becomes

$$\mathbf{Z}_{0} = (\mathbf{Q}\mathbf{V})^{-1}\mathbf{y}(0)\mathbf{y}^{T}(0)((\mathbf{Q}\mathbf{V})^{-1})^{T}$$
(C.13)

The left hand side of equation C.12, denoted \mathbf{H} is given by

$$\mathbf{H} = \mathbf{Q}\mathbf{V}\left(\frac{1}{M}\sum_{k=0}^{M} e^{\mathbf{L}k\Delta t} (\mathbf{Q}\mathbf{V})^{-1} \mathbf{y}(0) \mathbf{y}^{T}(0) [(\mathbf{Q}\mathbf{V})^{-1}]^{T} e^{\mathbf{L}^{*}k\Delta t}\right) (\mathbf{Q}\mathbf{V})^{T}$$

The matrix \mathbf{QV} is proportional to the POD modes, as stated before, but it has to be modified in order to match the dimensions and the norm of $\boldsymbol{\Phi}$. To this end, denoting $\overline{\mathbf{QV}}$ as a matrix containing the norms of each column of \mathbf{QV} , one defines $\boldsymbol{\Psi}$ as the normalized \mathbf{QV} , i.e.

$$\mathbf{QV} = \mathbf{\Psi} \circ \overline{\mathbf{QV}}$$

where the symbol \circ denotes the *Hadamard* product, i.e. the element-by-element product of two matrices with the same dimensions. The matrix **H** becomes:

$$\mathbf{H} = \boldsymbol{\Psi} \circ \overline{\mathbf{QV}} \left(\frac{1}{M} \sum_{k=0}^{M} e^{\mathbf{L}k\Delta t} \boldsymbol{\Psi}^{-1} \circ \overline{\mathbf{QV}^{-1}} \mathbf{y}(0) \mathbf{y}^{T}(0) \boldsymbol{\Psi}^{-1} \circ [\overline{\mathbf{QV}^{-1}}]^{T} e^{\mathbf{L}^{*}k\Delta t} \right) \left(\boldsymbol{\Psi} \circ \overline{\mathbf{QV}} \right)^{T}$$

Here $\overline{\mathbf{QV}^{-1}}$ denotes the matrix containing the inverse of the norms of each elements of the columns of \mathbf{QV} . These terms cancel with the $\overline{\mathbf{QV}}$, leading to

$$\mathbf{H} = \boldsymbol{\Psi} \left(\frac{1}{M} \sum_{k=0}^{M} e^{\mathbf{L}k\Delta t} \boldsymbol{\Psi}^{-1} \mathbf{y}(0) \mathbf{y}^{T}(0) (\boldsymbol{\Psi}^{-1})^{T} e^{\mathbf{L}^{*}k\Delta t} \right) \boldsymbol{\Psi}^{T}$$

This expression of **H** still contains too much information: the complex conjugates of the eigenvectors and eigenvalues of the system. The size of Ψ is equal to $N \times 2N$, and contains twice each eigenvectors while **L** is $2N \times 2N$ and contains twice each eigenvalue. The selection of one of the complex conjugate elements is equivalent to removing all even columns and lines. Define ${\bf P}$ as another selection matrix of the form

$$\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & \dots & 0 & 0 \end{pmatrix}_{2N \times N}$$

Mathematically, the selection consists in the right multiplication of Ψ by **P** and the right and left multiplications of **L** and **L**^{*} by **P** and its transpose. These manipulations lead to the definition of the matrix $\tilde{\mathbf{H}}$ having the same dimensions as **H**:

$$\tilde{\mathbf{H}} = \mathbf{\Psi} \mathbf{P} \left(\frac{1}{M} \sum_{k=0}^{M} \mathbf{P}^{\mathbf{T}} e^{\mathbf{L}k\Delta t} \mathbf{P} (\mathbf{\Psi} \mathbf{P})^{-1} \mathbf{y}(0) \mathbf{y}^{T}(0) \left[(\mathbf{\Psi} \mathbf{P})^{-1} \right]^{T} \mathbf{P}^{\mathbf{T}} e^{\mathbf{L}^{*} k\Delta t} \mathbf{P} \right) (\mathbf{\Psi} \mathbf{P})^{T}$$

Notice that $\mathbf{P}^{\mathbf{T}} e^{\mathbf{L}k\Delta t} \mathbf{P} = e^{\mathbf{P}^{\mathbf{T}}\mathbf{L}\mathbf{P}k\Delta t}$, and denoting the projections: $\mathbf{P}^{\mathbf{T}}\mathbf{L}\mathbf{P}$ by $\mathbf{L}_{\mathbf{s}}$ and $\mathbf{\Psi}\mathbf{P}$ by $\mathbf{\Psi}_{\mathbf{s}}$, one obtains

$$\tilde{\mathbf{H}} = \boldsymbol{\Psi}_{\mathbf{s}} \left(\frac{1}{M} \sum_{k=0}^{M} e^{\mathbf{L}_{\mathbf{s}} k \Delta t} \boldsymbol{\Psi}_{\mathbf{s}}^{-1} \mathbf{y}(0) \mathbf{y}^{T}(0) \left[(\boldsymbol{\Psi}_{\mathbf{s}})^{-1} \right]^{T} e^{\mathbf{L}_{\mathbf{s}}^{*} k \Delta t} \right) \boldsymbol{\Psi}_{\mathbf{s}}^{T}$$

It can be shown that $\tilde{\mathbf{H}}$ approximates the matrix \mathbf{H} via $\tilde{\mathbf{H}} \approx \frac{1}{2}\mathbf{H}$, yielding

$$\boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^{-1} \approx \frac{1}{2}\boldsymbol{\Psi}_{\mathbf{s}} \left(\frac{1}{M} \sum_{k=0}^{M} e^{\mathbf{L}_{\mathbf{s}}k\Delta t} \boldsymbol{\Psi}_{\mathbf{s}}^{-1} \mathbf{y}(0) \mathbf{y}^{T}(0) \left[(\boldsymbol{\Psi}_{\mathbf{s}})^{-1}\right]^{T} e^{\mathbf{L}_{\mathbf{s}}^{*}k\Delta t} \right) \boldsymbol{\Psi}_{\mathbf{s}}^{T}$$
(C.14)

This expression is the basis of the present damping identification technique based on the POD method. It expresses a relation between the eigenvalues of \mathbf{D} and the diagonal
matrix $\mathbf{L}_{\mathbf{s}}.$ The modal identification through the use of POD can be summarized by

$$\Phi = \Psi_{\rm s} \tag{C.15a}$$

$$\mathbf{Z} \equiv \boldsymbol{\Phi}^{-1} \mathbf{y}(0) \mathbf{y}^{T}(0) (\boldsymbol{\Phi}^{-1})^{T}$$
(C.15b)

$$\mathbf{\Lambda} \approx \frac{1}{2M} \sum_{k=0}^{M} e^{\mathbf{L}_{\mathbf{s}} k \Delta t} \mathbf{Z} e^{\mathbf{L}_{\mathbf{s}}^* k \Delta t}$$
(C.15c)

The summation term in the RHS of equation C.15c leads to a full complex and symmetric matrix. Without loss of generality, the approach is demonstrated on a 2-dof system. The unknown matrix L_s can be expressed as

$$\mathbf{L}_{\mathbf{s}} = \left(\begin{array}{cc} a+ib & 0\\ \\ 0 & c+id \end{array} \right)$$

Variables a, b, c and d are four unknown real values to be determined. Using the property of the matrix exponential of a diagonal matrix, equation C.15c can be written as

$$\frac{1}{2M}\sum_{k=0}^{M} \begin{pmatrix} e^{(a+ib)k\Delta t} & 0\\ 0 & e^{(c+id)k\Delta t} \end{pmatrix} \begin{pmatrix} z_{11} & z_{12}\\ z_{12} & z_{22} \end{pmatrix} \begin{pmatrix} e^{(a-ib)k\Delta t} & 0\\ 0 & e^{(c-id)k\Delta t} \end{pmatrix} \approx \begin{pmatrix} \lambda_1 & 0\\ 0 & \lambda_2 \end{pmatrix}$$
(C.16)

where the $z_{k,l}$ are the elements of the real symmetric matrix **Z**. After multiplications, the equation C.16 becomes

$$\frac{1}{2M} \sum_{k=0}^{M} \begin{pmatrix} z_{11}e^{2ak\Delta t} & z_{12}e^{[(a+c)+i(b-d)]k\Delta t} \\ z_{12}e^{[(a+c)+i(d-b)]k\Delta t} & z_{22}e^{2ck\Delta t} \end{pmatrix} \approx \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$$
(C.17)

where the diagonal terms are real and equal to the eigenvalues of \mathbf{D} . The first element is shown in equation C.18 and the method for computing the other diagonal terms is equivalent.

$$\frac{1}{2M} \sum_{k=0}^{M} e^{2ak\Delta t} \approx \lambda_1 / z_{11} \tag{C.18}$$

Equation C.18 represents a sum of exponentials with k increasing to very large values. Hence, if the time step Δt is small enough, the discrete summation can be written as a continuous integral, where $t = k\Delta t$.

$$\lim_{\Delta t \to 0} \frac{1}{2M} \sum_{k=0}^{M} e^{2ak\Delta t} = \frac{1}{2M\Delta t} \int_{0}^{M} e^{2at} dt = \frac{1}{4aM\Delta t} (e^{2an} - 1)$$

The value of a is negative since it represents the damping of the first mode of the system. Furthermore the value of n must be large, hence the exponential term e^{2aM} decreases rapidly and the integral becomes

$$\lim_{n \to \infty} \frac{1}{4aM\Delta t} (e^{2aM} - 1) = \frac{-1}{4aM\Delta t}$$

Leading to the solution for the value of a

$$a \approx -\frac{z_{11}\Delta t}{4M\lambda_1}$$

The off-diagonal terms of the left hand side in equation C.17 are complex conjugates and both equal to zero. The element of the second line and the first column is

$$\frac{1}{2M} \sum_{k=0}^{M} z_{12} e^{[(a+c)+i(d-b)]k\Delta t} = 0$$

This equality can be expressed in terms of the real and imaginary parts, which are both equal to zero:

$$\begin{cases} \frac{1}{2M} \sum_{k=0}^{M} e^{(a+c)k\Delta t} \cos((d-b)k\Delta t) = 0\\ \frac{1}{2M} \sum_{k=0}^{M} e^{(a+c)k\Delta t} \sin((d-b)k\Delta t) = 0 \end{cases}$$

As stated before, both summations converge to zero, because of the exponential term with a negative coefficient (a + c in this case). Hence the equalities above are respected. Nevertheless, no information about the values of b and d have been obtained.

The present analysis succeeded in obtaining an approximate expression for the real part of the eigenvalues of the system:

$$a_k \approx -\frac{z_{kk}\Delta t}{4M\lambda_k} \tag{C.19}$$

Extension to Common-base POD

As presented in chapter 2, the CPOD approach is an extension of the classical POD method.

Several responses to different sets of initial conditions are assembled into a global

response matrix \mathbf{Y} . The advantage of using CPOD is that the global matrix of responses contains more information than the single response used in the POD approach, i.e. it is more likely to excite all the modes of the system when applying several sets of initial conditions.

In this section, the co-variance matrix of the CPOD method is developed similarly to that of the POD method. The formulation of the approximation of the mode shapes by the CPOD modes and their damping ratios is initially presented.

One considers N_p different sets of responses of the system. The free response of the system corresponding to the initial displacement $\mathbf{x}_p(0)$ is denoted by $\mathbf{y}_p(k\Delta t)$ with $p = 1 \dots N_p$. The global matrix \mathbf{Y} contains all the responses and has dimensions of $N \times (M \times N_p)$:

$$\mathbf{Y} = \left(\begin{bmatrix} \mathbf{y}_1(0) & \mathbf{y}_1(\Delta t) & \dots & \mathbf{y}_1(M\Delta t) \end{bmatrix} \begin{bmatrix} \mathbf{y}_2(0) & \mathbf{y}_2(\Delta t) & \dots & \mathbf{y}_2(M\Delta t) \end{bmatrix} \cdots \\ \begin{bmatrix} \mathbf{y}_{N_p-1}(0) & \mathbf{y}_{N_p-1}(\Delta t) & \dots & \mathbf{y}_{N_p-1}(M\Delta t) \end{bmatrix} \begin{bmatrix} \mathbf{y}_{N_p}(0) & \mathbf{y}_{N_p}(\Delta t) & \dots & \mathbf{y}_{N_p}(M\Delta t) \end{bmatrix} \right)$$

The co-variance matrix \mathbf{D} is defined by

$$\mathbf{D} = \frac{1}{M \times N_p} \mathbf{Y} \mathbf{Y}^T = \frac{1}{M \times N_p} \sum_{k=0}^M \sum_{p=1}^{N_p} \mathbf{y}_p(k\Delta t) \mathbf{y}_p^T(k\Delta t)$$

The free response of the system $\mathbf{y}_p(k\Delta t)$ is

$$\mathbf{y}_p(k\Delta t) = \mathbf{Q}e^{\mathbf{A}k\Delta t}\mathbf{x}_p(0)$$

where the matrices \mathbf{Q} and \mathbf{A} have been defined in the beginning of the Appendix. After several manipulations, comparable to those performed for the POD method, one obtains

$$\mathbf{D} = \mathbf{Q}\mathbf{V}\left(\frac{1}{M \times N_p} \sum_{k=0}^{M} e^{\mathbf{L}k\Delta t} \mathbf{Z}_0 e^{\mathbf{L}^* k\Delta t}\right) \left(\mathbf{Q}\mathbf{V}\right)^T$$

where \mathbf{L} is defined as before and \mathbf{Z}_0 is defined by

$$\mathbf{Z}_0 = \mathbf{V}^{-1} \left(\sum_{p=1}^{N_p} \mathbf{x}_p(0) \mathbf{x}_p^T(0) \right) (\mathbf{V}^{-1})^{\mathbf{T}}$$

Note that this expression is equivalent to the expression for \mathbf{Z}_0 for the POD method (equation C.13), when N is equal to one.

A relation equivalent to equation C.14 can be deduced for the CPOD method:

$$\boldsymbol{\Phi}\boldsymbol{\Lambda}\boldsymbol{\Phi}^{-1} \approx \frac{1}{2}\boldsymbol{\Psi}_{\mathbf{s}} \left(\frac{1}{M \times N_p} \sum_{k=0}^{M} e^{\mathbf{L}_{\mathbf{s}}k\Delta t} \boldsymbol{\Psi}_{\mathbf{s}}^{-1} \left[\sum_{p=1}^{N_p} \mathbf{y}_{\mathbf{p}}(0) \mathbf{y}_{\mathbf{p}}^{T}(0) \right] \left[(\boldsymbol{\Psi}_{\mathbf{s}})^{-1} \right]^{T} e^{\mathbf{L}_{\mathbf{s}}^{*}k\Delta t} \right) \boldsymbol{\Psi}_{\mathbf{s}}^{T}$$

And the modal identification using CPOD is summarized by

$$\begin{split} \Phi &= \Psi_{\mathbf{s}} \\ \mathbf{Z} &= \Phi^{-1} \left[\frac{1}{N_p} \sum_{p=1}^{N_p} \mathbf{y}_p(0) \mathbf{y}_p^T(0) \right] (\Phi^{-1})^T \\ \mathbf{\Lambda} &\approx \frac{1}{2M} \sum_{k=0}^M e^{\mathbf{L}_{\mathbf{s}} k \Delta t} \mathbf{Z} e^{\mathbf{L}_{\mathbf{s}}^* k \Delta t} \end{split}$$

where $z_{k,l}$ denotes the elements of the real symmetric matrix **Z**. Note the division by N_p of the summation in the expression of **Z**, which can be interpreted as a mean value of the co-variance matrices of all the sets of initial conditions. The expression for **Z** shown here is identical to equation C.15b if N_p is equal to one.

The general expression of the approximation of the damping coefficient a_k of the modes is equivalent to equation C.19 for the POD method:

$$a_k \approx -\frac{z_{kk}\Delta t}{4n\lambda_k} \tag{C.20}$$

Appendix D Quasi-steady theory for galloping instability

The quasi-steady model of galloping of a structure with non-zero angle of attack is presented in this appendix. The aerodynamic forces are predicted from the variations of the static aerodynamic force coefficients with angle of attack.

The body is first considered static in the flow-field and the relation between its position and the resulting vertical aerodynamic force is presented. The analysis is further expanded to the case where the body undergoes vertical oscillation and the quasi-steady assumption is used in order to obtain an expression of the aerodynamic vertical force function of the motion of the body:



Figure D.1: Static body in a flow-field

If the body is static in the flowfield, the total angle of attack is defined by $\alpha' = \alpha_s + \alpha$, where α_s is the static angle of the body and α is the angle of attack of the flow velocity V_r with the horizontal axis (see figure D.1). The vertical aerodynamic force F_y is defined positive downwards. It is obtained as the sum of the vertical components of the lift and drag acting on the body.

$$F_y = -F_L(\alpha')\cos\alpha - F_D(\alpha')\sin\alpha \tag{D.1}$$

where F_L and F_D are the lift and drag respectively, per unit length. The non-dimensional force coefficients $C_L(\alpha')$ and $C_D(\alpha')$ are defined by

$$C_L(\alpha') = \frac{F_L(\alpha')}{1/2\rho V_r^2 B} \qquad \qquad C_D(\alpha') = \frac{F_D(\alpha')}{1/2\rho V_r^2 B}$$

where B denotes the chord of the body. The non-dimensional coefficient C_{F_y} , expressed in the vertical coordinate system, is defined relative to the projection of the incident airspeed V_r on the horizontal axis : $V = V_r \cos \alpha$, yielding

$$C_{F_y} = \frac{F_y}{1/2\rho V^2 B}$$

Using equation D.1, the definitions of the force coefficients above and the relation between V and V_r , one finds:

$$C_{F_y} = -\left[C_L(\alpha') + C_D(\alpha') \tan \alpha\right] \sec \alpha \tag{D.2}$$

Now consider the system as a mass-spring-damper with one degree of freedom, exposed to a free-stream with horizontal airspeed V_{∞} (figure D.2). The vertical displacement y(t)is defined positive downwards, similarly to $F_y(t)$, which is now time dependent.

The dynamic equation of motion is given by

$$m\ddot{y} + c\dot{y} + ky = F_y(t) \tag{D.3}$$

where *m* is the mass, *c* the damping and *k* the stiffness, expressed per unit length. In this case, the instantaneous angle of attack α' depends on the static angle of attack, α_s and on the motion-dependent angle of attack α , given by $\tan^{-1}(\dot{y}/V_{\infty})$ so that

$$\alpha' = \alpha_s + \alpha = \alpha_s + \tan^{-1} \frac{\dot{y}}{V_{\infty}}$$

The quasi-steady theory assumes that the lift and drag coefficients $C_L(\alpha')$ and $C_D(\alpha')$ are the same for the oscillating body and the static body. Then expression D.2 for C_{F_y}



Figure D.2: Aeroelastic galloping model

remains valid and the vertical force ${\cal F}_y$ is expressed by

$$F_y(t) = 1/2\rho V^2 B \ C_{F_y} = -1/2\rho V^2 B \ [C_L(\alpha') + C_D(\alpha') \tan \alpha] \sec \alpha$$

where the airspeed V appearing in this expression corresponds to the free stream velocity V_{∞} , yielding

$$F_y(t) = -1/2\rho V_{\infty}^2 B \left[C_L(\alpha') + C_D(\alpha') \tan \alpha \right] \sec \alpha$$
 (D.4)

Note that if the body is static, i.e. $\dot{y} = \alpha = 0$, equation D.4 expresses the static lift force acting on the body :

$$F_{y0} = -1/2\rho V_{\infty}^2 BC_L(\alpha_s) \tag{D.5}$$

Equation D.4 represents the nonlinear expression of the quasi-steady vertical force applied to the body. For small values of $\alpha \approx \frac{\dot{y}}{V_{\infty}}$, the term sec α can be approximated by

$$\sec \alpha \approx \sqrt{\left(\frac{\dot{y}}{V_{\infty}}\right)^2 + 1} \approx 1 + \frac{\dot{y}}{V_{\infty}}$$
 (D.6)

And the corresponding equation of motion of the quasi-steady system is

$$m\ddot{y} + c\dot{y} + ky = -1/2\rho V_{\infty}^2 B\left[C_L(\alpha') + C_D(\alpha')\frac{\dot{y}}{V_{\infty}}\right] \left(1 + \frac{\dot{y}}{V_{\infty}}\right)$$

In order to carry out a stability analysis of this equation of motion, the stability of the underlying linear system is explored. The nonlinear functions $C_L(\alpha')$ and $C_D(\alpha')$ are linearized using a Taylor expansion around α_s for $\alpha = \tan^{-1} \dot{y}/V_{\infty} \approx \dot{y}/V_{\infty}$ so that

$$C_L(\alpha') = C_L(\alpha_s) + \left. \frac{\partial C_L}{\partial \alpha} \right|_{\alpha_s} \frac{\dot{y}}{V_{\infty}} + \ldots + \frac{1}{n!} \left. \frac{\partial^n C_L}{\partial \alpha^n} \right|_{\alpha_s} \left(\frac{\dot{y}}{V_{\infty}} \right)^n + \mathcal{O}(\alpha^n)$$

An equivalent expression is found for $C_D(\alpha')$. To first order, the equation of motion becomes

$$m\ddot{y} + c\dot{y} + ky = -1/2\rho V_{\infty}^2 B \left[C_L(\alpha_s) + \frac{\partial C_L}{\partial \alpha} \Big|_{\alpha_s} \frac{\dot{y}}{V_{\infty}} + \left(C_D(\alpha_s) + \frac{\partial C_D}{\partial \alpha} \Big|_{\alpha_s} \frac{\dot{y}}{V_{\infty}} \right) \frac{\dot{y}}{V_{\infty}} \right]$$

$$= -1/2\rho V_{\infty}^2 B C_L(\alpha_s) - 1/2\rho V_{\infty}^2 B \left[\frac{\partial C_L}{\partial \alpha} \Big|_{\alpha_s} + C_D(\alpha_s) \right] \frac{\dot{y}}{V_{\infty}} \qquad (D.7)$$

Note that the term $-1/2\rho V_{\infty}^2 bC_L(\alpha_s)$ produces a static displacement $y_0 = -\frac{1}{2k}\rho V_{\infty}^2 bC_L(\alpha_s)$ and plays no role in the dynamic behaviour of the system. It can be removed from equation D.7, leading to

$$m\ddot{y} + c\dot{y} + ky = -1/2\rho V_{\infty} B\left(\left.\frac{\partial C_L}{\partial \alpha}\right|_{\alpha_s} + C_D(\alpha_s)\right)\dot{y}$$
(D.8)

The equation of motion of the quasi-steady system is

$$m\ddot{y} + c\dot{y} + ky = 1/2\rho V_{\infty}^2 B C_{F_y} \tag{D.9}$$

where the coefficient C_{F_y} defined in the static case by equation D.2 is expressed for the oscillating body around the static position α_s by

$$C_{F_y} = -\left(\frac{\partial C_L}{\partial \alpha}\Big|_{\alpha_s} + C_D(\alpha_s)\right)\frac{\dot{y}}{V_{\infty}}$$
(D.10)

The total damping coefficient, sum of the structural and aerodynamic components, is equal to

$$c_t = c + 1/2\rho V_{\infty} B\left(\left.\frac{\partial C_L}{\partial \alpha}\right|_{\alpha_s} + C_D(\alpha_s)\right)$$

The critical airspeed V_{crit} is defined as the value of V_{infty} for which the total damping vanishes:

$$V_{crit} = \frac{-c}{1/2\rho B\left(\left.\frac{\partial C_L}{\partial \alpha}\right|_{\alpha_s} + C_D(\alpha_s)\right)}$$

This equation yields Den Hartog's galloping criterion, which states that the system

$$A_1 = -\left(\frac{\partial C_L}{\partial \alpha}\Big|_{\alpha_s} + C_D(\alpha_s)\right) > 0$$

and the critical airspeed is positive:

$$V_{crit} = \frac{2c}{\rho B A_1} \tag{D.11}$$

Appendix E

Aerodynamic center

This appendix presents the analytical solution of the differential equation

$$\sum_{k=0}^{N} \left[A_k k \alpha^{k-1} - \frac{\partial r}{\partial \alpha} B_k \alpha^k - r(\alpha) B_k k \alpha^{k-1} \right] = 0$$
 (E.1)

which can be written as

$$T_1(\alpha)\frac{\partial r}{\partial \alpha} + T_2(\alpha)r + T_3(\alpha) = 0$$
(E.2)

Adopting the following notations:

$$T_{1}(\alpha) = \sum_{k=0}^{N} B_{k} \alpha^{k}$$

$$T_{2}(\alpha) = \sum_{k=0}^{N} B_{k} k \alpha^{k-1} = \frac{\partial T_{1}(\alpha)}{\partial \alpha}$$

$$T_{3}(\alpha) = -\sum_{k=0}^{N} A_{k} k \alpha^{k-1}$$
(E.3)

and using the appropriate changes of variable:

$$P(\alpha, r) = \frac{\partial u}{\partial \alpha} = T_2(\alpha)r + T_3(\alpha)$$
(E.4)

$$Q(\alpha, r) = \frac{\partial u}{\partial r} = T_1(\alpha)$$
 (E.5)

equation E.2 can take the form of an absolute differential equation

$$\frac{du}{d\alpha} = \frac{\partial u}{\partial r}\frac{\partial r}{\partial \alpha} + \frac{\partial u}{\partial \alpha} = 0$$
(E.6)

whose solution is $u(\alpha, r) = cst_1$, under the condition

$$\frac{\partial P}{\partial r} = \frac{\partial Q}{\partial \alpha} \tag{E.7}$$

From the definition of $P(\alpha, r)$ and $Q(\alpha, r)$, the condition expressed by equation E.7 becomes

$$\frac{\partial}{\partial r} \left(T_2(\alpha) r + T_3(\alpha) \right) = \frac{\partial T_1(\alpha)}{\partial \alpha} \to T_2(\alpha) = \frac{\partial T_1(\alpha)}{\partial \alpha}$$

which is satisfied according to the definitions of $T_1(\alpha)$ and $T_2(\alpha)$ (equations E.3).

Hence $u(\alpha, r) = cst_1$ is a solution of the differential equation and it is now necessary to obtain the analytical expression of $u(\alpha, r)$ from equations E.4 and E.5. The integration over α of equation E.4 gives

$$u(\alpha, r) = r \left(T_1(\alpha) - B_0 \right) + T_3(\alpha) - A_0 + \Phi(r)$$
(E.8)

where $\Phi(r)$ is an unknown function of r. The differentiation of E.8 with respect to r yields

$$\frac{\partial u}{\partial r} = T_1(\alpha) - B_0 + \frac{\partial \Phi(r)}{\partial r}$$

This expression for $\frac{\partial u}{\partial r}$ is equivalent to equation E.5, hence

$$T_1(\alpha) - B_0 + \frac{\partial \Phi(r)}{\partial r} = T_1(\alpha) \rightarrow \frac{\partial \Phi(r)}{\partial r} = B_0 \rightarrow \Phi(r) = B_0 r + cst_2$$

where cst_2 denotes a constant value different from cst_1 . Then, equation E.5 becomes

$$u(\alpha, r) = r (T_1(\alpha) - B_0) + T_3(\alpha) - A_0 + B_0 r + cst_2 = cst$$

$$\rightarrow rT_1(\alpha) + T_3(\alpha) - A_0 = cst$$

and the analytical solution of the differential equation is

$$r = \frac{cst + A_0 - T_3(\alpha)}{T_1(\alpha)} = \frac{r_0 + \sum_{k=1}^N A_k \alpha^k}{\sum_{k=0}^N B_k \alpha^k}$$
(E.9)

Appendix F

Aerodynamic coefficients

Figure F.1 presents the static moment coefficient and the force coefficient C_z for the generic bridge section and the rectangular cylinder. These curves have been partially presented in sections 4.2 and 4.3 but are shown here in order to compare the aerodynamic characteristics of the two structures. Note that the values of the moment coefficient for the rectangular cylinder come from the work of Nakamura [109].

In addition, the A_k and B_k coefficients appearing in the polynomials of C_M and C_z (equations 4.12 and 4.13) are presented for the two structures in table F.1. The order of the polynomials is set to N = 9.

	A_0	A = A	$A_1 A_2$	A_3	A_4	A_5	A_6	A_7	A_8	A_9
]	3 0.2	2 0.0	08 -2.8	15.5	-66.1	-413.5	754.2	$8.1e^3$ –	$8.2e^{3}$ -	$-5.8e^{4}$
]	R 0.00)2 -1	.3 -0.1	86.6	-71.4	$-2.5e^{3}$	$5.8e^3$	$2.7e^4$ –	$1.1e^{5}$	$1.2e^{5}$
	B_0	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8	B_9
В	0.6	4.4	-35.9	-29.4	843.1	$-1.3e^{3}$	$-1.2e^{4}$	$5.2e^{4}$	$6.4e^{4}$	$-4.5e^{5}$
R	-0.02	11.9	-5.9	-867.6	$1.1e^{3}$	$2.9e^{4}$	$-6.4e^{4}$	$-3.4e^{5}$	$1.1e^{6}$	$-5.8e^{5}$

Table F.1: Coefficients of the polynomials of the aerodynamic coefficients generic bridge section (B) - rectangle (R)



Figure F.1: Force coefficients - generic bridge (circles) and rectangle (squares)

Appendix G

Publications

The list below presents the conference papers, journal publications and book chapter that have originated from the present thesis:

Journal publications

- T. Andrianne and G. Dimitriadis, Torsional flutter oscillations of a 4:1 rectangular cylinder: Part I - Experiments, submitted to Journal of Fluids and Structures, December 2011
- T. Andrianne and G. Dimitriadis, Torsional flutter oscillations of a 4:1 rectangular cylinder: Part II - Discrete Vortex Simulations, submitted to Journal of Fluids and Structures, December 2011
- T. Andrianne and G. Dimitriadis, Damping identification of lightly damped linear dynamic systems using Common-base Proper Orthogonal Decomposition, Journal of Mechanical Systems and Signal Processing, 28, 492-506, 2012

Book chapter

 T. Andrianne, N. Abdul Razak and G. Dimitriadis, *Flow Visualization and Proper* Orthogonal Decomposition of Aeroelastic Phenomena, in Wind Tunnels, edited by Satory Okamato, ISBN 978-953-307-295-1, InTech, 2011

Conference publications

 T. Andrianne and G. Dimitriadis, Discrete Vortex Simulation and PIV measurements of the flow-field around a 4:1 rectangular cylinder undergoing stall flutter oscillations, Proceedings of the 53th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Hawaii, USA, April 2012

- T. Andrianne and G. Dimitriadis, Experimental Analysis of the Bifurcation Behaviour of a Bridge Deck Undergoing Across-Wind Galloping, Proceedings of the 8th International Conference on Structural Dynamics, EURODYN 2011, pp. 1341-1347, Leuven, Belgium, July 2011
- 3. T. Andrianne, B. Korbahti and G. Dimitriadis, Wind tunnel analysis of separated aerodynamics leading to different types of Torsional Flutter in bluff-bodies, Paper number IFASD2011-137, 15th International Forum on Aeroelasticity and Structural Dynamics, IFASD 2011, Paris, June 2011
- 4. T. Andrianne and G. Dimitriadis, Etude du comportement aéroélastique des structures soumises à des écoulements décrochés, 6ème Séminaire de l'Association de l'Ingenierie du Vent France-Belgique-Suisse (AIV), CNAM Paris, March 2010
- T. Andrianne and G. Dimitriadis, Using the discrete vortex method to simulate the stall flutter phenomenon, International Forum on Aeroelasticity and Structural Dynamics, IFASD 2009, Seattle, Washington, June 2009
- T. Andrianne and G. Dimitriadis, Experimental Analysis of Stall Flutter Phenomena of a Bridge Deck, 8th National Congress on Theoretical and Applied Mechanics, NCTAM 2009, Brussels, Belgium, May 2009

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