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Numerical simulation of rivulet dynamics associated with rain-wind induced vibration

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Abstract

An aeroelastic instability which can affect the cables of cable stayed bridges, Rain-Wind Induced Vibration (RWIV) has been the subject of considerable research. Yet despite determining the characteristics of, and the conditions under which, such a response occurs, the physical mechanism underlying this phenomenon is not fully understood. This thesis summarises previous RWIV literature before taking the first steps towards its numerical simulation. Concentrating, in particular, on modelling rivulet dynamics which are thought to play an important role in causing this instability.

To achieve this two numerical solvers are developed and validated for two different problems, each representing an aspect of RWIV previously studied. A Discrete Vortex Method solver which determines the aerodynamic loading on a body of given geometry, is validated with artificial rivulet wind-tunnel studies and a number of aspects including how the location, form and size of this rivulet affect the overall response are examined. A pseudo-spectral solver which determines the geometric evolution of a thin film given an aerodynamic loading, is validated with previous numerical and analytical solutions for related problems. Likewise, a number of aspects including how loading magnitude and distribution, affect the evolutionary response are examined.

These are subsequently combined to form a numerical model capable of simulating the formation and evolution of rivulets under the aerodynamic field these influence. Given the advancement this signifies however, comparative results to directly validate this are not available. A number of studies are undertaken, with rivulets of self limiting thickness being found to form in each case. The locations and sometimes periodic nature of which are in excellent agreement with previous literature as is the Karman vortex suppression detected under certain circumstances. By successfully capturing such characteristic features of RWIV, the solver presented is the most advanced computational tool for numerically simulating this phenomenon.

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Contents

\mathbf{A}	bstra	act	ii
A	ckno	wledgements	iii
Co	onter	nts	iv
Li	st of	Figures	ix
Li	st of	Tables 2	cvii
N	omer	nclature	xix
1	Intr	roduction	1
	1.1	Motivation	1
	1.2	Outline of Thesis	4
	1.3	Aims and Objectives	6
	1.4	Availability of Data	8
2	Lite	erature Review	10
	2.1	Flow Around Circular Cylinders	11
		2.1.1 Flow Regimes	12
	2.2	Characteristic Features of RWIV	21
	2.3	Other Phenomena Causing Cable Vibration	25
		2.3.1 Vortex Induced Vibration	26
		2.3.2 Galloping	29

		2.3.3	Vortex Induced Vibration at High Reduced Velocity	31
		2.3.4	Dry Inclined Galloping	34
	2.4	Possibl	le Mechanisms of RWIV	37
		2.4.1	Influence of Rivulet	38
		2.4.2	Varying Cross Sectional Geometry	42
		2.4.3	Interaction with Axial Vortex	44
		2.4.4	Interaction with One Bubble Regime	45
	2.5	Full Sc	eale 'In-Situ' Studies	46
		2.5.1	Data Acquisition	47
		2.5.2	Data Interpretation	48
		2.5.3	An Actual RWIV Response	49
	2.6	Wind-	Tunnel Studies	51
		2.6.1	Artificial and Natural Rivulet Studies	51
		2.6.2	Other Parameters	54
	2.7	Analyt	cical Studies	55
		2.7.1	The First Representation	56
		2.7.2	Other Representations	57
	2.8	Previo	us Numerical Studies	60
	2.9	Design	ing Against RWIV	63
		2.9.1	Minimum Scruton Number Criteria	64
		2.9.2	Methods of Mitigation	66
	2.10	Summa	ary	73
3	Aero	odynar	mic Solver - Discrete Vortex Method	75
	3.1	The D	iscrete Vortex Method	76
		3.1.1	Numerical Aspects of the DVM	78
		3.1.2	Issues with the DVM	82
	3.2	Additi	ons and Refinements to DIVEX Implementation	84
		3.2.1	Shear Stress Distribution	84
		3.2.2	Local Modelling of Rivulet	88

		3.2.3	Allowing for Changes in Geometry	90
	3.3	Circula	ar Cylinder Basics	94
	3.4	Forced	Oscillation VIV	102
	3.5	Additi	on of Artificial Rivulets	106
		3.5.1	Initial Validation	108
		3.5.2	Single Rivulet - Effect of Location	110
		3.5.3	Multiple Symmetric Rivulets - Effect of Location	124
		3.5.4	Multiple Antisymmetric Rivulets - Effect of Location	129
		3.5.5	Form and Size of Rivulet	130
		3.5.6	Oscillating Rivulet	136
	3.6	Summ	ary	138
4	Thi	n Film	Solver - Pseudo-Spectral Method	140
	4.1	Thin I	Film Approximation	141
	4.2	The P	seudo-Spectral Method	143
	4.3	Mathe	matical Model	146
		4.3.1	Model Description	147
		4.3.2	Evolution Equation	149
	4.4	Impler	nentation of Present Code	151
		4.4.1	Parameter Selection	152
	4.5	Initial	Validation	157
		4.5.1	Gravity and Surface Tension Loading	157
		4.5.2	Constant Shear Loading Only	160
	4.6	Test C	ases	163
		4.6.1	Gravity and Surface Tension Loading	164
		4.6.2	Shear and Surface Tension Loading	168
		4.6.3	Pressure and Surface Tension Loading	172
		4.6.4	Full Loading	182
		4.6.5	Comparison with Aerodynamic Solver	185
	4.7	Summ	ary	190

5	Cou	pled S	Solver	192
	5.1	Constr	ruction of Coupled Solver	193
		5.1.1	Varying Nodal Location	194
		5.1.2	Smoothing Data	196
		5.1.3	Fourier Series Representation of Data	197
		5.1.4	Transient Aerodynamic Loading	203
		5.1.5	Area Conservation	206
	5.2	Solver	Parameter Selection	208
	5.3	Physic	al Parameter Selection	213
	5.4	Result	S	214
		5.4.1	Shear and Surface Tension Loading	214
		5.4.2	Pressure and Surface Tension Loading	220
		5.4.3	Combined Pressure and Shear Loading	225
		5.4.4	Full Loading	227
	5.5	Case S	Studies	234
		5.5.1	Initial Thickness of Thin Film	235
		5.5.2	Angle of Attack in Plane	239
		5.5.3	Final Study	246
	5.6	Summ	ary	248
6	Con	cludin	g Remarks	252
	6.1	Summ	ary	252
	6.2	Future	e Research	257
Aj	ppen	dices		261
\mathbf{A}	Der	ivatior	n of Modified Angles	261
в	Par	ticular	s of DIVEX Implementation	264
	B.1	Cut-O	ff Functions	264
	B.2	Opera	tion Count	265

		B.2.1	V	orte	ex N	ſer	gin	g						•				 •		266
		B.2.2	Ζ	ona	l D	eco	$^{\mathrm{mp}}$	osi	tior	1.							 •	 •		266
	B.3	Surfac	ce '	Vort	ex	Dis	stril	but	ion									 •		268
~		~					_													
С	Mea	an Cur	rva	tur	e o	fF	ree	e S	burf	ace										270
	C.1	Deriva	atio	n.	•	•								•	•			 •		270
D	Ana	lvtical	18	olu	tio	n f	or	Cc	onst	tant	S	hea	r							272
_			- ~	010			01						-							
	D.1	Deriva	atio	n.	•	•		•	•••					•	•	 •	 •	 •	•	272
	D.2	Examp	ple		•	•								•	•		 •	 •		274
Re	References 274						274													

List of Figures

1.1	Artists impression of proposed Forth crossing	2
1.2	Flowchart for construction of coupled solver	5
2.1	Variation of pattern of vortex shedding with Re	13
2.2	Schematic defining flow regions around a circular cylinder	14
2.3	Schematic of TrW, TrSL and TrBL regimes	15
2.4	Variation of \bar{C}_L , \bar{C}_D , C'_L and C'_D with flow regime $\ldots \ldots \ldots$	16
2.5	Variation of St with Re over TrSL regime $\ldots \ldots \ldots \ldots \ldots$	17
2.6	Variation of St and \bar{C}_D with Re in the TrBL range $\ldots \ldots \ldots$	19
2.7	Configurations of TrBL flow regimes	21
2.8	Definition of stay cable system geometry, α , β and θ	24
2.9	Variation of f_s with U_{∞} , displaying 'lock-in' region $\ldots \ldots \ldots$	27
2.10	Variation of a and f_b with U for RWIV and VIV	28
2.11	Relationship between a, U and β for VIV	29
2.12	Interaction between Karman and axial vortices for HSV	32
2.13	Effect of θ on spectral response of HSV	33
2.14	Minimum value of Z_a for any α_m , variation of ϕ_m with Re	37
2.15	Schematic clarifying notation of upper and lower surfaces	39
2.16	Definition of additional angles, β^* and γ_{θ}	39
2.17	Axial variation of upper and lower rivulets	41
2.18	Variation of rivulet position with U	42
2.19	Proposed excitation mechanism 2.2	43
2.20	Divergent, velocity-restricted and hybrid responses	44

2	2.21	Mean cycle of one bubble mechanism	45
2	2.22	Typical RWIV cable stay event	50
2	2.23	Variation of θ with β . Stable and unstable regions	53
2	2.24	Commonly used wind-tunnel setup	55
2	2.25	Two D.O.F. system for analytically representing RWIV \hdots	57
2	2.26	Stationary points for two D.O.F. system	59
2	2.27	Pressure contours at two points in the shedding cycle	62
2	2.28	Variation of U_R with Sc. Proposed guidelines for DIG	65
2	2.29	Two types of dampers	67
2	2.30	Effect of viscous dampers on response	67
2	2.31	Variation of a with U for varying ζ	68
2	2.32	Typical cable cross-tie system	69
2	2.33	Modal frequency response curve	70
2	2.34	Various forms of cable surface treatment	71
0	2.35	Variation of a with U for varying rib height	73
2			10
3	3.1	Co-ordinate system and body orientation	80
33	3.30 3.1 3.2	Co-ordinate system and body orientation	80 81
4 00 00	3.1 3.2 3.3	Co-ordinate system and body orientation	80 81 81
ຊ (1) (1) (1) (1)	3.1 3.2 3.3 3.4	Co-ordinate system and body orientation	80 81 81 85
	3.1 3.2 3.3 3.4 3.5	Co-ordinate system and body orientation	80 81 81 85 86
	3.1 3.2 3.3 3.4 3.5 3.6	Co-ordinate system and body orientation $\dots \dots \dots$	 80 81 81 85 86 87
	3.1 3.2 3.3 3.4 3.5 3.6 3.7	Co-ordinate system and body orientation $\dots \dots \dots$	80 81 81 85 86 87 88
	3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8	Co-ordinate system and body orientation $\dots \dots \dots$	80 81 81 85 86 87 88 88 89
	3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9	Co-ordinate system and body orientation	80 81 81 85 86 87 88 89 92
	3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10	Co-ordinate system and body orientation $\ldots \ldots \ldots \ldots \ldots \ldots \ldots$ Regions and labels of flow domain $\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$ Surface vorticity discretisation and outline of control zone $\ldots \ldots$ Linear velocity distribution at wall $\ldots \ldots \ldots$ Position of additional points for calculation of wall shear stress $\ldots \ldots $	80 81 81 85 86 87 88 89 92 92
	3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11	Co-ordinate system and body orientation	 80 81 81 85 86 87 88 89 92 92 92 92
	3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12	Co-ordinate system and body orientation	 80 81 81 85 86 87 88 89 92 92 92 92 93
	3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12 3.13	Co-ordinate system and body orientation	 80 81 81 85 86 87 88 89 92 92 92 92 93 94
	3.1 3.2 3.3 3.4 3.5 3.6 3.7 3.8 3.9 3.10 3.11 3.12 3.13 3.14	Co-ordinate system and body orientation	 80 81 81 85 86 87 88 89 92 92 92 92 92 93 94 97

3.15	Power Spectral Density indicating St peak $\ldots \ldots \ldots \ldots$	98
3.16	Instantaneous picture of vortex particle location $\ldots \ldots \ldots$	99
3.17	Variation of \bar{C}_P with θ . Comparison with literature	99
3.18	Definition of C_{pb}, C_{pm}, θ_b and θ_m	100
3.19	Variation of C'_P with θ . Comparison with literature $\ldots \ldots \ldots$	100
3.20	Variation of u/U_{∞} with x/D	101
3.21	Schematic of centreline	101
3.22	Power Spectral Density below, during and above 'lock-in'	104
3.23	Variation of f_s/f_b with U_R for various a/D	104
3.24	Variation of \bar{C}_D with U_R	105
3.25	Form and size of base artificial rivulet	107
3.26	Three cases of rivulet geometry	108
3.27	Variation of \bar{C}_P with θ for single artificial rivulet	110
3.28	Variation of \bar{C}_L with θ for single artificial rivulet	111
3.29	Variation of \bar{C}_L and \bar{C}_D with θ for single artificial rivulet	112
3.30	Variation of \bar{C}_P with θ for single artificial rivulet at $\theta = 20^\circ$	113
3.31	Mean and RMS normalised velocity contours for single artificial	
	rivulet at $\theta = 20^{\circ}$	114
3.32	Mean and RMS normalised velocity contours for single artificial	
	rivulet at $\theta = 50^{\circ}$	115
3.33	Variation of \bar{C}_P with θ for single artificial rivulet at $\theta = 50^{\circ}$	116
3.34	Velocity vector plot at two points in the shedding sequence for	
	single artificial rivulet at $\theta = 50^{\circ}$	117
3.35	Time averaged mean velocity vectors for the symmetric rivulet case	
	at $\theta = 50^{\circ}$	118
3.36	Mean and RMS normalised velocity contours for single artificial	
	rivulet at $\theta = 75^{\circ}$	119
3.37	Variation of \bar{C}_P with θ for single artificial rivulet at $\theta = 75^{\circ}$	120
3.38	Variation of \bar{C}_P with θ for single artificial rivulet at $\theta = 150^\circ$	120

3.39	Mean and RMS normalised velocity contours for single artificial	
	rivulet at $\theta = 150^{\circ}$	121
3.40	Variation of θ_s with θ for single artificial rivulet $\ldots \ldots \ldots$	122
3.41	Variation of St with θ for single artificial rivulet $\ldots \ldots \ldots$	123
3.42	Power Spectral Density within and outwith $40^\circ < \theta < 60^\circ$ regime	123
3.43	Variation of $\frac{\partial \bar{C}_L}{\partial \theta} + \bar{C}_D$ with θ for single artificial rivulet	124
3.44	Variation of \bar{C}_L and \bar{C}_D with θ for symmetric artificial rivulets	125
3.45	Comparison with previous literature for symmetric artificial rivulets	s126
3.46	Variation of \bar{C}_L with θ for slight asymmetry $\ldots \ldots \ldots \ldots$	127
3.47	Variation of \bar{C}_L with flow angle for various rivulet locations	128
3.48	Variation of \bar{C}_L and \bar{C}_D with θ for antisymmetric artificial rivulets	129
3.49	Various forms of artificial rivulet studied	131
3.50	Variation of \bar{C}_L with θ for various rivulet forms $\ldots \ldots \ldots$	132
3.51	Variation of \bar{C}_D with θ for various rivulet form	132
3.52	Variation of \bar{C}_L with θ for various rivulet widths	134
3.53	Variation of \bar{C}_L with θ for various rivulet heights $\ldots \ldots \ldots$	135
3.54	Variation of \bar{C}_D with θ for various rivulet heights $\ldots \ldots \ldots$	135
3.55	C_P 'jumps' when oscillating rivulet is in or out of wake region $\ . \ .$	137
4.1	Notation used for thin film on circular cylinder	146
4.2	Variation of h/h_0 with θ . For varying N	153
4.3	Error in Fourier Series Truncation of \bar{C}_P	156
4.4	Variation of \bar{C}_P and \bar{C}_F^* with θ	157
4.5	Variation of τ_{max} with \hat{Bo}	159
4.6	Variation of h/h_0 with τ at $\theta = 90^\circ$ and 270°	160
4.7	Variation of h/h_0 with θ at $\tau = 0.2$ and 0.4	160
4.8	Variation of h/H with θ at $t = 1$ s and 1.6032 s	162
4.9	Variation of h/H with t at $\theta = 90^{\circ}$ and 270°	163
4.10	Temporal evolution of film thickness under gravitational and sur-	
	face tension loading	165

4.11	Variation of h_0/h with θ for various G at fixed t	166
4.12	Variation of h_0/h with θ for various \hat{G} at fixed τ	167
4.13	Temporal evolution of film thickness under shear and surface ten-	
	sion loading	168
4.14	Comparison of profile at $t = 6.9 \times 10^{-3} \mathrm{s}$	169
4.15	Variation of h/h_0 with θ for two different solvers	170
4.16	Variation of h/h_0 with θ for various U	171
4.17	Variation of h/h_0 with θ for shear and pressure loading with and	
	without surface tension	172
4.18	Temporal evolution of film thickness under pressure and surface	
	tension loading	173
4.19	Comparison of profiles from pressure and shear loading \ldots .	174
4.20	Comparison of profile at $t = 6.9 \times 10^{-3} \mathrm{s}$	174
4.21	Variation of h/h_0 with θ for two different solvers	175
4.22	Variation of \bar{C}_P with θ for four profiles of \bar{C}_P	176
4.23	Variation of h/h_0 with θ for four profiles of \bar{C}_P	178
4.24	Variation of h/h_0 with θ for varying Re , for shear and surface	
	tension loading	180
4.25	Variation of h/h_0 with θ for varying Re , for pressure and surface	
	tension loading	180
4.26	Temporal evolution of film thickness under full loading \ldots .	182
4.27	Variation of h/h_0 with t of upper and lower rivulets	183
4.28	Comparison of profile at $t = 6.9 \times 10^{-3} \mathrm{s}$	184
4.29	Comparison of profile at $t = 6.9 \times 10^{-3}$ s for three loading combi-	
	nations	185
4.30	Variation of \bar{C}_P and $\bar{C_F}^*$ determined from DVM code with θ	186
4.31	Variation of h/h_0 with θ for various loadings combinations \ldots	187
4.32	Variation of h/h_0 with θ , for varying \bar{C}_P profiles, pressure and	
	surface tension loading	189

4.33	Variation of h/h_0 with θ for varying \bar{C}_P profiles, full loading \ldots	189
5.1	Simplified flowchart of coupled solver	193
5.2	Nodal locations of different solvers	194
5.3	Variation of h/h_0 with θ , for interpolated versus actual results	195
5.4	Simplified flowchart of coupling between two solvers	196
5.5	Variation of smoothed C_P with θ at $t = 15 \times 10^{-3} \text{ s} \dots \dots$	197
5.6	Variation of smoothed and modelled C_P with θ at $t = 15 \times 10^{-3}$ s	199
5.7	Variation in \mathbb{R}^2 and n with timestep $\ldots \ldots \ldots \ldots \ldots \ldots$	200
5.8	Variation in \bar{C}_P with θ for actual, smooth and modelled data	201
5.9	Variation in \bar{q} with θ for pressure and shear loading	202
5.10	Variation in \bar{q} and σ_q with θ between original, smoothed and mod-	
	elled C_P	203
5.11	Variation of time to solve with timestep	205
5.12	Definition of h_{eff} for conservation of area	207
5.13	Variation in normalised body area and radius with timestep $\ . \ .$.	208
5.14	Variation of h/h_0 with θ , for varying Δt^*	210
5.15	Variation of h/h_0 with $\theta,$ for varying numbers of thin film nodes $% \theta$.	211
5.16	Variation of time to solve with timestep for different solvers \ldots	212
5.17	Temporal evolution of film thickness under shear and surface ten-	
	sion loading	215
5.18	Variation of h/h_0 with t of upper and lower rivulet under shear	
	loading	216
5.19	Variation of θ with t of upper and lower rivulet under shear loading	g216
5.20	Variation of h/h_0 with t at $\theta = 0^\circ$	217
5.21	Variation of θ and h/h_0 with t for fixed and varying C_F	219
5.22	Temporal evolution of film thickness under pressure and surface	
	tension loading	221
5.23	Variation of h/h_0 with t of upper and lower rivulet under pressure	
	and surface tension loading	221

5.24	Variation of θ and h/h_0 with t for fixed and varying C_P	223
5.25	Variation of \bar{C}_P with θ under pressure and surface tension loading	223
5.26	Power Spectral Density indicating St peak $\ldots \ldots \ldots \ldots$	224
5.27	Temporal evolution of film height under pressure, shear and surface	
	tension loading	225
5.28	Variation of h/h_0 with t of upper rivulet for three loading combi-	
	nations	226
5.29	Variation of θ with t for upper rivulet for three loading combinations	s227
5.30	Temporal evolution of film thickness under full loading \ldots .	228
5.31	Variation of h/h_0 with t of lower rivulet for all four loading com-	
	binations	229
5.32	Variation of h with t at $\theta = 67^{\circ}$	230
5.33	Temporal evolution of film thickness of upper rivulet region under	
	full loading	231
5.34	Variation of h with t of ripples	232
5.35	Variation of θ with t of ripples	232
5.36	Variation of θ and h/h_0 with t for fixed and varying C_P and C_F .	233
5.37	Variation of h/h_0 with t for varying $h_0 \ldots \ldots \ldots \ldots \ldots$	236
5.38	Variation of h with t for varying $h_0 \ldots \ldots \ldots \ldots \ldots \ldots$	236
5.39	Variation of h with θ for varying h_0 at $t = 1.4$ s	237
5.40	Schematic defining γ_{θ} and U_{eff}	239
5.41	Temporal evolution of film thickness under full loading at $\gamma_{\theta}=20^{\circ}$	240
5.42	Variation of h/h_0 with t for different γ_{θ}	241
5.43	Schematic showing effect of varying γ_{θ}	241
5.44	Variation of $\bar{\theta}_{ac}$ with γ_{θ}	241
5.45	Temporal evolution of film thickness under full loading at $\gamma_{\theta}=35^{\circ}$	242
5.46	Power Spectral Density at $\gamma_{\theta} = 10^{\circ}$ and $\gamma_{\theta} = 20^{\circ} \dots \dots \dots \dots$	245
5.47	Temporal evolution of film thickness under full loading at $\gamma_{\theta}=20^{\circ}$	
	with varied g_{eff} and U_{eff}	246

5.48	Variation of θ and h/h_0 with t for flat and inclined cases	247
A.1	Definition of α , β , β^* , U_{eff} and γ_{θ}	261
B.1	Vorticity and velocity distributions for various core functions	265
B.2	Sample flow field - Hierarchical zonal decomposition	267
B.3	Sample flow field - Series expansion	268

List of Tables

1.1	Cost estimates for possible types of proposed Forth crossing 2
2.1	Variation of flow regime with Re for circular cylinder $\ldots \ldots \ldots$
2.2	Cable-stayed bridges where RWIV has been reported
2.3	Mean wind speed ranges reported for RWIV
2.4	Maximum reported amplitude of RWIV by bridge
2.5	Time-frames of 'in-situ' studies by bridge
2.6	Cable stayed bridges which incorporate helical ribs
3.1	Comparison of C'_L , \bar{C}_D and St with previous literature $\ldots \ldots 96$
3.2	Comparison of C_{pb}, C_{pm}, θ_b and θ_m , with literature
3.3	Values for VIV parameter study
3.4	Comparison of St , C_{pb} , \overline{C}_D and C'_L with previous literature 106
3.5	Sizes and forms of artifical rivulets previously examined 108
4.1	Multiplication factors for a_k
4.2	Convergence study for N
4.3	Convergence study for Δt
4.4	Values of the standard parameters
4.5	Parameters and fluid properties of water, benzene and mercury 165
5.1	Increase in time to solve under different combinations of load 206
5.2	Increase in time to solve per timestep of coupled solver
5.3	Comparison of C'_L , \bar{C}_L , \bar{C}_D and St with previous cases $\ldots \ldots 219$

5.4	Comparison of $\bar{\theta}_{ac}$ under four loading combinations considered $~$.	229
5.5	Comparison of C'_L , \bar{C}_L , \bar{C}_D and St with previous cases $\ldots \ldots$	234
5.6	Comparison of C'_L , \bar{C}_L , \bar{C}_D and St for various $h_0 \ldots \ldots \ldots \ldots$	238
5.7	Combinations of α , β at which previous experiments determined	
	RWIV	243
5.8	Comparison of \bar{h} , $\bar{\theta}$ and $\bar{\theta} + \gamma_{\alpha}$ for varying γ_{θ}	244

Nomenclature

Latin symbols

A	Area of Body
В	Body region in flow field
Bo	Bond Number
C_D	Coefficient of drag
\bar{C}_D	Time averaged mean coefficient of drag
C_D'	Fluctuating (or RMS) coefficient of drag
C_F	Coefficient of friction
\bar{C}_F	Time averaged mean coefficient of friction
$\bar{C_F}^*$	Normalised time averaged mean coefficient of friction
C_L	Coefficient of lift
\bar{C}_L	Time averaged mean coefficient of lift
C_{L0}	Amplitude of coefficient of lift
C_L'	Fluctuating (or RMS) coefficient of lift
C_P	Coefficient of pressure
\bar{C}_P	Time averaged mean coefficient of pressure
C'_P	Fluctuating (or RMS) coefficient of pressure
C_{pb}	Pressure coefficient in wake region
C_{pm}	Minimum value of pressure coefficient
D	Cylinder diameter
F	Flow region
F_b	Control zone region of flow field

F_{body}	Body force	
F_D	Drag force	
F_L	Lift force	
F_w	Wake zone region of flow field	
G	Gravity Number	
G_0	Vortex merging parameter	
Н	Typical film thickness	
K_p	Number of panels	
K_{sp}	Number of sub-panels	
L	Typical length scale	
N	Number of nodes, particles or operations	
N_p	Number of vortex particles in zone, for zonal decomposition	
N_p	Number of terms in series expansion, for zonal decomposition	
Р	Pressure	
Q	Azimuthal volume flux	
R	Radius	
R_N	Residual in spectral method	
R^2	Coefficient of determination	
Re	Reynolds number	
S	Surface in flow field	
Sc	Scruton number	
St	Strouhal number	
T	Shear stress at wall	
U	Wind speed	
	Flow velocity	
\hat{U}	Typical velocity in thin film	
U_{eff}	Effective wind speed in plane normal to cable axis	
U_R	Reduced velocity	
U_{ic}	Velocity at reference point of body	

U_{crit}	Critical velocity for the onset of galloping	
U_n	Normal component of free stream flow velocity	
U_z	Velocity induced by given zone in zonal decomposition	
U_{∞}	Free stream flow velocity	
W	Incomplete elliptic integral of the first kind	
Z_a	Non-dimensional aerodynamic damping parameter	
Z_s	Non-dimensional structural damping parameter	
a	Vibration amplitude	
a_j	Coefficients of series expansion for zonal decomposition	
a_k	Coefficients of trial functions for pseudo-spectral method	
	Fourier Coefficients	
a_s	Normalised amplitude of function, analytical shear calculation	
С	Value to be modelled in coefficient of determination	
\bar{c}	Mean of value to be modelled in coefficient of determination	
cm	Value actually modelled in coefficient of determination	
d	Net damping coefficient	
$e_{(x)}$	Unit vector in given coordinate, in this instance x	
f_{axial}	Frequency of axial vortex	
f_b	Natural mechanical frequency of structure	
f_s	Vortex shedding frequency	
f_{Ω}	Undamped natural circular frequency	
g	Gravitational vector in thin film context	
g_{eff}	Effective gravity in plane	
h	Thickness of film	
\bar{h}	Mean thickness of film	
h_{eff}	Additional offset of film thickness to conserve area	
h_0	Initial film thickness	
i,j,k	Traditional Cartesian unit vectors	
j	Imaginary unit	

k	Multiplying coefficient	
m	Mass per unit length	
n	Normal vector	
n	Number of terms in Fourier series truncation	
n_s	Number of terms in series of analytical calculation	
\hat{p}	Pressure in thin film	
q	Original value of a function	
q^*	Smoothed value of a function	
q_m	Modelled value of function	
$ar{q}$	Mean difference between actual, smoothed and modelled distributions	
r	Radial co-ordinate	
r	Position vector	
r	Magnitude of position vector	
\hat{r}	Direction or unit position vector	
$oldsymbol{r}_{hd}$	Position vector to high definition reference point	
$oldsymbol{r}_{mb}$	Position vector of high definition vortex to high definition reference point	
t	Tangential vector	
t	Time	
t^*	Non-dimensional time for DVM code	
\hat{u}	Velocity vector in thin film	
u	Velocity parallel to wall for shear calculation	
	Local velocity	
u'	Fluctuating local velocity	
v	Function to be represented in pseudo-spectral method	
v'	Derivative of function in pseudo-spectral method	
v_N	Truncated series expansion representing function	
x_{hd}, y_{hd}	Local coordinate system for high definition region	
y	Transverse body displacement	
	Normal distance to wall, for shear model	

y_0	Amplitude of transverse body displacement
y_{step}	Step size for Adams Bashforth method
z	Third space co-ordinate, orthogonal to plane studied

Greek symbols

Γ	Circulation	
Φ	Velocity potential	
Ω	Rotational velocity	
α	Cable angle of inclination	
α_m	Angle of cable response for DIG	
α_{θ}	Angle of incident flow with respect to a fixed body	
α_w	Wind attack angle for dry inclined galloping	
β	Cable angle of yaw	
β^*	Relative yaw angle	
γ	Coefficient of surface tension	
γ_{θ}	Angle of attack in plane normal to the cable axis	
ϵ	Aspect ratio	
ϵ_z	Truncation error in series expansion for zonal decomposition	
ζ	Damping ratio	
ζ_a	Aerodynamic damping ratio	
ζ_s	Structural damping ratio	
η	Random walk	
θ	Angle clockwise from windward horizontal	
	Angular co-ordinate	
θ_{ac}	Angle anti-clockwise from windward horizontal	
θ_b	Angle where minimum value of C_P is reached	
θ_{le}	Angle of leading edge of artificial rivulet	

- θ_m Angle where C_{pb} first reached
- θ_s Separation angle
- κ Mean curvature of the free surface
- λ Magnitude of calculation point for determination of shear profile
- μ Dynamic viscosity
- ν Kinematic viscosity
- ξ Angular location for analytical shear calculation
- ρ Fluid density
- σ Core radius (for cut-off function)
- σ_q Standard deviation between DVM, smoothed and modelled distributions
- au Reduced time
- τ_{max} Final resolved reduced time for solver
- ϕ Phase angle
- ϕ_m Cable wind angle for DIG
- ψ Stream-function (Vector potential)
- ψ_k Trial function for pseudo-spectral method
- ω Vorticity vector
- ∇ Gradient operator
- $\nabla \cdot$ Divergence operator
- $\nabla \times$ Curl operator
- $\nabla^{2} \qquad \text{Laplace operator} \qquad \left(= \frac{\partial^{2}()}{\partial x^{2}} + \frac{\partial^{2}()}{\partial y^{2}} \right)$ $\frac{D}{Dt} \qquad \text{Material derivative} \qquad \left(= \frac{\partial()}{\partial t} + (\boldsymbol{U} \cdot \nabla) \right)$ $\frac{\partial}{\partial x} \qquad \text{Partial derivative with respect to x}$
- $\triangle U$ Difference in induced velocity
- $\triangle t$ Timestep
- $\triangle y$ Mesh size

Common Subscripts

- hd High definition region
- *i* Body identifier
- *ic* Reference co-ordinate of body
- j Identifier for terms in series expansion of zonal decomposition
- jp Panel identifier for conservation of circulation
- k Identifier for particles in zone in zonal decomposition algorithm
- mp Sub-panel identifier for conservation of circulation
- s Value when shock occurs analytical shear calculation
- p Point in the flow field
- t Differentiated with respect to t
- w Wall
- x Component in x direction Differentiated with respect to x
- *y* Component in y directionDifferentiated with respect to y
- z Particular zone, zonal decomposition
- zc Zone centre, zonal decomposition
- 0 Original value of parameter
- ∞ Far field Boundary
- θ differentiated with respect to θ
- ^ property of thin film
- * thin film scaling

Chapter 1 Introduction

Under the combined actions of wind and rain the cables of cable stayed bridges can undergo oscillations which, despite being potentially detrimental to the life of the structure, are not yet fully understood. This thesis details the present level of understanding of the physical mechanism which underlies these Rain-Wind Induced Vibrations (RWIV), the methods by which this was achieved and identifies areas still to be addressed. In response to the latter two numerical solvers are presented and validated for two distinct but different problems, each representing an aspect of RWIV previously studied. These are subsequently combined to form a coupled solver which, for the first time, provides a computational technique capable of numerically simulating the formation and evolution of rivulets under the external aerodynamic field these influence.

1.1 Motivation

Constructing bridges to span large distances is a major financial and temporal undertaking. To minimise these there is typically an optimum bridge type for a given distance between supports. For shorter spans (< 400 m) a cantilevered design reduces financial outlay by requiring less temporary support during construction. While a suspension bridge is typically the only design capable of crossing the largest spans (> 1200 m). In the interim range, cable-stayed bridges are typically the preferred choice as the increase in construction "costs per m^2 as a

function of span are considerably smaller than for any other type of structure" [1]. A pertinent example at the time of writing was the announcement [2] that the new Forth crossing, Firth of Forth, Scotland, was to be of cable stayed design, figure 1.1, due to the initially budgeted costs outlined in table 1.1 and other criteria such as operating restrictions, environmental impact and construction time. This figure subsequently decreased in December 2008 to £1.7 - 2.3 billion [3].



Figure 1.1: Artists impression of cable stayed bridge for the proposed new Forth crossing, from Transport Scotland [4].

Proposal	Cost estimate inc. VAT (\pounds)
Cable-stayed bridge	3.25 - 4.22 billion
Suspension bridge	3.62 - 4.70 billion
Bored tunnel	5.12 - 6.60 billion
Immersed tube tunnel	4.77 - 6.19 billion

Table 1.1: Initial cost estimates of possible designs considered for the proposed new Forth crossing, from Scottish Government [2].

Given the scale of these commitments, designers are keen to ensure such bridges are 'fit for purpose' and attain expected design life. However as bridges become longer they also consistently become taller and lighter, and as such more flexible. In so doing the possible occurrence of an aeroelastic phenomenon, an aerodynamic loading which affects a structural response, increases. As such a response can reduce serviceability, lifespan and public confidence in the bridge as a whole, calculation of the unsteady aerodynamic field, the force this induces and the structural response which emanates are now a major part of the design process. This is particularly true for cable-stayed bridges, as in addition to a possible aeroelastic response within the towers or decks faced by other bridge designs, the stay cables themselves are susceptible to vibration under wind loading.

Therefore, in an attempt to alleviate or eliminate these vibrations from the operating conditions, responses are classified as distinct phenomenon dependent upon the factors which cause the oscillation and the characteristics of the cable motion. This allows designers to better understand and hence to mitigate against each of these possible vibrations individually. The level of understanding of each of these phenomena however are not equal. Consequently those which are not sufficiently understood, while mitigable, may prove difficult to efficiently design against. RWIV is one such response.

Since first documented in the mid 1980's [5], RWIV has been the focus of a considerable body of research in the wind and bridge engineering communities. However, due to the complexity of the unsteady three-dimensional flow field, the three phase nature of the stay/rain/air interface, the non-linear dynamics of the response and the inherent transience of the environmental conditions, to date this has not provided a complete understanding of the underlying physical mechanism. That said, a range of conditions under which such vibrations commonly occur has been established and the manner of response identified. Several robust manners of mitigation have also been developed.

Other Fluid Structure Interaction (FSI) problems have shown that the incorporation of numerical simulations into the design process can reduce both the financial and temporal costs involved and can provide detailed information about the flow field that may not be easily extracted experimentally. The development of such a numerical model to analyse RWIV would therefore be a valuable tool for the designer in the long-term, and could help elucidate the underlying governing mechanism in the short term. Despite this, a body of work concerning numerical simulation of RWIV has not been undertaken or even initiated to date. The main motivation of this thesis was to provide the initial steps of such a program and to ascertain the feasibility for future work as part of a 'proof of concept' test. With the numerical solver created particularly focused on tracking the formation and evolution of rain water rivulets thought to play an important role in causing this response.

1.2 Outline of Thesis

Given the present state of numerical simulation of RWIV, or lack thereof, and the requirement to validate the present work against available literature, the progression undertaken to construct this solver comprised three main sections. First, an extensive literature review was undertaken, highlighting both the present understanding of the physical mechanism behind RWIV and the present state of numerical simulation thereof. Second, two solvers were developed, one to ascertain the aerodynamic field about a given body in the presence of artificial rivulets and one to ascertain the evolution of a thin film into rivulets given such an aerodynamic field. These were then validated against previous literature and a number of studies undertaken. Finally, these two distinct solvers are combined to form a coupled solver capable of numerically simulating the formation and evolution of rivulets under the external aerodynamic field these influence. Shown in figure 1.2, this progression is reflected in the structure of this thesis. The chapters which follow can therefore be summarised as:

Chapter 2 RWIV and the parameters which characterise this from other aeroelastic instabilities are introduced and placed in context. A review of the 'in-situ', wind-tunnel, analytical and previous numerical research which have determined these characteristics is conducted and a summary of the present methods of practical mitigation and design guidelines are given. Characteristics of bluff body flows around circular cylinders, are also summarised.



Figure 1.2: Flowchart for construction of final coupled solver.

Chapter 3 The Discrete Vortex Method (DVM) and the reasons for its selection in determining the present aerodynamic field are outlined. A brief review of the particular implementation used and its history in related fields of bluff body flow fields are given [6, 7], before the modelling updates implemented are discussed. The results of validation exercises for a static and oscillating plain circular cylinder and for similar exercises examining the addition of a rigid rivulet are presented and discussed. Various further studies regarding the form, location and size of these rivulets are then undertaken.

Chapter 4 A governing equation determining the evolution of a thin film of fluid on the outer surface of a circular cylinder subject to the effects of gravity and surface tension loading in addition to pressure and shear forces, which result from the aerodynamic flow field, is developed. A brief review of the pseudospectral method, the reasons for its selection in simulating the evolution of this film and details of the particular solver created are discussed. This is then validated against analytical [8] and previous numerical [9] solutions for various combinations of loading. Several further studies, including how the distribution and magnitude of aerodynamic loading influence the rivulets temporal evolution are then undertaken and discussed. **Chapter 5** The DVM and pseudo-spectral solvers are combined and pertinent details of the coupling process, including convergence studies, are presented. Without specific data against which to directly validate the final solver, tests regarding accuracy and potential effectiveness of the method are undertaken where possible and the results discussed. Four loading cases, including the full representative physical conditions, are examined and differences between these results and those from the independent thin film solver of chapter 4 are outlined. Two further studies into the effects that the magnitude of initial film thickness and the angle of attack in plane have on the resulting evolution are then undertaken and discussed.

Chapter 6 Conclusions regarding the effectiveness of both the individual DVM and pseudo-spectral solvers and the combined solver are drawn where possible, and the contributions made to research discussed. Suggestions for future research and potential improvements to the present method are then made.

1.3 Aims and Objectives

RWIV is a highly complex, non-linear, three-dimensional aeroelastic instability which to date has undergone only limited numerical modelling. Consequently it is likely that the creation of a solver capable of fully simulating this phenomenon will not occur in a single step, but through a series of incremental steps, each of increasing complexity and bearing more resemblance to the full fluid structure interaction. As such, this thesis does not aim to explicitly model RWIV but rather to focus on one particular aspect of the instability, rivulet formation which is thought to be central to the underlying governing mechanism. The objectives of this research are therefore to simulate, in two dimensions, the formation and evolution of rivulets from a thin film of fluid on the outer surface of a circular cylinder in a uniform flow and gravity field. Through examination of this simplified representation, the research presented aims to clarify the influence of fixed artificial rivulets on the aerodynamic response and the underlying physical mechanism and to highlight features associated with the coupled evolution of rivulets which have not previously been investigated. Accomplishing which, should create the most advanced computational tool for simulating aspects of RWIV, to date, while acting as a 'proof of concept' of both the solver and the methodology used This in turn allows the present work to act as the basis for subsequent development of a more complete numerical method.

The key aims and objectives of this research can therefore be summarised as the development and validation of:

- an aerodynamic solver capable of numerically simulating the flow field around a cylinder with and without artificial rivulets. Having achieved which, case studies are undertaken whose objective is to determine the effect that static rivulet size, location and form have on the time averaged response of the cylinder.
- a pseudo-spectral solver capable of numerically simulating the evolution of a thin film flow on the outer surface of this cylinder under fixed aerodynamic loading, gravity and surface tension. Having achieved which, case studies are undertaken whose objective is to determine the effect that Reynolds number and various magnitudes, distributions and combinations of loading have on the geometry and location of the rivulets formed.
- a coupled solver capable of numerically simulating the formation and evolution of rivulets from a thin film under uniform gravitational loading and a transient aerodynamic field that is dependent upon the rivulets themselves. Having achieved which, case studies are undertaken whose objective is to determine the effect that initial film thickness, angle of attack in plane and various combinations of loading, have on the geometry and location of the rivulets formed.

1.4 Availability of Data

Fortunately for bridge designers and the general public, the number of sites on which occurrences of RWIV have been reported to date are still limited, unfortunately for the present purpose, this is reflected in the breadth of 'in-situ' data available. Indeed given the uniqueness of an individual event, the limited range of conditions and the large time intervals between such vibrations, the financial and temporal requirements for such full scale studies are extensive. Given this prohibitive expense, 'in-situ' studies are typically only undertaken at the onset of a new occurrence, or with government backing and are primarily directed towards environmental and structural examinations of the response. These enable the identification of response characteristics and parameter ranges under which vibrations occur but leave the governing mechanism of RWIV unresolved. Consequently most research undertaken at academic institutions focuses on examination of a particular set of variables and the effects these have on the overall response, which are more practical to investigate experimentally. However even these do not provide the resolution of time and length scale necessary for validation of particular aspects of the present work. As such other sources of data, which may not be in this specific area of interest, must also be employed to ascertain confidence in the results of the numerical studies undertaken. To ensure this the numerical solvers presented here are evaluated using several different types of available data. These are:

In-Situ work Ideally this would provide the means to fully validate the numerical solvers created. However given; the proportion of variables tracked of those which could be tracked, the degree of uncertainty in capturing environmental data and its inherent transience, especially of key variables such as wind speed U, 'in-situ' data is mainly used in a qualitative sense within the present context. Analytical solutions These can be split into two categories. Exact solutions for the governing equation for the evolution of thin films and mathematical models generated to simulate the possible mechanism of RWIV. The former are the preferred method of validating the independent thin film solver of chapter 4, while due to the assumptions made not concentrating on rivulet formation and evolution, the latter are not of direct relevance and are only used in a qualitative sense for interpreting results.

Numerical results Given its status as a 'benchmark' test, there is a large body of research determining the flow field around circular cylinders and the multitude of variables therein. Where these numerical results are previously validated against experiment, a qualitative and quantitative comparison could be undertaken. However, while providing an interesting comparison, without such a background these results are not used to validate the results of the present simulation.

Experimental work Given that these provide the most accurate known data, experimental results are the preferred means of validation for the numerical simulations undertaken herein which involve either the DVM or the final coupled solvers of chapters 3 and 5 respectively. Therefore, where good quality, repeatable, experimental work from reliable sources is available, such results are used extensively. This however proved difficult for the final coupled solver.

However, before any of this data can be used, its origins should be clarified and put into a RWIV context. In so doing, the following chapter details the present understanding of the physical mechanism underlying RWIV and the characteristics of its response. It also outlines the present consensus as to the conditions under which this instability occurs, how this was achieved, how the phenomenon can be mitigated against and what numerical modelling has previously been undertaken.

Chapter 2 Literature Review

Investigations of RWIV can typically be classified into four categories distinguishable by the nature of the study and by method employed. Outlined in the introduction these can be summarised as :

- Full-scale 'in-situ' studies where actual climatic, structural and vibrational response data are gathered and countermeasures tested.
- Wind-tunnel studies where environmental conditions can be controlled and the inherent uncertainty, transience and non repeatability limited. Thus allowing the influence of specific variables to be examined.
- Analytical analyses where idealised mathematical representations of the system are constructed and the influence of specific variables on the response are examined.
- Numerical simulations where computational analysis replicates aspects of the phenomenon allowing the influence of specific variables to be examined.

This chapter highlights the present state of research in each of these categories and outlines the techniques and methods used to achieve this. Practical measures used to mitigate such vibrations should these arise and the codes proposed to eliminate these are also introduced.

The underlying mechanism of RWIV is not yet fully understood. As such determining which occurrences of cable oscillation are RWIV and which are the
result of other related aerodynamic instabilities can still be difficult. A detailed review of the characteristic features including established parameter ranges and a list of previous occurrences of RWIV are therefore given, as are overviews of closely related aeroelastic instabilities, including how these can be differentiated from RWIV.

The cross-section of a typical sheathed stay cable, such as those under investigation, are nominally circular [1]. The first section of the present chapter therefore summarises the large body of literature on bluff-body flows around such circular cylinders; a bluff flow being one which generates separated flow over a large proportion of the body surface. As separation does not occur at fixed locations for flow around circular cylinders and is dependent upon general flow conditions and the structure of the separated flow region, particular emphasis is given to literature undertaken at similar conditions as those to be simulated.

2.1 Flow Around Circular Cylinders

It will be assumed throughout this thesis that a stay cable can be represented as a circular cylinder. A common assumption amongst previous literature [10– 13]. Even that literature which does not explicitly model a circular section [14], accepts that the cross-section itself is circular but due to the three-dimensional geometry the cross-section parallel to the incident flow may be elliptical.

Two important variables for bluff-body flows, such as that under investigation, are the free stream flow velocity U_{∞} and the characteristic dimension. The latter of which although sometimes dependent upon the body geometry and parameter under investigation, is always the diameter D for a circular cylinder. Together with kinematic viscosity ν these can be combined into a non-dimensional parameter, the Reynolds Number Re, which provides a measure of the ratio of inertial to viscous forces.

$$Re = \frac{DU_{\infty}}{\nu} \tag{2.1}$$

By the act of flowing around the bluff body the fluid generates a force on

the body. This can be decomposed into orthogonal components, a lift force F_L perpendicular to the incident flow and a drag force F_D parallel to the flow. These in turn can be non-dimensionalised into a lift C_L and drag C_D coefficient (2.2) using the aforementioned parameters and the density of the fluid ρ , and which are most often reported as time averaged means \bar{C}_L and \bar{C}_D .

$$C_L = \frac{2F_L}{\rho D U_\infty^2} \qquad C_D = \frac{2F_D}{\rho D U_\infty^2} \tag{2.2}$$

A characteristic feature of bluff body flows, and therefore that around a circular cylinder, is the periodic shedding of vortices of opposite sense from alternate sides of the body at a given frequency f_s . The separation points of which are dependent upon the exact flow conditions as are the vortices themselves. These may form an organised, if unsteady, wake pattern or vortex street. The nature of which is complex and dependent upon several parameters including Re, figure 2.1. Suffice to say that for a particular geometry vortex shedding typically occurs at a regular frequency. If this is assumed time invariant then a non-dimensional frequency termed the Strouhal number St can be defined.

$$St = \frac{f_s D}{U_{\infty}} \tag{2.3}$$

2.1.1 Flow Regimes

Laminar flow is characterised by fluid flowing in undisturbed layers with no disturbances [16]. In contrast turbulent flow is characterised by chaotic disturbances and fluctuations [16]. The flow around a circular cylinder does not transition from a fully laminar to a fully turbulent state in all regions simultaneously. Rather a succession of transitions occur within different regions of the flow allowing the definition of interim transition states [17]. Primarily characterised by the Reynolds number, which governs the state of the disturbance free flow over a smooth cylinder, these transition regimes are however very sensitive to small disturbances in the flow such as those arising from turbulence, surface roughness or oscillatory



Figure 2.1: Variation of pattern of vortex shedding with Reynolds number, *Re*, for a circular cylinder, from Simiu and Scanlan [15].

motion [18]. Such disturbances can; initiate transitions at lower Re than would be the case for disturbance free flow, inhibit the occurrence of certain flow regimes or bypass them altogether.

As the system to be modelled however is a smooth cylinder in smooth flow, and will attempt to limit such disturbances, this section will focus on disturbance free flow around a singular circular cylinder. Particular attention being given to the flow regimes at the range of Reynolds numbers, $50 \times 10^3 < Re < 150 \times 10^3$ [19], over which RWIV has been typically found to occur. Reviews of turbulence and surface roughness effects can be found in papers [20–24] or in Zdravkovich [18].

A large body of research has been undertaken in identifying; the ranges of these transition regimes, the mechanisms which cause these and the systems response. The seminal review however and that commonly used by other papers in RWIV [19, 25] is that presented by Zdravkovich [18]. A significant proportion of the following discussion, including the names of the transition regimes and the accompanying abbreviations, is based upon this work. Before proceeding, figure 2.2 clarifies the regions which constitute the flow field :

- The upstream region, or free-stream, where the flow is not influenced by the body. Considered here to be uniform and turbulence-free.
- The boundary layer, where due to the viscosity of the fluid and the proximity to the body, the flow is retarded such that the no-slip condition can be satisfied at the interface.
- Separation points where the boundary layer detaches from the surface of the body.
- Free shear layers, continuations of the separated boundary layer which form between the surrounding flow and the wake.
- The wake region behind the body and between the two shear layers which contains separated unsteady flow.



Figure 2.2: Schematic defining flow regions around a circular cylinder.

Zdravkovich [18] identified three stages in the progression from fully laminar to fully turbulent flow. Each transition to turbulence occurring in a different location and with increasing Reynolds number. These are shown in figure 2.3 and can be summarised as:

- Transition in wake (TrW) where transition occurs within laminar vortices shed from the body. The point at which this happens moving further upstream as *Re* increases, until only turbulent vortices are shed.
- Transition in shear layer (TrSL) where transition occurs in the shear layers. The point at which this happens moving further upstream as *Re* increases.
- Transition in boundary layer (TrBL) where transition occurs in the boundary layers. Initially at the separation points before moving upstream with increasing *Re* until the entire boundary layer is turbulent.



Figure 2.3: Schematic of TrW, TrSL and TrBL regimes, indicating laminar, turbulent and transitions regions of the flow. Upper: Transition in Wake. Middle: Transition in Shear Layer. Lower: Transition in Boundary Layer.

In conjunction with the fully laminar (L) and fully turbulent (T) states, these describe five distinct flow regimes for a circular cylinder. Table 2.1 highlights a characteristic range of Re for each, although definitive boundaries cannot be given due to the sensitivity of the flow to very small disturbances and the inherent variation between experiments.

Regime	Description	Range of <i>Re</i>		
L	Laminar Flow	0	to	180 - 200
$\mathrm{Tr}\mathrm{W}$	Transition in wake	180 - 200	to	350 - 400
TrSL	Transition in shear layers	350 - 400	to	$1\times 10^5 - 2\times 10^5$
TrBL	Transition in boundary layers	$1\times 10^5 - 2\times 10^5$	to	$6\times 10^6 - 8\times 10^6$
Т	Turbulent flow	$6\times 10^6 - 8\times 10^6$	to	∞

Table 2.1: Variation of flow regime with Reynolds number for a single smooth circular cylinder in an undisturbed flow. Based on Zdravkovich [18].

By identification of distinct flow structures, further subdivision of these five regimes is possible [18]. However as only flows within the range $50 \times 10^3 < Re < 150 \times 10^3$ are to be studied, even accounting for the possibility of early transition, these should fall within the TrSL and TrBL regimes. As such only the subdivisions of these two regimes and the effect these have on aerodynamic response are discussed further, figure 2.4.



Figure 2.4: Variation of mean and fluctuating aerodynamic coefficients (\bar{C}_L , \bar{C}_D , C'_L and C'_D) for different flow regimes in order of increasing Reynolds number, from Zdravkovich [18].

Transition in Shear Layers (TrSL)

Within the shear layers, transition to turbulence progresses through three stages as the Reynolds number increases:

- TrSL1 Transition waves, (350 500) < Re < (1000 2000)
- TrSL2 Transition vortices, (1000 2000) < Re < (20000 40000)
- TrSL3 Turbulent bursts, (20000 40000) < Re < (100000 200000)

In the first stage, TrSL1, transition waves appear along the free shear layers. The period of which decreases as Re increases, causing an elongation and stabilisation of the near wake region [26] and a decrease in \bar{C}_D (figure 2.4). The resulting rise in vortex shedding frequency f_s , is for practical purposes in direct relation to the rise in free stream velocity. As such the Strouhal number remains approximately constant throughout this regime, $St \simeq 0.2$, as can be seen from figure 2.5.



Figure 2.5: Compilation of experiments showing variation of Strouhal number with Reynolds number over TrSL regime, from Zdravkovich [18]). Variation of St with Re.

Figure 2.5 also illustrates that the Strouhal number is approximately constant in TrSL2, where "these transitions waves roll up into discrete eddies along the free shear layer before becoming turbulent" [18]. This reduces the pressure P behind the body and thus increases \bar{C}_D (figure 2.4). While as the formation of these turbulent vortices are accompanied by large fluctuations in pressure and therefore in fluctuating lift coefficient C'_L , the closer these form to the rear of the cylinder the larger this becomes. Hence why C'_L increases in TrSL2 (figure 2.4).

With further increase in Reynolds number, the TrSL3 regime is reached. Here these transition eddies disappear altogether and the distance to the transition point to turbulence within the shear layers remains constant. This 'burst' to turbulence [18] is therefore characterised by an invariance in \bar{C}_D and C'_L (figure 2.4) and constant St (figure 2.5). However as the wake region closest to the body also becomes truly three-dimensional, due to temporal variation and inherent uncertainty this causes there is an innate scatter of approximately 10% in this latter parameter. As such, only a range of Strouhal number can be determined 0.18 < St < 0.22, (figure 2.5).

Transition in Boundary Layers (TrBL)

Within the cylinder boundary layer the transition to turbulence progresses through five stages as the Reynolds number increases:

- TrBL0 Pre-critical regime, $(1 \times 10^5 2 \times 10^5) < Re < (3 \times 10^5 3.4 \times 10^5)$
- TrBL1 One-bubble regime, $(3 \times 10^5 3.4 \times 10^5) < Re < (3.8 \times 10^5 4 \times 10^5)$
- TrBL2 Two-bubble regime, $\left(3.8\times10^5-4\times10^5\right) < Re < \left(5\times10^5-1\times10^6\right)$
- TrBL3 Super-critical regime, $(5 \times 10^5 1 \times 10^6) < Re < (3.5 \times 10^6 6 \times 10^6)$
- TrBL4 Post-critical regime, $(3.5 \times 10^6 6 \times 10^6) < Re < (6 \times 10^6 8 \times 10^6)$

Using the more widespread terminology, the latter two would be classified as 'super-critical', while the former (TrBL0) would join the aforementioned TrSL regimes as being 'sub-critical'. It is this 'sub-critical' and the 'critical' regime, of TrBL1 and TrBL2, which are of most relevance to the present study.

As a result of the 'burst' to turbulence which characterised the previous TrSL3 regime, the flow was locally accelerated beyond the separation point in turn becoming more stable. Increasing the Reynolds number further, into the TrBL0 regime, causes the boundary layer to become progressively perturbed, therefore remaining attached to the surface for longer. This results in a gradual displacement of the separation points downstream and an aligning of the shear layers; in turn causing the wake to narrow and vortices to form further downstream [18]. Drag coefficient, fluctuating lift coefficient and the negative pressure behind the body are therefore all reduced, figure 2.4. The Strouhal number however remains approximately constant, $St \simeq 0.2$, until close to the end of this regime where it begins to increase [27] before there is a 'jump', marked A in figure 2.6, indicating the transition to the TrBL1 or one bubble regime.



Figure 2.6: Variation of Strouhal number St and time averaged mean drag coefficient \bar{C}_D , with Reynolds number Re in the TrBL range, from Schewe [28].

The abrupt transition to $St \simeq 0.32$, shown in figure 2.6, which accompanies this change of regime arises as the separated shear layers suddenly become sufficiently turbulent to be able to re-attach to the cylinder surface before subse-

quently separating once again, forming a separation bubble in the process. This initially happens on just one side of the cylinder [27] the selection of which is random in the absence of disturbances or defects and which is chosen due to small perturbations inherent in the boundary layer [29]. Once formed however, the separation bubble remains fixed on that side unless flow conditions change. The formation of this separation bubble introduces an asymmetry and a circulation into the flow, generating a mean lift \bar{C}_L to the side on which this is located, as shown in figure 2.7. Taken from Schewe [28] this illustrates a generic flow configuration for both possibilities of TrBL1, in addition to the direction and relative magnitude of the resulting force and the location of the transition to turbulence. Figure 2.7 also highlights the narrowing of the wake which arises from the later final separation of the re-attached turbulent flow after the separation bubble. This causes a decrease in \bar{C}_D , the 'jump' in magnitude of which is similar to that of St, as can be seen in figures 2.4 and 2.6.

A further increase in Reynolds number results in the sudden formation of a second separation bubble, this one on the opposite side of the cylinder from the first. The appearance of which eliminates the circulation and asymmetry of the one bubble regime and consequently no net lift force is generated on the body [27, 29], figure 2.4. As was true of the formation of the first separation bubble the appearance of this second bubble is likewise accompanied by 'jumps' in both St and \bar{C}_D . These are marked B in figure 2.6 and also illustrated in figure 2.4. The magnitude of these 'jumps' are similar to those between the pre-critical and one-bubble regimes as the wake is now narrowed on both sides. As such the Strouhal number increases to $St \simeq 0.46$ [28], while although \bar{C}_D decreases the exact magnitude is experiment dependent.

In the next flow regime by increasing Reynolds Number, the TrBL3 or supercritical regime, the separation bubbles are disrupted along the span-wise length of the cylinder and the flow can no longer be regarded as essentially two-dimensional. These variations in separation angle with span-wise location affect the shape of



Figure 2.7: Configurations of sub-critical, two possible configurations of the one bubble TrBL1 and super-critical flow regimes, from Schewe [28].

separation bubble and near wake region which in turn causes a cessation of periodic vortex shedding [27]. This regime is therefore characterised by a fluctuating \bar{C}_D and C'_L [18] which can be seen in figure 2.4. Regular vortex shedding reappears in TrBL4, where the transition to turbulence occurs upstream of where the initial separation of the separation bubbles had occurred and continues to progress upstream as Re increases, until the boundary layer becomes full turbulent.

2.2 Characteristic Features of RWIV

Since the phenomenon was initially reported on the Meiko-Nishi West bridge near Nagoya, Japan, by Hikami and Shiraishi [5], RWIV has been recorded on several bridges worldwide. Table 2.2 provides a list of these including the year that the bridge was opened and the paper which reported the RWIV. While this list is comprehensive, it is by no means complete as an exact definition of RWIV is not yet available. Therefore there is still some doubt as to whether certain occurrences of cable vibration are RWIV or are another aeroelastic phenomenon. As such those oscillations reported as RWIV but which do not fall within the standard range of parameters to be outlined, e.g. those in near vertical hangers [30, 31], have been omitted. However those occurrences which arose prior to 1986 and the initial paper of Hikami [32] but which have subsequently been identified as RWIV have been included.

Bridge	Location	Year	Reference
Köhlbrand bridge	Hamburg, Germany	1974	[33]
Pont de Brotonne	Caudebed-en-Caux, France	1977	Op cit. [34]
Meiko-Nishi West Bridge	Nagoya Harbour, Japan	1985	[5]
Faroe Bridge	Sjaeiland, Denmark	1986	Op cit. [34]
Aratsu Bridge	Fukuoka, Japan	1988	[33]
Tenpozan Bridge	Osaka, Japan	1990	[33]
Talmadge Memorial Bridge	Savannah, Georgia, USA	1990	[35]
Cochrane Bridge	Mobile, Alabama, USA	1991	[35]
Nanpu Bridge	Shanghai, China	1991	[35]
Yangpu Bridge	Shanghai, China	1992	Op cit.[36]
Fred Hartman Bridge	Baytown, Texas, USA	1995	[35]
Erasmus Bridge	Rotterdam, Netherlands	1996	[11]
Nanjing Bridge No.2	Nanjing, China	2001	Op cit.[36]
Dongting Lake Bridge	Dongting Lake, China	2002	[37]
Sidney Lanier Bridge	Brunswick, Georgia, USA	2003	[35]

 Table 2.2: List of names, locations and opening dates of cable-stayed bridges

 which have reported RWIV.

Using data from these locations and from related studies under controllable conditions, parameter ranges for the conditions under which occurrences of RWIV

have been most prevalent can be outlined. Details from specific cases will be discussed in sections 2.5 to 2.8 but in summary this data suggests that the range for each parameter is restricted. For wind speed this is typically between $5 \text{ m/s} \leq$ $U \leq 15 \text{ m/s}$, table 2.3. For Reynolds numbers this is between $50 \times 10^3 < Re <$ 150×10^3 [19, 33]. While for reduced velocities this is between $20 \leq U_R \leq 90$ [38]. Where Re is based upon the wind speed normal to the cable and U_R is a non-dimensionalised, reduced velocity defined as

$$U_R = \frac{U_\infty}{fD} \tag{2.4}$$

Wind-speed Range	Bridge / Wind Tunnel	Reference
$\simeq 7$ to 14m/s	Meiko-Nishi West Bridge	Hikami [5]
6 to 17 m/s	Three Bridges	Matsumoto [38]
7 to $12m/s$	Wind Tunnel	Flamand [39]
$\simeq 14 \mathrm{m/s}$	Erasmus Bridge	Geurts [11]
12 to $17m/s$	Wind Tunnel	Gu [40]
7 to $12m/s$	Wind Tunnel	Cosentino [19]
6 to 14 m/s	Dongting Lake Bridge	Ni [37]
$\simeq 5$ to $15 \mathrm{m/s}$	Fred Hartman Bridge	Zuo [41]

Table 2.3: Mean wind speed ranges reported for RWIV, whether this occurred ona bridge or in a wind-tunnel and the author who reported these.

Moderate rainfall is required [5] and although vibrations identified as RWIV have been observed in dry conditions it is postulated that due to differences in the conditions that these are a result of a different but related physical phenomenon, namely vortex induced vibration at high reduced velocity [42]. This will be discussed further in the following section. Even so a recognised rainfall range is not yet forthcoming, as many studies do not provide or do not measure exact flow rates.

Using the angles of inclination in the cable pylon plane, α , and yaw angle, β ,



Figure 2.8: Definition of orientation of stay cable system geometry. Left: Cablepylon plane, angle of inclination (α) and angle of yaw (β). Right: Rivulet angle clockwise from windward horizontal (θ).

displayed in the configuration of stay cable geometry, figure 2.8, it can be said that RWIV typically occurs in cables which descend in the windward direction [5] at yaw angles between $20^{\circ} \leq \beta \leq 60^{\circ}$ [19, 39] and at angles of inclination between $20^{\circ} \leq \alpha \leq 45^{\circ}$ [5, 43]. While vibrations have also been noted in near vertical hangers ($\alpha \simeq 90^{\circ}$) [30, 31] and cables which ascend in the windward direction ($\beta < 0^{\circ}$) [34, 44] these are again thought to result from a different mechanism [19]. From figure 2.8 the angle from the windward horizontal θ should also be noted as this is the angular co-ordinate system that will be used in this thesis when referring to the cylinder. Under the flows considered this is is also typically the mean stagnation point of the incident flow.

The cables which undergo vibration are typically found to fall within the diameter range 100 mm $\leq D \leq 250$ mm [19, 33], to have low structural damping $\zeta \leq 0.5\%$ [11, 43], where ζ is the damping ratio of the cable, and are normally coated in Polyethylene [33, 39]. The response has typically been found to occur in the 0.6 to 3 Hz frequency range [5, 33], to occur in more than one mode [41, 45] and to vibrate at an angle aligned to the cable-pylon plane, figure 2.8. The magnitude of this vibration a, although amplitude limited is significant with peak to peak responses of up to 2 m being reported, table 2.4.

Environmental conditions should present low free-stream turbulence [19, 33],

Bridge	Maximum Peak to Peak Amplitude
Pont de Brotonne	$600\mathrm{mm}$
Köhlbrand bridge	$1000 \mathrm{mm}$
Faroe Bridge	$2000~\mathrm{mm}$
Meiko-Nishi West Bridge	$260\mathrm{mm}$
Aratsu Bridge	$600\mathrm{mm}$
Tenpozan Bridge	$1950\mathrm{mm}$

Table 2.4: Maximum reported peak to peak amplitude of cable vibration caused by RWIV on several bridges, from Matsumoto [33].

the flow should be well aligned with the ground and the cables should be sufficiently coated in contaminant to overcome water repellency and allow the formation and oscillation of rivulets [19, 39]. Practical quantification of these last parameters has however, not yet been attempted to the authors knowledge.

Through these characteristics RWIV can thus be identified as a distinct FSI, with particular features that distinguish it from other phenomena causing cable vibration. These other instabilities are summarised in the following section before returning to RWIV in section 2.4 where the various interpretations of the underlying mechanism are discussed.

2.3 Other Phenomena Causing Cable Vibration

A stay cable is susceptible to several types of wind induced vibration in addition to RWIV. These include, but are not limited to:

- Vortex Induced Vibration
- Galloping
- Vortex Induced Vibration at High Reduced Velocity
- Dry Inclined Galloping

- Wake Galloping
- Buffeting
- Parametric Excitation

Some are more probable than others, but each displays individual characteristics that should allow differentiation between themselves and RWIV, and between one another.

Wake galloping transpires from fluid interactions between two or more inline bodies. As the cables in the present context are typically located at sufficient distance to maximise the stability condition determined by Cooper [46], which provides an excellent review of this phenomenon, and as this instability has yet to be encountered on cable-stayed bridges [47] differentiating between wake-galloping and RWIV is relatively straightforward. Likewise buffeting, which gradually evolves from the random variations in turbulence present within the free-stream becoming significant enough to induce a response during high wind speeds [48], is readily distinguishable from RWIV. As is parametric excitation which occurs when excitation of a natural mode of vibration in another bridge component, or the bridge as a whole, induces a large amplitude response within a cable. Therefore only the mechanisms, characteristics and features which differentiate the four former phenomena listed from RWIV are expanded upon.

2.3.1 Vortex Induced Vibration

The regular shedding of vortices of opposite sense from alternate sides of a bluff body is one of the predominant features of flow over a circular cylinder (section 2.1). This vortex street induces an oscillating pressure load on the body with the same frequency as vortex shedding f_s perpendicular to the flow and twice the frequency of vortex shedding $2f_s$ parallel to the flow. If the body is flexibly mounted then this fluctuating pressure load can effect a motion either transverse to (across-wind), or in-line with the free-stream. Such motions are called vortex induced vibrations (VIV). That said, in-line oscillations are more commonly found in fluids with significant density. Given that in the present study the fluid is air, which has relatively low density, the remainder of this review will therefore focus on motion in a single degree of freedom (D.O.F) transverse to the free-stream which is of more relevance.

While regular vortex shedding induces a transverse oscillation, this motion is small in amplitude unless the frequency of alternating pressure loading f_s approaches the natural mechanical frequency of the structure in this direction f_b . When this occurs the structure begins to interact strongly with the fluid passing over it and the body motion increases significantly in magnitude, although remaining self-limited. This body motion begins to control the manner of vortex shedding, the frequency of which is offset to that of body oscillation. The vortex shedding frequency is therefore no longer a linear function of free-stream velocity and remains constant over a range of velocities, an effect known as "lock-in" which is illustrated in figure 2.9.



Figure 2.9: Variation of vortex shedding frequency f_s with mean flow velocity U_{∞} over an elastic structure displaying 'lock-in' region, from Simiu & Scanlan [15].

Extensive reviews of the large body of literature on VIV are given by Bearman [49] and Sarpkaya [50] amongst others. These reflect on many aspects of the phenomenon including the effect of geometry, orientation of oscillation and free

versus forced vibration. However as the response varies with these and with Reynolds number, despite many attempts to analytically model VIV from basic flow principles, none as yet have been sufficiently general to represent the full range of behaviour possible [15, 51].

What can be said is that during the lock-in region both the fluctuations in lift coefficient C_L and transverse oscillation y are approximately sinusoidal but occur out of phase with one another. These can therefore be represented as

$$y = y_0 \sin(2\pi f_s t),$$

 $C_L = C_{L0} \sin(2\pi f_s t + \phi),$ (2.5)

where t is time, C_{L0} and y_0 are the amplitudes of lift coefficient and transverse oscillation respectively and ϕ is the phase angle by which lift force leads the motion. VIV can only occur when this latter parameter is positive ($\phi > 0$), i.e. when lift force precedes and hence contributes to the motion.



Figure 2.10: Comparison of conditions and responses of RWIV and VIV. Left: Variation of vibration frequency f_b with wind speed U. Right: Variation of mean vibration amplitude a with wind speed U, from Hikami [5].

RWIV and VIV are both self-limiting instabilities [5, 41]. That said, the amplitude of vibration of VIV is considerably less than that of RWIV. VIV also tends to occur at lower wind speeds and higher frequencies than RWIV and in direct relation to the Strouhal number [5, 41] as can be seen in figure 2.10.

Furthermore although RWIV primarily occurs over a narrow region of yaw angles β , as outlined in section 2.2, VIV is not so limited and as can be seen from figure 2.11 can arise over a broad range of yaw angles. This figure further illustrates that the response due to VIV is smaller in amplitude and higher in frequency than RWIV. The former through comparison of the size of circle indicating each response with the cable diameter, shown top right; the latter through the mode number of a given response indicated by the hatching using. 'Through these differences VIV can be readily distinguished from RWIV.



Figure 2.11: Comparison of amplitudes and frequencies of VIV with the wind speed and yaw angle under which these occur, from Zuo [41]. Variation of a and β with U.

2.3.2 Galloping

Galloping instabilities can take several forms. Both across-wind and torsional single D.O.F. galloping are possible, while a two D.O.F. combination of these can also be instigated [51]. Although all three have been used in analytical models, as will be discussed in section 2.7, this outline will focus on the first of these, across-wind galloping.

A self-excited single D.O.F. aeroelastic instability, across-wind galloping is found in bodies with an asymmetry in the aerodynamic forces associated with motion perpendicular to the incident flow. Wherein an initial perturbation normal to this flow receives an additional force contributing to an increase in this motion. This can lead to large amplitude oscillations many times larger than the across-wind body dimension, with cases in excess of ten times this ratio being recorded [15]. These divergent oscillations occur at frequencies much lower than those of vortex shedding from the same section [51]. Given this low frequency response and that the angle of attack α_{θ} , the angle between the incident flow and the normal face of the body due to body motion, is directly dependent upon the across-wind velocity. Experience has shown this phenomenon can be represented in a quasi-steady manner by aerodynamic forces determined for a static section using instantaneous relative velocities. An analytical representation of a galloping instability can thus be formulated based upon the quasi-steady assumption and a rigidly fixed body [15, 52]. For small amplitude motion, this simplifies to the Glauert-Den Hartog criterion

$$\left(\frac{d\bar{C}_L}{d\alpha_\theta} + \bar{C}_D\right) < 0, \tag{2.6}$$

which is a necessary though not sufficient condition for galloping. The latter is achieved if the net damping coefficient of the system d given by equation (2.7)

$$d = 2m\zeta f_{\Omega} + \frac{1}{2}\rho UD\left(\frac{d\bar{C}_L}{d\alpha_{\theta}} + \bar{C}_D\right)_0$$
(2.7)

is negative, d < 0. Here the two terms on the RHS correspond to the mechanical and aerodynamic damping respectively, m is mass per unit length, f_{Ω} is the natural circular frequency and the aerodynamic damping is assessed at zero angle of attack. Likewise if the net damping coefficient is positive, d > 0, the system is stable. A critical velocity U_{crit} which determines the minimum flow velocity for onset of this instability can be found when d = 0,

$$U_{crit} = \frac{-4m\zeta f_{\Omega}}{\rho D \left(\frac{d\bar{C}_L}{d\alpha_{\theta}} + \bar{C}_D\right)_0}.$$
(2.8)

As circular cylinders are axisymmetric about their centre, $\frac{dC_L}{d\alpha_{\theta}} \equiv 0$, they cannot gallop [15, 51]. Thus for the onset of this instability on stay cables, the basic circular cross-section must somehow be altered. One manner to achieve this is through the formation of accretions on the body surface, as in the case of "icing" of overhead power lines [51]. For RWIV these accretions may not be ice but rain-water rivulets, outlined in section 2.4.1. Yamaguchi [12] considered this but due to the size, location and circumferential oscillation of these rivulets, found that "the one D.O.F galloping theory . . . is not useful for explanation of the rainvibration mechanism." Yamaguchi [12] did however show that by consideration of rivulet oscillation in combination with the plunge motion of the cable as a two D.O.F. galloping model, many of the features of RWIV were replicated and this remains a common contention to this day, section 2.4. Therefore while RWIV is a distinct aeroelastic instability from across-wind galloping, as it could be the result of a more complicated galloping mechanism, determining between the two has proven difficult to date.

2.3.3 Vortex Induced Vibration at High Reduced Velocity

While VIV is characterised by the shedding of vortices at the Strouhal frequency (section 2.3.1) should the cylinder be yawed or inclined to the incident flow then a similar but different aeroelastic instability may occur. Termed vortex induced vibration at high reduced velocity (HSV) by Matsumoto [42, 53] these oscillations typically occur at reduced velocities of $U_R = 20, 40, 60, 80$, which as multiples of the reciprocal of the Strouhal number, $1/St \simeq 5$, indicates a link to VIV.

In addition to a component of flow perpendicular to the body, inclined or yawed cylinders also have a component parallel to the surface. The former produces the Karman vortices so important in VIV while the latter can result in a low frequency axial vortex forming behind the body [33]. As HSV is found to cease if the flow is conditioned such that this axial vortex is eliminated, Matsumoto [42] postulated that HSV arises from the interaction between these two planes of vortices [53]; the Karman vortices parallel to the incident flow and the axial vortices parallel to the body. Due to the difference in shedding frequency between these, $f_s = 3f_{axial}$ where f_{axial} is the frequency of axial vortex, only every third Karman vortex is affected and strengthened [42] and therefore the reduced velocity of the amplified vortex is $U_R \simeq 20$. A simplified illustration of this interaction is given in figure 2.12. Recent numerical work by Yeo and Jones [54] has confirmed the existence of such "moving peaks of forces" due to the interaction between the Karman and axial vortices.



Figure 2.12: Illustration of interaction and amplification of Karman and axial vortices as possible mechanism of vortex induced vibration at high reduced velocity, from Matsumoto [42].

Given the three-dimensional nature of this interaction, considerable variations in the pressure distribution and the corresponding magnitude and spectrum of loading are generated along the cylinder length as was confirmed by Yeo and Jones [54]. As such differences in the inclination and yaw angle, free-stream turbulence intensity, end conditions or any other parameter which affects the strength, frequency or location of these vortex interactions are significant in the stability of the response [42, 55]. Of particular relevance to the present work is that the addition of an artificial rivulet to the cylinder in certain locations can enhance this instability. That is, while HSV can occur for a dry cable [56, 57], the presence of rainwater accretions at angles between 68° and 75° clockwise from the incident flow could significantly increase the magnitude of the response [42, 53]. Figure 2.13 highlights the two distinct peaks of HSV and VIV in the power spectral density (PSD) and the amplification of these that the addition of an artificial rivulet can provide.



Figure 2.13: Effect of artificial rivulet location on the PSD of unsteady lift force on the upstream side of a stationary circular cylinder yawed at $\beta = 45^{\circ}$ to smooth incident flow of U = 4 m/s, from Matsumoto [53].

Matsumoto [42, 53] goes further and proposes that "rain-wind induced vibration might be explained as vortex induced vibration at high reduced velocity." Therefore while these two phenomena have been distinguished as distinct in the present work it should be noted that demarcation is far from complete and at least a considerable overlap is inherent. Therefore if it is accepted that these are indeed different aeroelastic instabilities as it was by Cheng [55, 58], due to the commonality of many features such as amplitude and velocity limited vibrations and both displaying a 'beating' type response [53, 55, 56] differentiating between HSV and RWIV proves difficult. Two differences that can be identified however are that HSV has been found to occur over a wider range of yaw angles than RWIV [57, 59] and also to occur under dry conditions [42, 55, 57].

2.3.4 Dry Inclined Galloping

Like HSV, dry inclined galloping (DIG) is an aeroelastic instability which can occur on yawed and inclined cables under dry conditions. Unlike HSV however, DIG is a divergent self-excited phenomenon, the parameter range for occurrence of which is limited even in comparison with that of HSV [55, 60]. This sensitivity to test conditions is such that even factors such as moisture content of the air may have an effect on reproducibility of results [55]. That said a close relationship between the two instabilities has been suggested and strong interactions discerned [42]. One possibility being that both arise from the same mechanism but at sufficient amplitude of vibration physical conditions either inhibit or amplify further growth [55].

What is clear is that DIG is a truly three-dimensional instability [60] that occurs at high wind speeds on smooth cables with little structural damping. The level of this structural damping is normally given in non-dimensionalised form by the Scruton number Sc (2.9) which by combining a structures inherent damping characteristics and the mass ratio between the structure and the fluid it displaces can be interpreted as an aerodynamic stability parameter providing the susceptibility to structural response for a given loading.

$$Sc = \frac{m\zeta}{\rho D^2},\tag{2.9}$$

As was true for across-wind galloping (section 2.3.2), DIG is thought to result from negative aerodynamic damping [60]. Therefore a condition similar to the Den Hartog criterion (2.6) to indicate onset conditions would aid in the identification and prevention of this response. Cheng [58] proposed just such a modified criteria, which in the present notation of angles of inclination (α) and yaw (β) is given by

$$\frac{d\bar{C}_L}{d\alpha_w} + \bar{C}_D < 0 \tag{2.10}$$

where the wind attack angle α_w is given by

$$\delta \alpha_w = \sqrt{1 - \left(\frac{\sin\beta\,\tan\alpha}{\cos^2\beta + \tan^2\alpha}\right)^2 \delta\beta} \tag{2.11}$$

within which the aerodynamic coefficients \bar{C}_L and \bar{C}_D are based upon the normal component of mean wind velocity, $U_n = U \sin\beta$. A representation which is valid for transition regimes upto and including TrBL0, but no longer holds for regimes TrBL1 and above [61]. This omission notwithstanding, the modified Den Hartog criteria (2.10) has been found to agree well the experimental onset of divergent response [58].

What is not clear however is within which transition regime or regimes identified in section 2.1 DIG occurs. Honda [62] and Saito [63] both found DIG in the sub-critical state, TrBL0 in the present notation. While Miyata [64] found DIG in the transition state (TrBL1) and Cheng [58] found DIG in the critical state (TRBL2).

Macdonald [60] contends that the rapid decreases in drag and appearance and disappearance of a stationary lift which accompanies the formation of separation bubbles on one and then both sides of the cable in these regimes (section 2.1.1) are at least partially responsible for DIG. Using a quasi-steady approximation Macdonald and Larose derived a general theoretical expression for the aerodynamic damping ζ_a of such an inclined/yawed cylinder in any given plane. First for a single degree of freedom [60], then for two coupled and tuned degrees of freedom [65] and then for two arbitrary frequency ratios [66]. By defining the respective lift and drag forces as functions of both Reynolds number and incident cable-wind angle ϕ_m and response plane angle α_m (outlined in [65]), these also included three-dimensional and Re effects for any cross-sectional shape. For a circular cylinder this aerodynamic damping ζ_a is given by

$$\begin{aligned} \zeta_a &= \frac{\mu R e}{4m f_\Omega} \cos \alpha_m \Big[\cos \alpha_m \Big\{ \bar{C}_D \left(2 \sin \phi_m + \frac{\tan^2 \alpha_m}{\sin \phi_m} \right) + \frac{\partial \bar{C}_D}{\partial R e} R e \sin \phi_m \\ &+ \frac{\partial \bar{C}_D}{\delta \phi_m} \cos \phi_m \Big\} \pm \sin \alpha_m \Big\{ \bar{C}_L \left(2 \sin \phi_m - \frac{1}{\sin \phi_m} \right) \\ &+ \frac{\partial \bar{C}_L}{\partial R e} R e \sin \phi_m + \frac{\partial \bar{C}_L}{\partial \phi_m} \cos \phi_m \Big\} \Big] \end{aligned}$$
(2.12)

From this a non-dimensional aerodynamic damping parameter Z_a and corresponding structural damping parameter Z_s can be defined as

$$Z_i = \frac{\zeta_i m f_b}{\mu}, \qquad i = a, s \tag{2.13}$$

from which a condition for instability can be determined as

$$Z_a + Z_s < 0. \tag{2.14}$$

Using this condition, plots of stability can be constructed [60] for given environmental and structural conditions. Figure 2.14 shows one such plot displaying the minimum value of aerodynamic damping possible Z_a for any α_m for a particular set of conditions. This as expected confirms that DIG is most likely to occur over Re in the TrBL0 to TrBL2 regimes which coincides well with experimental results [58, 62, 64] and the modified Den Hartog criteria (2.10) previously discussed. Further confirmation of which is given by Cheng [58] who by measuring the pressure field on the cylinder surface found that while certain cross-sections of the cylinder might be unstable below critical Re, once this value is reached the flow becomes better conditioned and aligned, making the whole cylinder more unstable [58].

DIG therefore shows significant similarities to RWIV in that both occur over a limited range of wind speeds, both have possible *Re* effects and the mode, frequency and plane of vibration response of both display distinct similarities [65]. However RWIV and DIG can be differentiated due to the response of the



Figure 2.14: Minimum value of non-dimensional aerodynamic damping parameter Z_a for any response plane angle α_m , on variation of cable-wind angle ϕ_m with Reynolds number, from Macdonald and Larose [60]. Variation of Z_a with α_m and Re

former being amplitude limited and occurring in wet conditions while the response of the latter is divergent and occurs when the cables are dry.

2.4 Possible Mechanisms of RWIV

To date there have been three prevailing interpretations proposed within the literature as to the underlying cause of the RWIV mechanism. These are:

- the formation and oscillation of a rivulet of water on the upper surface of the cable. This alters the cross-section of the body and therefore the aerodynamic profile of the cable, instigating a 'galloping' type phenomenon.
- the interaction of Karman vortex shedding with an axial vortex formed in the wake of the cable. This instigates a three-dimensional 'HSV' type phenomenon which is enhanced by the addition of rivulets of water.
- the rivulet induces a 'one bubble' regime altering the loading on the cable. In conjunction with restorative elastic forces this instigates a cyclic motion

which triggers the appearance and disappearance of this flow transition.

While all three proposed mechanisms are different, what is clear is the important role played by the rain water rivulet in each and the similarity these have to the aeroelastic instabilities outlined in section 2.3. The following section therefore highlights the influence of this rivulet before discussing each of these three mechanisms in greater depth.

2.4.1 Influence of Rivulet

Under the sole influence of gravity, rain which falls on the surface of a stay gathers along the underside of this cable before tracking down to the deck below [5]. Under the additional influence of wind loading however this process can be interrupted and one or more accumulations of water, or rivulets, can form at different circumferential locations on the cable [5]. In general two such rivulets are formed, one each on the upper and lower surfaces, figure 2.15. However while the literature agrees that it is the upper rivulet which is primarily responsible for RWIV and that the lower rivulet has negligible effect [5, 33, 39, 67], due to differences in the coordinate systems used, discrepancies arise whether this upper rivulet is located on the windward [5] or leeward face [68].

The location of these rivulets are dependent upon both the angles of inclination and yaw of the cable and the velocity of the incident free-stream. Furthermore as the rivulets also oscillate circumferentially and hence vary with time [5, 12, 39], these are only mean positions.

To aid in the determination of the locations of these rivulets and to simplify the three-dimensional frame of reference, two additional parameters are commonly used within the literature to simplify the angles involved. These are the relative yaw angle β^* , which defines the angle between the wind direction and the cable axis in the π plane in which the stay cable sits

$$\beta^* = \arcsin(\cos\alpha\,\sin\beta),\tag{2.15}$$



Figure 2.15: Schematic clarifying notation of the windward and leeward directions, the upper and lower surfaces and the rivulets that form on these. Further highlighting the angular notation clockwise from windward horizontal used (θ) .

and γ_{θ} the angle of attack in the plane normal to the cable axis

$$\gamma_{\theta} = \arcsin\left(\frac{\sin\alpha\,\sin\beta}{\sqrt{\cos^2\beta + \sin^2\alpha\,\sin^2\beta}}\right),\tag{2.16}$$

which defines the angle between stagnation point of the incident flow and the windward horizontal. Originally proposed by Matsumoto [34] although in modified forms, both are illustrated in figure 2.16. A full derivation of these is also given in appendix A.



Figure 2.16: Definitions of simplified angles. Left: Relative yaw angle β^* . Right: Angle of attack in plane normal to cable axis γ_{θ} .

Any water which is imparted upon the upper surface in cables which incline in the direction of the free-stream, $-180^{\circ} < \beta < 0^{\circ}$, directly encounters the incident free-stream and is blown off the cable with no rivulet forming [68]. However in cables which decline in the direction of the free-stream, $0^{\circ} < \beta < 180^{\circ}$, three different states can exist:

- At velocities lower than the onset of RWIV, water imparted upon the upper surface slides down the cable and the upper face remains dry [5, 39].
- At velocities within the RWIV range, a pressure force overcomes the forces of gravity and friction and fluid can ascend the cable surface to form an upper rivulet [5].
- At velocities higher than the cessation of RWIV this upper rivulet is 'pulled away' from the surface by the magnitude of the aerodynamic loading [39].

The importance of the upper rivulet and the velocity limited nature of RWIV can thus be seen to be related.

Both Bosdogianni [67] and Zhou [69] proposed that these rivulets would form close to the flow separation points of a dry circular cylinder as a result of the lower pressures associated with these regions. For a horizontal cylinder at the Reynolds numbers presently under investigation, $50 \times 10^3 < Re < 100 \times 10^3$, these separation points form at $\theta_s \simeq \pm 80^\circ$ [61] but oscillate between $\theta_s \simeq 75^\circ$ and 85° [70] during the shedding cycle. Where θ_s is the angle clockwise from windward horizontal which in this case is the effective stagnation point, as was shown in figures 2.8 and 2.15. Both Bosdogianni [67] and Wang [68] found that after the effective angle of attack in plane γ_{θ} was offset, when inclined and yawed cylinders were considered, that rivulets did indeed form and oscillate at approximately these locations throughout the range of yaw angle studied, when the cable declined in the direction of the wind. The frequency of this oscillation coinciding with that of the cable motion [5, 19, 71].

While both rivulets form close to the separation points of a dry cylinder, the upper rivulet was found to form windward of this position and the lower rivulet was found to form leeward of this position [12, 40], figure 2.15. Therefore while

the former can influence the flow directly, the latter has negligible influence due to its location in the separated flow of the wake. This is reflected in the axial profile assumed by both rivulets, where the lower rivulet maintains a straight profile and small amplitude of oscillation throughout the range of yaw angles studied [68], as illustrated in figure 2.17. While the upper rivulet follows a '*pseudo-sinusoidal* wave' of axial wavelength $\simeq 2D$ which is postulated to result from a variation in phase of vortex shedding along the cylinder axis [68] (figure 2.17). The amplitude of oscillation of which reduces with increasing yaw angle [68].



Figure 2.17: Axial variation of rivulets at constant inclination, yaw, velocity and flow-rate. Left: Lower rivulet. Right: Upper rivulet, from Wang [68].

For a given geometry (fixed values of α and β) the angle of the upper rivulet from the stagnation point $\theta + \gamma_{\theta}$, increases with increasing free-stream velocity and the rivulet ascends the cable face [5, 67, 68]. The lower rivulet also moves further from the stagnation point with increasing velocity but at a reduced rate due to the greater influence of gravity [67, 68]. Both of which can be seen in figure 2.18. The location of rivulets on the cable are therefore dependent upon both wind and adhesion forces and the momentary acceleration of the cable [19], possibly explaining why these oscillate at the same frequency as the cable itself.



Figure 2.18: Variation of rivulet position, from upper and lower hemispherical points with wind speed U, from Bosdogianni [67].

2.4.2 Varying Cross Sectional Geometry

As was found for HSV (section 2.3.3) and as has been found in a RWIV context through wind-tunnel tests, which will be elaborated upon in section 2.6.1, at certain locations rivulets can significantly affect the aerodynamic loading [71]. As the rivulets oscillate at the same frequency as the cable [5, 19, 71], if this loading occurs at the same frequency and with positive phase to the motion, work can be introduced into the system and a self excited instability can ensue. Verwiebe [71] proposed that it was this permanently changing cross section of the aerodynamic profile of the cable and attached rivulet which plays a predominant role in RWIV. In so doing proposing three classifications of response which occur at different geometries, wind speeds and orientations. These excitation mechanisms [71] can be summarised as:

• Type 1 - Streamwise vibration - if the two rivulets on the upper and lower surfaces oscillate symmetrically with respect to the incident flow, then the separation lines and therefore the wind force will also oscillate with the same frequency. The inherent symmetry of the system ensures that while the resulting force will fluctuate, and at sufficient magnitude vibrate, it will do so in the streamwise direction. For such a confluence of events to occur the cable must descend parallel with the flow, $\beta \simeq 90^{\circ}$, with sufficient wind

speed to split the fluid into two rivulets, $U \ge 23 \,\mathrm{m/s}$.

- Type 2.1 Transverse vibration (twin rivulets) at similar yaw angles ($\beta \simeq 90^{\circ}$) but lower wind speeds ($15 \leq U \leq 22 \text{ m/s}$) to the previous case, the two rivulets were found to oscillate anti-symmetrically about the separation points. While as one rivulet moves windward towards the stagnation point the other moves leeward towards the wake. This produces an asymmetric cross section and corresponding pressure profile, which in turn produces a force transverse to the incident flow which changes sign in rhythm with the motion. If this force has sufficient magnitude then this can induce a transverse (across-wind) vibration.
- Type 2.2 Transverse vibration (single rivulet) in cables which do not decline parallel to the incident flow, β ≠ 90°, a single rivulet forms on the underside of the cable. This rivulet changes shape and location during cable motion resulting in a fluctuating force in both the streamwise and transverse direction. The magnitude of the latter is greater than the former however, and motion is therefore primarily undertaken in the transverse (across-wind) direction if this fluctuating force is sufficient. Figure 2.19 illustrates this pattern of rivulet motion, the resulting forces and the work imparted.



Figure 2.19: Excitation mechanism 2.2 proposed by Verwiebe, transverse vibration caused by the motion of a single rivulet, from Verwiebe [71].

2.4.3 Interaction with Axial Vortex

Through wind-tunnel studies with artificial rivulets, which will be discussed in further depth in section 2.6.1, Matsumoto [33, 38, 42, 53] determined that three types of rivulet laden cable response were possible. These three responses were designated, divergent, velocity-restricted and hybrid respectively, and are illustrated in figure 2.20. Which response occurred for a given system was dependent upon the angles of inclination and yaw, the location of the rivulet, wind speed, turbulence and the Scruton number.



Figure 2.20: Divergent, velocity-restricted and hybrid responses of yawed and inclined circular cylinder with artificial rivulet in smooth flow. Variation of non dimensional ratio of double amplitude to cylinder diameter 2a/D with reduced velocity U_R , from Matsumoto [38].

The divergent response is thought to correspond to a 'galloping' type instability, where due to the artifical rivulet being located at a certain circumferential location, a net lift force is generated on the body and Karman vortex shedding is suppressed [38]. The sharp onset of this lift results in a large negative lift curve slope ($\frac{d\bar{C}_L}{d\alpha_{\theta}} < 0$) which in turn means the Den-Hartog criterion (2.6) can be satisfied. Therefore at any wind speed greater than the critical velocity (2.8) this response is instigated, the amplitude of which is divergent and is only restrained once physical testing limits are reached. The particular response illustrated to the left of figure 2.20 occurred at $\alpha = 0^{\circ}$, $\beta = 35^{\circ}$, $\theta = 58^{\circ}$ and Sc = 1.64 and occurred at reduced velocities greater than $U_R \simeq 80$.

The velocity-restricted response is thought to correspond to a 'HSV' type instability [38, 42, 53] where due to the artifical rivulet being located at a certain circumferential locations a velocity and amplitude restricted response were found to occur. The particular response illustrated in the centre of figure 2.20 occurred at $\alpha = 0^{\circ}$, $\beta = 45^{\circ}$, $\theta = 72^{\circ}$ and Sc = 1.06.

The hybrid response is a combination of the previous responses with an amplitude limited response occurring over a restricted range of lower reduced velocities and a galloping type divergent response occurring at reduced velocity greater than critical. The particular response illustrated to the right of figure 2.20 occurred at $\alpha = 0^{\circ}$, $\beta = 0^{\circ}$, $\theta = 63^{\circ}$ and Sc = 5.50.

2.4.4 Interaction with One Bubble Regime

By using a 'mean-cycle' approach, where numerous individual vibrations are superimposed to form a single time averaged vibration cycle, Cosentino [19] proposed a mechanism for RWIV based on the critical flow regime properties outlined in section 2.1. Illustrated in figure 2.21 the interpretation forwarded by Cosentino et al. [19] for this mechanism can be summarised as follows.



Figure 2.21: RWIV mechanism based upon one-bubble regime proposed by Cosentino. 'Mean-cycle evolution of pressure distribution, rivulet thickness (amplified for clarity) and displacement vs. time, from Cosentino [19].

During ascending cable motion, inertial effects due to the negative acceleration, cause the fluid to accumulate into a rivulet on the upper surface of the cable. The increased thickness of this rivulet causes transition of the flow to the one-bubble regime. This in turn disturbs the pressure field reducing the negative suction pressure on the upper surface of the cable. The elastic restorative force of the cable is thus augmented with an additional component adding work to the system throughout the downward motion.

As the cable proceeds downward, the rivulet flattens and moves toward the stagnation point becoming increasingly "spread and irregular" [19], in a reversal of the rivulet accumulation during upward motion. In so doing the rivulet no longer presents a sufficient obstacle to the flow to trigger the one bubble regime and the flow therefore transitions back to its previous state. The restorative forces then cause the cable to begin to rise once again causing the rivulet to ascend back toward the separation point, albeit in a disorganised manner, and to increase in thickness, completing the cycle.

It should be noted for future reference that the authors themselves [19] state that using such a 'mean-cycle' approach may smooth the random pressures fluctuations intrinsic to this response but that while these are both significant and sudden they are also stochastic in nature and may be treated as being superimposed upon the underlying mechanism outlined here.

2.5 Full Scale 'In-Situ' Studies

As mentioned at chapter outset, due to the unpredictability and the infrequence of RWIV responses and the expense of the instrumentation required to measure these, full-scale 'in-situ' testing of RWIV is both costly and time consuming. To illustrate this table 2.5 outlines the time scales involved in the studies referenced herein. Given the extent of backing thus required, these studies typically concern aspects which can attract such funding, either the identification of characteristic parameter ranges of RWIV or measuring the effectiveness of countermeasures in mitigating the response. Typical parameter ranges were discussed previously in section 2.2 while countermeasures will be discussed in section 2.9 in conjuction with design guidelines. This section therefore focuses on how such data is acquired and how this may be manipulated and interpreted. A typical RWIV response is then discussed.
Location	Author	Time-Frame
Erasmus Bridge	Persoon et al. [72]	17 months
Fred Hartman Bridge	Main and Jones [57]	16 months
East Huntingdon Bridge	Main and Jones [57]	2 years
Custom Tower	Matsumoto et al.[53]	2 months
Veterans Memorial Bridge	Phelan et al. $[73]$	2 years
Dongting Lake Bridge	Ni et al. $[37]$	$45 \mathrm{~days}$

Table 2.5: A list of 'in-situ' studies and the time-frames involved.

2.5.1 Data Acquisition

Each data acquisition system is unique, however a typical package used for the collection of vibration data can be summarised from the studies of Main and Jones [57], Phelan [73] and Ni [37]. Such a system consists of:

- Two three-axis anemometers to ascertain wind speed and direction at deck and tower height respectively.
- An electronic rain gauge or precipitation sensor to obtain rainfall measurement. Some systems also obtain temperature and pressure data [41, 57].
- A data acquisition system with a large number of channels, each tracking an individual parameter.
- Computing equipment with sufficient storage.
- At least one accelerometer mounted to the deck or tower to obtain a baseline structure motion. Ideally multiple measurements would be taken at different locations and in different planes simultaneously.
- A multi-axis accelerometer to determine both the in-plane and out-of-plane motion for every stay to be tracked. Ideally two locations per stay would be instrumented to ensure detection of multiple modes and to minimise the effect of accelerometer location.

A given stay will only produce a wind-induced response for a certain range of wind speeds and directions. Therefore to increase the probable number of responses on an instrumented stay over a stipulated period of time, several different stays in different fans are typically instrumented in each study. Main [57] and Zuo [41] examined eleven stays in three different fans, while Phelan [73] examined four stays in two different fans. As such a substantial amount of data is being acquired from several channels simultaneously.

To reduce the quantity of non-relevant data to be reviewed, most systems [41, 57, 73] only permanently store data once one of a pre-set list of 'triggers', typically a threshold wind speed or acceleration value [73], is reached and continue to record for set time intervals until this condition is no longer sampled. This technique however may omit key events and some shorter period studies still run continuous recording triggered by a manual response [59].

Trying to record exclusively the relevant data is only one of a number of possible problems that such 'in-situ' equipment must endure. Others previously encountered include corrosion of components, impact damage, seal failure and lightning strikes [45]. Furthermore the locations of these anemometers and accelerometers are typically difficult to access, as an example Phelan [73] mounted accelerometers at locations a quarter and a third of the cable length from its base. As such these systems must be sufficiently robust to withstand the rigours of the operating environment but sensitive enough to provide sufficiently detailed data. This latter point can prove particularly difficult given the large operating range required of certain components, ± 10 g in the case of the accelerometers used by Phelan [73]. Should these requirements be satisfied however such systems can produce a large quantity of data without significant human interaction.

2.5.2 Data Interpretation

After acquisition some form of transformation may be required to allow a more ready interpretation of the raw data, therefore a typical system samples at a rate significantly greater than is required. This 'oversampling' aids in anti-aliasing and allows for higher resolution analogue to digital conversions. It also means that a programmable digital filter can be applied to restrict the signal to the bandwidth of interest, typically the lower frequency range of RWIV response. The exact sampling rate and filter used vary with system but as illustrative examples, Main [57] used a sampling rate of 40 Hz and a 10 Hz filter, while Matsumoto [53] used a sampling rate of 200 Hz and a 20 Hz filter.

As the accelerometers are fixed to the stays themselves, these will report in a local coordinate system specific to that stay. Therefore it may be advantageous to transform this data into a global system for analysis. To ease this process however it has been shown that the variation of local coordinate system with instantaneous cable position during motion has negligible effect and can therefore be ignored [73]. Thus considerably simplifying this process. Likewise the relative motion of the stay can be determined by subtracting the base-line response of the structure obtained from the accelerometers located on the bridge deck or tower.

Typically the acceleration trace is integrated twice to determine both the velocity and the displacement at a particular location. This process however does introduce additional errors. Therefore several systems use a displacement transducer to acquire this latter result directly thereby allowing a degree of self verification [57, 73]. Finally by using two anemometers at two different heights within the system, a relation between the wind speeds at these locations can be determined and the variation of wind profile with height established, as this may effect loading. Local variations in wind speed can also be determined, allowing effects due to instrument location or wind direction to be identified and filtered out if necessary.

2.5.3 An Actual RWIV Response

When this data is then collected, processed and plotted individual RWIV events can be examined. Figure 2.22 portrays one such large amplitude event from the study undertaken by Phelan [73]. Every occurrence of RWIV is unique however and this is only given as a representative example of typical data obtained.



Figure 2.22: 5g RWIV event from Veterans Memorial Bridge, showing simultaneous wind speed and direction, rainfall and stay acceleration, from Phelan [73].

Due to atmospheric conditions both the wind speed and direction changed rapidly over the duration of response. The ranges and mean values of both however are typical of the characteristics outlined in section 2.2 for RWIV. Specifically the 1-minute mean wind speed was found to lie between 7.6 m/s $\leq U \leq 9.0$ m/s while the wind direction was found to range between $20^{\circ} \leq \beta \leq 50^{\circ}$ under the present coordinate system with a mean of 32° .

The cable itself was also indicative of ranges outlined for RWIV within section 2.2, having an angle of inclination of $\alpha = 21^{\circ}$, a diameter of D = 114 mm and a damping ratio of $\zeta \simeq 0.3\%$ as was the motion undertaken. Given by the acceleration trace, the lowest of those shown in figure 2.22, this comprises more than one mode as evidenced by the 'beating' type response and is only significant

over the period during which rain fell. Building up rapidly from first occurrence at 300 s before returning to a negligible amplitude upon cessation. It should be noted that while there appears to be a sudden drop in rainfall at 700 s in figure 2.22, this is due to cumulative rainfall being illustrated. The measurement was reset at this time and rainfall was indeed measured from 200 s to 1700 s as indicated on the figure.

2.6 Wind-Tunnel Studies

To allow closer examination of the influence of a given parameter of RWIV greater control must be exerted upon the testing conditions. Wind-tunnel studies reduce the inherent uncertainty and transience present under environmental loading and allow flow conditions to be consistently repeated and results to be verified. Such studies have therefore comprised a significant proportion of the RWIV literature. Particularly pertinent to the present work are those studies which identify the effect of the rivulets on the external aerodynamic field and vice versa. These investigations can be broadly separated into two distinct classes:

- those where the rivulet is replaced by a fixed, static, rigid protuberance or 'artificial rivulet' [5, 38, 40, 67].
- those where a film of water is sprayed onto the surface of the cable and the rivulets are allowed to form 'naturally' [19, 39, 43, 71, 74].

As many of the results of these investigations are detailed elsewhere in this literature review, this section; highlights the differences between these cases, discusses some of the determinations made and outlines other aspects of practical wind-tunnel testing which may affect response.

2.6.1 Artificial and Natural Rivulet Studies

Both artificial and natural rivulet studies indicate that the presence of the rivulet on the upper surface is largely responsible for the motion of the cable [5, 38]. The latter class also confirms that when free to do so, these rivulets oscillate circumferentially at the same frequency as the cable vibrates [43, 71]. Differentiating between the effect of circumferential oscillation and rivulet position has however proven more difficult and led to discrepancies within the literature. Bosdogianni [67] contended that it is rivulet location and not circumferential oscillation which initiates the response. Verwiebe [71] however determined that circumferential oscillation of the rivulets is a primary cause of RWIV. Flamand [39] examined both the fixed artificial rivulet and naturally forming rivulet classes and found that "rivulets act through their movement" as the former class did not display any excitation. Zhan [74] likewise differentiated between the two classes, determining the static artificial rivulet class to be a 'galloping' type response. Gu [36] using a movable artifical rivulet also found that for a certain wind speed and level of damping that the upper rivulet would adopt a certain mean angle and then vibrate about this point.

In the case of an artificial rivulet displaying a 'galloping' type response, circumferential location has been found to be the most significant parameter. For a given orientation of cable a range of critical angles can be determined which display this divergent response. In the case of flow occurring normal to the cylinder, zero inclination and yaw angles α and $\beta = 0^{\circ}$, this was found to be between $48^{\circ} \leq \theta \leq 58^{\circ}$ by Gu [40] and at $\theta \simeq 63^{\circ}$ by Matsumoto [33]. This location was further found to change as the angles of inclination and yaw were varied. That said, the results of Matsumoto [38] and Zhan [74] however do show a good level of agreement where these overlap. Figure 2.23 shows the case of variation of rivulet angle with yaw angle and indicates the areas where an unstable (possible RWIV) response were detected.

In comparison with location, the shape of the artificial rivulet has been found to have significantly less impact [40, 67], with elliptical, rectangular and square forms all used. This led Bosdogianni [67] to state that "to a large extent it is simply the presence of a protuberance ... and not its magnitude or shape that



Figure 2.23: Regions of stable and unstable response, for artifical rivulet attached to circular cylinder. Variation of rivulet position clockwise from windward horizontal θ with yaw angle β , from Matsumoto [38]).

develops galloping." The effect of size however is less well known as several definitions are possible, and the ratio of this rivulet dimension to cable diameter varies considerably within the literature. Gu, however, investigated rivulets of similar form but different dimension in the same study [40] and found "the size effects of rivulet ... are insignificant." Determining whether size and shape are significant for 'natural' rivulets however has proven difficult due to the time and length scales involved, because the fluid disperses upon contact and as the form, size and location of these cannot be directly controlled. Cosentino [19] did determine however that the rivulet formed in that experimental setup was composed of a "base carpet", $\simeq 0.25 \times 10^{-3}$ m thick, over which a superimposed "oscillating wave" slides.

Using an artificial rivulet and a cable aligned normal to the flow, Gu [40] ascertained that the relationship between the natural frequency of the cable and the onset wind speed is approximately linear for a given mass and damping ratio. However with a natural rivulet the same author found no such relationship [43]. The response being velocity restricted when it did occur, with the largest amplitude of response at 1 Hz and almost no response at 2.6 Hz. This range corresponds with the general characteristics of RWIV outlined in section 2.2. The same author determined a similar pattern for the effect of damping ratio on both

natural and artifical rivulets [40, 43], with an almost linear relationship between the onset wind speed and the damping ratio when artificial rivulets were used and no relationship when naturally forming rivulet were used. The response again being velocity restricted when it did occur, with the larger amplitudes of response at smaller damping ratio and almost no response once damping reached $\zeta = 0.6\%$. This range again corresponds well with the general characteristics of RWIV outlined in section 2.2.

2.6.2 Other Parameters

By studying RWIV in the wind-tunnel several other parameters which could not be easily measured in situ can be ascertained. These include the pressure distributions around the cable with and without rivulets [19, 33, 75], velocities at points on the surface and in the wake of the cable [53, 68] and aerodynamic forces on the cable section [19, 38, 40].

That said, while wind-tunnels are helpful in determining the characteristics of the RWIV response, the characteristics of these vibrations are significantly affected by how the body is supported and restrained [34, 74]. Care must therefore be taken to replicate physical conditions as the method of support can significantly modify; system damping, gravitational and restorative forces, and both the plane, amplitude and frequency of response. The most commonly used setup [5, 43] is to suspend a rigid cable model at either end to a steel frame using two springs of sufficient length to replicate the low frequency of an actual stay. The cable is then connected by ancillary wires to the surroundings to ensure that it vibrates in the correct plane and to balance the component of gravity force along the cable axis, figure 2.24. This setup however has not been universally adopted and several other rigs have been used. Flamand [39] and Cosentino [19] replaced these ancillary wires with a driving cable and two lateral springs to remove limitations on model inclination and yaw. Phelan [73] suspended the whole section model transversely using springs fixed to a pair of force balances attached to two separate frames. While to more accurately represent the restorative forces as resulting from cable tension Zhan [74] adopted a model pulled axially at both ends by pre-tensioned springs connected to a steel frame. The variation in the wind-tunnel setup should therefore be considered when results using different experimental setups are compared.



Figure 2.24: Commonly used wind-tunnel setup, from Zhan [74].

Another aspect to be considered, if this is to be considered, is how simulated rainfall is introduced into the system. Both Verwiebe [71] and Gu [43] use showers upstream of the cylinder to spray water in a matter reminiscent of rain. While Flamand [39] and Cosentino [19] use a peacock-tail device similar to the shower method. In addition to such showers Zhan [74] also introduces water directly onto the upper end of the cylinder at a controllable flow rate to reproduce the effect of the upstream rivulet running down the cable. Wang [68] only uses this method. Whether this affects the overall response however is yet to be determined.

2.7 Analytical Studies

By theoretically modelling a simplified representation of the physical conditions present during an RWIV event, analytical studies attempt to clarify the influence of a given parameter on a particular mathematical model. Should corroborating results be achieved for a number of different representations, confidence as to the role played by that parameter in the actual RWIV mechanism, can be achieved. Consequently a significant proportion of the literature has focused on analytical modelling RWIV.

Given that the same phenomenon is to be represented, many of these mathematical models adopt similar approaches and make similar assumptions; individual nuances being determined by the nature of the specific investigation. To outline this approach, the common assumptions made and the reasons for these, the mathematical model proposed by Yamaguchi [12], which is indicative of these, is initially discussed. Several, of the numerous, other studies are then outlined and the major results reviewed.

2.7.1 The First Representation

The earliest mathematical representation of RWIV was provided by Yamaguchi [12]. This assumed that both the cable and the artificial rivulet could be represented by two-dimensional, rigid, horizontal cylinders without structural damping and that the circumferential motion of the rivulet produced an inertia moment equal to the aerodynamic moment of the combined geometry. The mass of the rivulet was also assumed to be negligible in comparison with that of the cable. To reduce the number of potential D.O.F., the cable was constrained to only translate across-wind, while the rivulet was constrained to only translate circumferentially at a fixed distance from the cable axis. This simplified two-dimensional, single mass, two D.O.F. system is shown in figure 2.25.

By applying the quasi-steady assumption to the aerodynamic forces and subsequently linearising these and the trigonometric functions involved, a homogeneous second-order linear differential equation which governs the system was derived. The quasi-steady assumption postulates that forces are without memory, such that the forces on a fixed body at the same instantaneous velocity as the ac-



ANALYTICAL MODEL

Figure 2.25: Two D.O.F. system to analytically represent RWIV, from Yamaguchi [12].

tual body are identical to the forces on the actual body. Linearisation meanwhile replaces actual functions with simplified linear functions over short intervals. Provided that steady flow conditions can develop throughout the vibration cycle and that the angles involved are small, then these assumptions are widely used in analytical representations of other aeroelastic phenomenon previously discussed. Particular examples being, during the derivation of the Den Hartog criterion for galloping (2.6) in section 2.3.2 and within the construction of similar criterion for DIG (2.10) in section 2.3.4.

By solving this differential equation Yamaguchi [12] was able to determine that when the position of the upper rivulet was fixed, a 'galloping' type response (section 2.3.2) could be determined but that this was was not "useful" for explaining the RWIV mechanism. However when free to move if the frequency of upper rivulet oscillation matched that of the across-wind motion of the cable then aerodynamic damping became negative, indicating an instability could occur. This agrees well with the experimental results outlined in section 2.4.1 regarding rivulet oscillation frequency during RWIV matching that of cable oscillation.

2.7.2 Other Representations

By using the two dimensional representation, quasi-steady and other assumptions used by Yamaguchi [12], and by determining the aerodynamic coefficients $(\bar{C}_L \text{ and } \bar{C}_D)$ as a Taylor series expansion of rivulet angle θ , Wang [13] proposed a linearised two D.O.F. model for rivulet and cable movement. By assuming the former was harmonic and by specifying both the amplitude and frequency of this oscillation, this analytical model captured a response that was both velocity and amplitude restricted agreeing well with physical findings as to the nature of RWIV. The reasons for this limited response were primarily attributed to the identification of a critical rivulet angle, such as those found in artificial rivulet wind-tunnel tests (sections 2.4.3 and 2.6.1), where the gradients of the aerodynamic coefficients $\left(\frac{d\bar{C}_L}{d\alpha_{\theta}} \text{ and } \frac{d\bar{C}_D}{d\alpha_{\theta}}\right)$ change rapidly in magnitude, and sometimes sign. Passing through this location therefore significantly affected both the magnitude and sign of the damping ratio and thus the resulting force. As such when the incident wind-speed caused rivulet oscillation to passed through this point, an amplitude restricted vibration of significant magnitude resulted while at other wind speeds the magnitude of any amplitude was found to be small.

Wilde [10] and Gu [36] also used similar representations, but by simplifying the system to a single D.O.F. Wilde [10] illustrated that this could determine many of the same features ascertained by more complex systems. Gu [36] meanwhile identified ranges of rivulet location θ under which the system was stable and unstable. Determining that as the ratio of cable vibration amplitude to rivulet vibration amplitude increased, the unstable range moved from an upper bound determined by no cable movement to a lower bound determined by no rivulet movement. The latter corresponding to the galloping case identified analytically by Yamaguchi [12] and the wind-tunnel studies of Matsumoto [38] amongst others. This representation further determined that this range of unstable rivulet locations decreased with increasing damping ratio and frequency but increased with increasing wind speed. The former two findings agreeing well with windtunnel studies by the same author [43].

Peil [76] used a similar linear stability criteria to Gu [36] for a two mass three D.O.F. system to study the stable locations at which rivulets could form. This determined that at wind speed below $U \simeq 8 \text{ m/s}$ while a stable rivulet could exist

at the lowest point, labelled A in figure 2.26, due to the effect of gravity no stable upper rivulet could exist. While at greater wind speeds rivulets could form in four possible locations although only two of these locations, A and D, were stable. The latter of which is only fixed at U > 15 m/s and oscillates about this point in the interim range $8 \leq U \leq 15$ m/s. This demonstrates a velocity restricted phenomenon with an oscillating upper rivulet is in good correspondence with the general characteristics of RWIV outlined in section 2.2. A further parametric study highlighted that the range of wind speeds causing oscillation of the upper rivulet decreases with increasing damping ratio in good agreement with Gu [36] and wind-tunnel work studies (section 2.6.1).



Figure 2.26: Variation of location of possible stationary points with wind speed U, from Peil [76].

Seidel [77] likewise investigated stable rivulet locations, here however the focus was in determining the differences between sub-critical and critical regime flow. A two rivulet, three mass, four D.O.F. system was constructed which found similar results to Peil [76] for subcritical flows, although exact rivulet locations did differ somewhat due to a different cable orientation being reported. Within the critical regime, the locations of these rivulets was found to move leeward, corresponding closely to the movement of separation points outlined in section 2.1.1. The authors postulate that this is the upper limit of wind speed which could induce RWIV; the lower limit being found to vary with the angle of cable inclination and yaw examined. Furthermore stable rivulets were only found to form on cables which decline in the direction of the wind. These general characteristics again correspond well with those physically found for RWIV.

Burton [78] also examined stability but for the system as a whole. Using two different methods to calculate aerodynamic forces, one using time averaged windtunnel data and one using numerically calculated values based on static separation points, this study ascertained stability plots and displacement time histories of various cable-rivulet configurations. The "common overlap" [78] between these results verifying the linearisation process and the quasi-steady assumption outlined in section 2.7.1 and used in most of the other analytical studies reported.

Peil [79] complemented his earlier work [76] by applying the aerodynamic forces based on rivulet movement determined in two dimensions to a threedimensional cable model. The results displaying both a velocity and amplitude restricted response thus comparing well with other literature and general RWIV characteristics. While with increasing turbulence intensity and damping ratio this study found that wind speed range and amplitude of response were found to decrease. The former partially verifying the decision in the present model to consider smooth flow conditions, the latter agreeing well with other analytical [36, 76] and physical studies [43] previously outlined. Cosentino [25] meanwhile included the effect of internal rivulet damping, the effect of rivulets on the local pressure field and the effect of skin friction for the first time, the results of which were also in keeping with previous studies.

2.8 Previous Numerical Studies

In contrast to the 'in-situ', experimental and analytical studies of RWIV summarised in the previous three sections, comparatively few studies have specifically concentrated on numerical simulation of RWIV. As outlined in the introduction to this thesis (section 1.1) this disparity was one of the primary motivations for the present work being undertaken. There is however a significant body of literature concerning the numerical simulation of flow over a circular cylinder. As such, of the studies to have numerically modelled some aspect of RWIV, given its close resemblance to this problem, most have focused on simulating the addition of fixed rigid artifical rivulets to the stay-cable in two or three dimensions [75, 80].

Using Large Eddy Simulation (LES) Liu [80] examined the effect that rivulet position θ has upon the flow pattern and overall system response. The results indicate that the flow separates from the surface at the leading edge of the rivulet but dependant upon the location of this rivulet and the position in the shedding cycle the flow may re-attach to the surface before separating once again. This has significant effects on both the magnitude and mechanism of the system response and agrees well with the wind-tunnel studies (section 2.6.1) and analytical studies (section 2.7) in that rivulet location has a major influence on body response. Specifically the flow was found to re-attach throughout the shedding cycle at small angles $\theta \leq 40^{\circ}$, to re-attach only at certain points in the cycle at angles between $46^{\circ} < \theta < 56^{\circ}$ and to remain separated throughout the cycle at large angles $\theta \geq 56^{\circ}$.

Figure 2.27 highlights this for a rivulet at $\theta = 46^{\circ}$, where the flow remains separated at one instant in the shedding cycle while re-attaching at a later instant. From this study Liu [80] was able to detect a divergent 'galloping' type response further agreeing with previous experimental and analytical studies. This therefore is a problem within a RWIV context for which there is both previous numerical and experimental data [81, 82] available. This problem will therefore be examined extensively in chapter 3, first to validate the aerodynamic solver constructed and second to act as the test cases for this solver outlined in figure 1.2.

Li and Gu [75] also examined the effect of rivulet position upon flow pattern, but in both two and three dimensions. Using a rigid artificial rivulet of similar size and shape under comparative flow conditions in two dimensions, the results were found to correspond well with those of Liu [80] with re-attachment of separated



Figure 2.27: Pressure contours for two locations in the shedding cycle of a cable with single artificial rivulet at $\theta = 46^{\circ}$, from Liu [80].

flow occurring at small θ , although exact angles were not reported. While a sudden drop in \bar{C}_L at angles between $40^\circ < \theta < 45^\circ$ in three dimensions was likewise interpreted as indicative of a 'galloping' type response. This was further emphasised as shedding of Karman vortices was found to become highly irregular. The authors [75] also detected a sharp change in the pressure distribution in the immediate vicinity of the rivulet location. This was also attributed to the sudden separation the rivulet causes but was not discussed in detail. Like the results of Liu [80], the results from this study will be used in validating the numerical aerodynamic solver created in the following chapter.

Rocchi and Zasso [83] did not explicity model a cable with artificial rivulets but considered the addition of a helical strake instead. The results determined via LES compared well qualitatively with those obtained from experiment but displayed discrepancies in the absolute quantitative values of \bar{C}_L and \bar{C}_D . While Yeo and Jones [54], used Detached Eddy Simulation (DES) to examine the threedimensional flow field generated behind a plain cable yawed and inclined to the incident flow at sub-critical Reynolds numbers. The results identified "multiple moving peaks of force" progressing down the axial length of the cable in a similar manner to the axial vortex Matsumoto [42] identified as important to the HSV instability outlined in section 2.3.3 whose temporal movement, low frequency and periodic nature were found to result from a coherent three-dimensional flow structure. This agrees well with the HSV phenomenon as did the delayed separation and strengthening of certain vortices. The level of the features captured and the quality of results predicted highlighting the potential benefits of numerical simulation for a related instability, HSV, further emphasising the need for such work for RWIV.

In contrast to the studies which specify the geometry of the rivulet to ascertain the resulting aerodynamic flow field, Lemaitre [9] derived a governing evolution equation which given aerodynamic loading on the body tracked how a thin film of fluid evolves, i.e. does a rivulet form. Specifically a constant distribution based upon the time averaged aerodynamic loading for a dry cylinder was used. Without accounting for temporal variation of the effect of rivulet growth on aerodynamic loading the results showed that for the flow conditions prescribed, two distinct rivulets formed at approximately the separation points of a dry circular cylinder at the same Re under all the loadings examined. Like the work of Liu [80] which will be used in the validation of the thin film solver developed in chapter 4, where these results will be discussed in more detail.

As such, while some computational modelling of RWIV has been undertaken, these studies are limited in number. There is therefore a distinct need not only for a computational model to resolve issues raised by in-situ, wind-tunnel or analytical studies, but for more numerical simulation of aspects of RWIV in general. The present work intends to address these by taking the first steps towards the creation of such a solver, specifically focusing on the prediction of rivulet formation and evolution under an external aerodynamic field these influence.

2.9 Designing Against RWIV

While areas such as aerodynamic loading on bridges and gust factors have been incorporated in a succession of prescribed design codes, a definitive code for the mitigation of wind induced vibration of stay cables including RWIV does not yet exist. Indeed guidelines which include RWIV have only become available relatively recently with the publication of the 2001 PTI Guide Specification [84] and the 2007 Federal Highways Authority report FHWA-HRT-05-083 [47]. The following section examines; these, the criteria proposed and the countermeasures outlined.

2.9.1 Minimum Scruton Number Criteria

As the Scruton number (2.9) can be interpreted as an aerodynamic stability parameter providing the susceptibility to structural response for a given loading "most types of wind-induced oscillation tend to be mitigated by increasing the Scruton number" [47]. As such, this would therefore appear to be the logical choice of parameter on which to establish a stability criterion.

From a series of wind-tunnel tests on an actual section of stay cable which underwent DIG, Saito [63] determined just such a relation between the onset reduced velocity for DIG and the Scruton number as

$$U_R = \frac{U_{crit}}{fD} = 40\sqrt{Sc}.$$
(2.17)

The performance of which, in determining onset velocity, was reiterated by Matsumoto [85] at considerably larger Scruton number than previously investigated. From this relation Irwin [86] and subsequently the PTI guide [84] proposed that RWIV could be mitigated to a satisfactory level if the following criteria was met

$$Sc = \frac{m\zeta}{\rho D^2} > 10. \tag{2.18}$$

To clarify the effectiveness of these criteria, tests similar to those performed by Saito [63] were undertaken as part of the FHWA publication [47]. Similar vibrations were found, but these were attributed to HSV rather than DIG. Large vibrations were only observed at the lowest damping ratios $\zeta < 0.1\%$, and at $\zeta >$ 0.3% there was no occurrence of an oscillation greater than 10 mm in amplitude. Figure 2.28 compares these results with the instability line determined by the Saito criteria (2.17) and the test results of Miyata [64] and Saito [63].



Figure 2.28: Variation of reduced wind velocity U_R with Scruton number Sc. Response of dry inclined cables and proposed design guidelines, from FHWA report [47].

The PTI guide [84] specifies that the level of damping required for satisfactory performance of each cable is controlled by a provision for DIG that is more stringent than that established for RWIV. Figure 2.28 however illustrates that if even a low amount of structural damping is provided, $\zeta > 0.3\%$, corresponding to a $Sc \simeq 3$, then the reverse is true and that if enough damping is provided to mitigate against RWIV then HSV and DIG should also be suppressed. The FHWA [47] therefore upheld the Irwin/PTI criteria but proposed an addendum to account for the use of countermeasures which have proven effective in the mitigation of the wind-induced vibrations,

$$Sc = \frac{m\zeta}{\rho D^2} > \begin{cases} 10, & \text{for regular cables} \\ 5, & \text{for cables with effective surface treatment} \end{cases}$$
(2.19)

where surface treatment is one such countermeasure. These are discussed in the next section.

2.9.2 Methods of Mitigation

Cable response can be mitigated by numerous methods, some of which are more practical than others. Increasing either cable spacing (to reduce wake interactions) or mass density (to increase the Scruton number) by more than a small amount impacts other design constraints. As does increasing cable tension in an attempt to increase natural frequency. The latter can however also be achieved by reducing the effective length of a given cable by the use of cross-ties [34, 87]. These can also assist in raising cable damping, the often insignificant level of which is a principal reason for cable vibration. Thus the provision of additional mechanical damping is "one of the most effective ways of suppressing aerodynamic instability, or postponing it to higher wind speeds and thus making it rare enough not to be of concern" [47]. The most prevalent method of these methods, is to attach dampers at cable anchorages [44, 57]. In contrast to these measures which act to suppress motion, surface treatment of the cable acts to suppress the governing mechanism of RWIV by altering the shape or surface roughness of the cable to prevent the circumferential oscillation of the rivulet [39, 88].

Dampers

Dampers can adopt several forms, can be fitted internally or externally to the cable and can be linear or non-linear in nature. All however act to provide additional mechanical damping to the system and are typically attached near the lower anchorage for reasons of aesthetic design and practical limitations of installation [47]. Figure 2.29 illustrates two such examples.

Linear dampers, where the force generated is directly proportional to the imparted velocity, have been shown to be effective in mitigating response on a number of cable stayed bridges to date [33, 57]. Figure 2.30 illustrates a typical example of this via cable response on the Fred Hartman Bridge pre- and post-viscous damper installation [44, 57]. As linear dampers are tuned to provide optimal damping in a particular mode of vibration, by nature providing less



Figure 2.29: Typical dampers attached to stay cable near lower anchorage. Left: Elastomeric internal damper. Right: External hydraulic damper, from Felber [89].

damping in other modes, selecting the correct mode to design against is essential in obtaining an effective result. The FHWA report [47] suggests that this should typically be the second mode given the prevalence of this response during 'insitu' testing [44, 73]. However as outlined in section 2.2 the RWIV response of a particular cable is typically composed of a number of modes, the predominant of which varies with cable design and environmental conditions. As such modes other than the second have also been found to be dominant [56, 57, 73].



Figure 2.30: Effect of viscous dampers on one cable (AS-16) of the Fred Hartman Bridge. 1 minute root mean square acceleration vs. incident free-stream velocity. Left: Pre-damper. Right: Post-damper, from FHWA report [47].

In contrast non-linear dampers, where the force generated is not directly proportional to the imparted velocity, have been shown analytically to provide more effective damping over a larger range of modes [90]. This arises as optimal performance in different modes is obtained at different amplitudes of vibration. Nonlinear dampers would therefore appear to have significant potential design advantages over conventional linear dampers. To date however these have not yet been subjected to the same degree of practical testing as linear dampers.

Several other aspects of damper design must also be considered in addition to the choice between linear and non-linear response. The FHWA report [47] provides a more extensive review of this. It should however be noted that both Ruscheweyh [30] and Gu [43] determined that although increasing damping reduced the amplitude of oscillation it did not significantly alter the wind speed range under which such a vibration would occur if a certain damping ratio was not met. As can be seen from figure 2.31 Gu [43] determined this to be $\zeta = 0.6\%$ in the wind-tunnel, while Geurts [11] determined a value of $\zeta = 0.5\%$. These are both significantly higher than the value of $\zeta = 0.3\%$ found earlier in the establishment of the FHWA criteria [47]. Furthermore both Gu [43] and Geurts [11] state that to ensure a sufficient margin of safety that a value of $\zeta \simeq 0.8\%$ is to be targeted in practise.



Figure 2.31: Comparisons of amplitude of response with wind speed for varying damping ratios ζ , from Gu [43]. Variation of *a* with *U* for varying ζ .

Cross-Tie Systems

By connecting stay cables to one another via a series of transverse secondary cables, or cross-ties, a more complex cable network with increased in-plane stiffness is formed, figure 2.32. These have proven a popular and commonly used method of vibration mitigation due to the ease by which such systems can be implemented and maintained [11, 33, 72].



Figure 2.32: Typical cable cross-tie system. Left: Cross-tie as part of network. Right: Close up of individual connection, from Felber [89].

Such a cable network allows redistribution of vibrational energy between stays. Cross-ties therefore promote two forms of response, local modes and global modes [87, 91, 92]. The former refers to vibration of a specific cable wherein the wavelength of response is governed by the distance between constraints. By reducing this length, cross ties increase the frequency of vibration while limiting the maximum amplitude of an individual cable. The latter refers to vibration of a significant proportion, or all, of the cable network as a single entity. Figure 2.33 illustrates the 'step' response pattern typical of such a network. Where a series of fundamental global modes is followed by a large number of localised modes with little frequency separation at the 'modal plateau' [87, 91, 92]. These are subsequently followed by a series of higher network modes before a second plateau of localised modes of higher order.

Adding cross-ties however is a complicated process and the benefits attainable



Figure 2.33: Modal Frequency response curve for three cable networks on Fred Hartman Bridge which display a typical 'step' response and 'modal plateau', from FHWA report [47].

through manipulation of cable frequencies must be balanced against the potential for undesirable behaviour in local modes. This can be achieved by ensuring the frequency of the modal plateau is kept as high as possible [47] and by ensuring that the number of unrestrained modes is limited by careful spacing of the crossties. Symmetric modes are preferable with evenly spaced configurations to be particularly avoided [47, 87]. Furthermore to ensure that cross-ties do not become slack and lose effectiveness during an event these must be provided with sufficient initial tension, the level of which is dependant upon the properties of the cable system and the design wind event [87]. Therefore if properly specified cross-ties can be an effective method for suppressing cable vibration within the cable plane, however these provide only marginal benefit for out of plane response [87].

Cable Surface Treatments

Countermeasures which disrupt rivulet movement or eliminate rivulet formation act to suppress the underlying governing mechanism of RWIV. This is typically achieved by some form of additional surface roughness or through small protrusions on the cable sheath, common examples of which can be seen in figure 2.34.



Figure 2.34: Various forms of cable surface treatment, from FHWA report [47].

The design to the far left of figure 2.34, with small axially aligned protrusions running parallel to the cable axis, was found to be successful in suppressing oscillations by Matsumoto [33, 34]. It was subsequently implemented and has proven effective on the Higashi-Kobe Bridge. This design however was found to raise the drag coefficient of a cable from $\bar{C}_D \simeq 0.6$ to $\bar{C}_D > 1.05$ [39] or $\bar{C}_D \simeq 1.35$ [88] dependant upon the exact geometry and flow conditions examined. As the drag force on cables represents a significant proportion of the total drag force for longer span bridges, 50% for a 900 m main span [93], unduly raising this drag coefficient provides additional loading which could readily be eliminated.

To counteract this increase in drag coefficient while maintaining aerodynamic stability Miyata [94] examined cables with lumped surface roughness elements, top right of figure 2.34. The wind-tunnel study undertaken found that if these 'dimples' were of the order of 1 percent cable diameter, aerodynamic stability was maintained with no appreciable increase in drag force [94]. A result which has seen dimpled cables successfully incorporated 'in-situ' on the Tatara bridge [88].

The design most commonly used on new bridges however is that of a helical rib or strake, bottom right of figure 2.34. A sample of the bridges to use such a design can be found in table 2.6. These ribs act to disturb the flow separation points with axial location and hence disorganise rivulet formation [34]. Flamand [39] tested various sizes and configurations of this design and found that a double helix design of a 1.3×10^{-3} m high, 2.0×10^{-3} m wide rib at a pitch of 0.6 m provided the optimal balance between stability, drag reduction ($\bar{C}_D \simeq 0.6$) and ease of manufacture. This led to successful incorporation of this specific design on the Normandie Bridge [39].

Bridge	Location	Country
Bai Chay Bridge	Ha Long Bay	Vietnam
Bill Emerson Memorial Bridge	Cape Girardeau, Missouri	USA
Greenville Bridge	Greenville, Mississippi	USA
Leonard P. Zakim Bridge	Boston, Massachusetts	USA
Maysville-Aberdeen Bridge	Maysville, Kentucky	USA
Millau Viaduct	Millau	France
Normandie Bridge	Le Havre	France
Uddevalla Bridge	Uddevalla	Sweden
U.S. Grant Bridge	Portsmouth, Ohio	USA
Vasco Da Gama Bridge	Lisbon	Portugal
William Natcher Bridge	Maceo, Kentucky	USA

Table 2.6: A list of some cable stayed bridges to employ helical ribs on stay cables.

Gu [43] likewise found that helical wires of $\simeq 1 \times 10^{-3}$ m height successfully reduced vibration amplitude, while larger diameters wires, 3 or 7×10^{-3} m, were found to stop rivulet formation and correspondingly completely eliminated vibration. In contrast smaller diameter wires, 0.5×10^{-3} m, were found to be too thin to stop rivulet formation and actually produced a vibration of larger amplitude, figure 2.35. This investigation also highlighted the importance of rib directionality and pitch. As under the same configuration a rib twisted in the anti-clockwise sense produced a far greater response than one twisted in the clockwise sense [43], while only at a pitch of less than 0.3 m was a single rib found to halt vibration [43] agreeing well with the study of Flamand [39].



Figure 2.35: Comparisons of amplitude of response with wind speed for varying rib heights and configurations, from Gu [43]. Variation of a with U.

Other surface treatments to alter the cross-sectional diameter along the cable axis have also been shown to be effective. Phelan [73] demonstrated the viability of the addition of circular rings at axial intervals in the wind-tunnel and on the Veterans Memorial Bridge. While Flamand [39] did likewise with sharp interfaces at the locations where the cable sheath overlapped on the Normandie Bridge. However these treatments are not as common as the designs previously outlined.

Surface treatments, such as these, are so effective in the suppression of RWIV that the Scruton condition (2.19) proposed by the FHWA [47] has a modifier if such a treatment is included. Therefore major cable suppliers [95–97] now all provide stay sheaths which include some form of surface modification and these are now integral to the design process of cable-stayed bridges.

2.10 Summary

In summary, it is clear that RWIV while not fully understood has been the subject of a significant body of research. This can be typically categorised as either a fullscale 'in-situ' investigation, as an experimental examination undertaken within the wind-tunnel, as an analytical analysis or as a numerical simulation; each of which have been discussed within this section. From these, the conditions under which RWIV typically presents and the form of the response have been identified. This in turn has enabled RWIV to be identified from other forms of cable instability and methods of mitigation determined.

What is also clear is that numerical simulation of RWIV has been the subject of significantly fewer studies than the other categories identified and as such there is a considerable need to develop this further. Modelling such an instability however is both challenging and computationally intensive due to the complexity of the problem outlined and would have to draw on traditionally disparate simulation techniques. Therefore an incremental approach, whereby key features that can be validated are incorporated within individual solvers before being combined into a more complete numerical model to effectively simulate aspects of RWIV, would appear well suited. To achieve this with regard to rivulet formation and evolution, it is necessary to understand both the effect that the geometry of the body has on the aerodynamic field and the effect that the aerodynamic field has on the body. The following chapters present the development and validation of the numerical model proposed to achieve this.

Chapter 3

Aerodynamic Solver - Discrete Vortex Method

Having reviewed RWIV literature and the limited number of previous studies which have computationally modelled some aspect of this, the requirement for the creation of a numerical solver capable of simulating RWIV was established in the previous chapter. The objective of this thesis was to undertake the first steps of this process by creating a solver capable of predicting rivulet formation and evolution within a RWIV context. To achieve this two individual solvers were constructed and validated for problems relating to RWIV for which data is available, as outlined in figure 1.2, one to determine the aerodynamic field around a body given its geometry and one to determine the geometry of a thin film of fluid on a body given an aerodynamic field. It is the former which is the subject of the present chapter. The focus of the work undertaken in this chapter was therefore to develop and validate a numerical solver to determine the aerodynamic field around a given body within a RWIV context such that it can be coupled with the thin film solver created in the following chapter.

The present solver is based upon a pre-existing code, DIVEX, originally developed by Vezza [98], Lin [6, 99] and Taylor [7]. This uses the Discrete Vortex Method (DVM). As such a brief theoretical background of this method is given before salient details specific to the DIVEX code, the modelling updates implemented and the reasons for each of these are presented. The remainder of the chapter focuses on applying the created solver to problems related to RWIV with existing results; in so doing validating the solver and gaining a greater understanding of these problems. To achieve this a number of problems including static and oscillating circular cylinders and the addition of rigid artificial rivulets were examined. Given its prevalence in the previous literature (section 2.8) further studies regarding the form, location and geometry of these rivulets were undertaken, many for the first time. The new understanding gained as a consequence of this work is discussed and a final determination as to the validity and suitability of the numerical method chosen is made.

3.1 The Discrete Vortex Method

Most forms of Computational Fluid Dynamics (CFD) typically track two variables within a given flow field, the pressure P and the velocity U. Vortex methods however only solve for one variable, the vorticity $\boldsymbol{\omega}$ where

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{U},\tag{3.1}$$

which in two dimensions can be represented as $\boldsymbol{\omega} = \boldsymbol{k}\omega$, where \boldsymbol{k} is the unit vector in the third dimension and where vectors are given in bold. The Discrete Vortex Method (DVM) accomplishes this based on the principle that the vorticity field within a given flow field can be discretised into a number of vortex particles. This premise, forwarded by Hermholtz [100] and Rosenhead [101] amongst others, suggesting that areas of vorticity, within inviscid flows, could be represented by a number of particles of "appropriate circulation and infinitely small cross section" [102]. Discretisation of the vorticity field into a distribution of vortex particles embedded within a potential flow thus became the origin from which vortex methods evolved. Many different models have now been developed and a vast body of literature has been constructed; comprehensive reviews of which are given by Leonard [103], Spalart [104] and Sarpkaya [102]. A potential flow is one where the velocity field can be given as gradient of a scalar function, $\mathbf{U} = \nabla \Phi$, the velocity potential Φ . The latter occurs when an irrotational fluid, $\nabla \times \mathbf{U} = 0$, occupies a simply-connected region [16].

As these vortex particles are tracked in a Lagrangian manner through the flow field they collectively induce, the requirement for a calculation mesh is eliminated. This represents a major advantage over traditional CFD methods where it has been estimated that "over 50% of the time spent in industry ... is devoted to ... grid generation" [105]. By eliminating this mesh, issues which directly arise from its creation such as numerical diffusion, sufficient resolution and movable or deformable geometries are also removed. While ensuring the concentration of computational resources in the areas of greatest interest, predominantly the wake and near rivulet regions in the present case. This allows small scale vortical features to be satisfactorily resolved.

The velocity of each vortex particle can be determined as a function of all the others through Biot-Savart integration and application of Green's Theorem. Therefore given the distribution and the velocities of these vortex particles and knowledge of the imposed boundary conditions, the entire flow field can be defined. The velocity therefore only has to be calculated at the centre of every vortex particle. Given this, the lack of mesh and the concentration of resources in the areas of most interest, vortex methods were therefore thought to represent the optimum choice of computational method for the present problem. This was further reinforced by the previous success of vortex methods [102–104], and the DIVEX code itself [7, 106, 107], with related bluff body aeroelastic problems.

Vortex methods do have several disadvantages however. These include singularities which arise from the use of point vortices, computational cost being proportional to the square of the number of vortex particles and the inherent inaccuracy of discretising a continuous field into distinct particles. Methods to circumvent these will be briefly discussed in section 3.1.2 after theoretical and numerical aspects of the DVM are considered. Particular emphasis in both cases being placed on the present formulation.

3.1.1 Numerical Aspects of the DVM

Two-dimensional incompressible flow is governed by the continuity (3.2) and Navier-Stokes equations (3.3),

$$\nabla \cdot \boldsymbol{U} = 0, \qquad (3.2)$$

$$\frac{D\boldsymbol{U}}{Dt} = -\frac{1}{\rho}\nabla P + \nu\nabla^2 \boldsymbol{U} + F_{body}, \qquad (3.3)$$

where $\frac{DU}{Dt}$ is the material derivative defined as $\frac{\partial U}{\partial t} + (U \cdot \nabla)U$ and F_{body} represents additional body forces such as gravity. Given the definition of the vorticity (3.1), that both density ρ and kinematic viscosity ν are assumed constant, and with only conservative body forces acting, equation 3.3 can be reformulated to give the vorticity transport equation

$$\frac{D\boldsymbol{\omega}}{Dt} = \nu \nabla^2 \boldsymbol{\omega}.$$
(3.4)

While by definition of a vector potential known as the stream-function $\boldsymbol{\psi}$, where $\boldsymbol{U} = \nabla \times \boldsymbol{\psi}$ and $\nabla \cdot \boldsymbol{\psi} = 0$ and which in two dimensions can be represented as $\boldsymbol{\psi} = \boldsymbol{k} \boldsymbol{\psi}$, then (3.2) may be reformulated as

$$\nabla^2 \psi = -\omega. \tag{3.5}$$

For an inviscid flow (3.4) and (3.5) establish that vorticity is a kinematic property of any given vortex particle, while for viscous flows these describe how any vorticity produced is convected and diffused within a given flow field. As the fluid examined herein is assumed homogeneous such vorticity production only occurs "*at the boundaries of the fluid region*" [102]. Therefore boundary conditions have extensive influence on the overall problem.

Herein the no-slip and no-penetration conditions are implemented at the body surface. While the condition for flow undisturbed by the vorticity shed by the body is applied in the far field. Using the present notation these are given by

$$\boldsymbol{U} = \boldsymbol{U}_i \quad \text{on} \quad \mathbf{S}_i \quad \text{or} \quad \nabla \boldsymbol{\psi} = \nabla \boldsymbol{\psi}_i \quad \text{on} \quad \mathbf{S}_i$$
$$\boldsymbol{U} = \boldsymbol{U}_{\infty} \quad \text{on} \quad \mathbf{S}_{\infty} \quad \text{or} \quad \nabla \boldsymbol{\psi} = \nabla \boldsymbol{\psi}_{\infty} \quad \text{on} \quad \mathbf{S}_{\infty}, \tag{3.6}$$

where S denotes a body surface and the subscripts i and ∞ are body and far field identifiers respectively. Proper definition of the problem only allows one of the normal and tangential boundary conditions to be applied explicitly at the body surface. In the present formulation this is the normal component or no-penetration condition, although the tangential, no-slip condition is implicitly satisfied by representing the internal kinematics of each body as solid (3.7). This states that the velocity at any point p on the surface or within the body i can be described as

$$\boldsymbol{U}_p = \boldsymbol{U}_{ic} + \boldsymbol{\Omega}_i \times (\boldsymbol{r}_p - \boldsymbol{r}_{ic}), \qquad (3.7)$$

where \mathbf{r}_p is the position vector of this point and \mathbf{r}_{ic} is the position vector of a fixed reference point on the body, figure 3.1. U_{ic} is the velocity at this reference point and Ω is the rotational velocity vector of the body which in two dimensions can be represented as $\Omega = \mathbf{k}\Omega$. Equation (3.7) can also be represented in stream function form as

$$\nabla^2 \psi_i = -2\Omega_i. \tag{3.8}$$

The velocity field can then be determined using the Biot-Savart law and application of Green's theorem to (3.5) for the flow region F and (3.8) to the body region B, figure 3.2. Originally derived by Vezza [98] for the DIVEX solver, with a more detailed discussion given in Lin [6, 99], this relationship states that for a point p within the flow region, the velocity is given as



Figure 3.1: Co-ordinate system and body orientation, from Taylor [7].

$$U_{p} = U_{\infty} + \frac{1}{2\pi} \int_{F_{b}} \omega \frac{\boldsymbol{k} \times (\boldsymbol{r}_{p} - \boldsymbol{r})}{\|\boldsymbol{r}_{p} - \boldsymbol{r}\|^{2}} dF_{b} + \frac{1}{2\pi} \int_{F_{w}} \omega \frac{\boldsymbol{k} \times (\boldsymbol{r}_{p} - \boldsymbol{r})}{\|\boldsymbol{r}_{p} - \boldsymbol{r}\|^{2}} dF_{w} + \frac{1}{2\pi} \int_{B_{i}} 2\Omega_{i} \frac{\boldsymbol{k} \times (\boldsymbol{r}_{p} - \boldsymbol{r})}{\|\boldsymbol{r}_{p} - \boldsymbol{r}\|^{2}} dB_{i}.$$
(3.9)

where $F = F_b \bigcup F_w$ and $F_b \bigcap F_w = 0$. Wherein the contributions resulting from the free-stream flow U_{∞} , the vorticity within a small control zone around the body labelled F_b , the vorticity within the remaining flow or wake, labelled F_w , and the vorticity within the solid region labelled B_i , due to the rotational movement of the body can be individually identified.

Two contributions arise from the vorticity within the flow region as this is split in two for numerical reasons [6, 7, 98, 99]; specifically that each region discretises vorticity differently, figure 3.3. The control zone contains nascent vortex sheets and uses a panel/sub-panel method with piecewise linear distributions [6] to represent the vorticity creation and reabsorption at the boundaries. The wake zone contains the remaining vorticity which although originally created in the control zone has been convected and diffused from this region.

The pressure distribution on the body surface can then be evaluated by integrating the pressure gradient along the body contour, which at a specific point pis given by



Figure 3.2: Regions and labels of flow domain, from Taylor [7].



Figure 3.3: Surface vorticity discretisation and outline of control zone, from Taylor [7].

$$\frac{1}{\rho}\frac{\partial P}{\partial s} = \boldsymbol{t} \cdot \frac{D\boldsymbol{U}_c}{Dt} - \boldsymbol{n} \cdot (\boldsymbol{r} - \boldsymbol{r}_p)\frac{D\Omega}{Dt} + \boldsymbol{t} \cdot (\boldsymbol{r} - \boldsymbol{r}_p)\Omega^2 + \nu \frac{\partial \omega}{\partial n}, \quad (3.10)$$

where n and t are normal and tangential vectors respectively. The resulting pressure distribution can then be integrated around the body surface to calculate the aerodynamic forces on the body and the moment about the body reference point r_{ic} at a particular computational time step. From these, further measures such as time-averaged mean coefficients and fluctuating components may then also be determined.

3.1.2 Issues with the DVM

Several techniques have been proposed to circumvent the disadvantages of vortex methods outlined in section 3.1 and are summarised in the reviews of Leonard [103], Spalart [104] and Sarpkaya [102]. The focus here however is on those used within the DIVEX code and in particular on those of most relevance to the present work.

By using a cut-off function which represents a distribution of vorticity within a 'core radius' σ around the vortex centre, DIVEX overcomes the singularities inherent in the use of point vortices of infinite vorticity. While to reduce the computational cost prescribed by the Biot-Savart law, DIVEX uses both vortex merging and a zonal decomposition algorithm. The former reduces the operation count by limiting the number of the particles within the flow, while the latter achieves similar reductions by determining the velocity through a series expansion of smaller zones rather than by direct summation. For completeness further details of these procedures are given in appendix B as these are not the focus of the present study.

To overcome issues arising from the diffusion term, $\nu \nabla^2 \omega$, present in the vorticity transport equation (3.4) various techniques have been proposed and utilised to modify an inviscid model to allow for viscous calculations [102]. Amongst the most common is the operator splitting technique, where the vorticity transport equation (3.4) is divided into two parts, a convection equation and a diffusion equation, which are then solved sequentially rather than simultaneously. Again a more complete review of the various methods can be found within Taylor [7] or Sarpkaya [102] suffice to say within the present formulation a random vortex method, as proposed by Chorin [108], is used. This assumes that the diffusion equation can be modelled in a statistical sense with a sufficient number of vortices each undergoing a stochastic local movement.

The procedure itself is such that at every timestep t the position of each vortex particle is first calculated according to the convection term
$$\frac{\partial\omega}{\partial t} + (\boldsymbol{U}\cdot\nabla)\omega = 0, \qquad (3.11)$$

of the vorticity transport equation. To simulate the diffusion term

$$\frac{\partial\omega}{\partial t} = \nu \nabla^2 \omega, \qquad (3.12)$$

a random displacement or walk, of Gaussian distribution with zero mean and a standard deviation of $\sqrt{2\nu\Delta t}$ is then applied in each of the two orthogonal directions, where Δt is the timestep size. The position vector of a given vortex particle, \mathbf{r}_p , is therefore given by

$$\boldsymbol{r}_{p}(t + \Delta t) = \boldsymbol{r}_{p}(t) + U(t)\Delta t + (\eta_{x} + \eta_{y})$$
(3.13)

where η_x and η_y are the components of the random walk in the x and y directions respectively.

While this method has been repeatedly shown to provide quantitatively and qualitatively correct solutions [6, 7, 104, 106–108] and to converge under laminar conditions in the absence of boundaries [109–112], it does have inherent limitations [102]. Two of which are of particular interest for the present study.

- Due to the statistical nature of the method, quantities can display significant spatial variations [102, 103] at a specific time instant and temporal variations at a given location. These may therefore require smoothing or averaging.
- As equation (3.12) represents viscous diffusion it has negligible effect in comparison with that of turbulence within regions of highly turbulent flow.

However given that these are inherent to the method and thus within the pre-existing code and that to date no satisfactory method has been proposed to eliminate these, within the present context these limitations were considered acceptable at this stage in the overall evolution of numerical modelling of RWIV.

3.2 Additions and Refinements to DIVEX Implementation

The original DIVEX code presented by Lin [6] was developed specifically for analysis of the dynamic stall phenomenon of pitching aerofoils. For use with a wider variety of potential problems, in particular aerodynamic stability of bridge deck sections, Taylor [7] generalised this method while introducing several improvements including sharp corner modelling, wake decay calculations and the zonal decomposition algorithm. The focus of the present work was not to create a new vortex method code but to develop the existing DIVEX code such that it can be used in problems related to RWIV. This involved numerous changes, the most pertinent of which will be discussed in greater detail in the following sections. Several other minor modifications involved with modelling near circular geometries and not the sharp edge bodies previously examined were also made but are not explicitly discussed.

3.2.1 Shear Stress Distribution

Prior versions of the DIVEX code did not calculate a wall shear stress as within the bodies considered separation generally occurred at the sharp edges and thus the friction distribution was not as significant as the pressure distribution. The results of Lemaitre [9] however highlighted that pressure and friction were of similar importance in the generation of rivulets. One modification made within the present code was therefore to determine a distribution of wall shear stress.

Traditional grid based methods typically use wall boundary conditions to model regions near the surface of a body and the boundary layers that these contain. Such conditions are complicated however and vary dependant upon whether this region is assumed laminar or turbulent, which CFD package and which, if any, turbulence model is to be used. The latter adding further complexity. As such several wall functions have been created, of which Versteeg and Malalasekera [105] provide a more detailed review. Typically however these comprise three distinct layers. A laminar sub-layer where the fluid is in direct contact with the wall, the turbulent log-law layer and the outer inertia dominated layer far from the wall [105] each of which has its own empirical relationship relating velocity to shear stress.

Creating specified layers would eliminate the benefits of a mesh-free code, especially when dealing with a deformable geometry, and would increase the time to solve. Furthermore no explicit turbulence model exists within the DIVEX code and successfully implementing such a model is a substantial body of work in its own right and outwith the scope of the present thesis. Therefore only the model to obtain wall shear stress for laminar flow, the linear sub-layer, was implemented. As figure 3.4 illustrates this assumes that velocity varies linearly with distance from the wall and that the shear stress T can therefore be given as

$$T = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0},\tag{3.14}$$

where μ is the dynamic viscosity, u is the flow velocity parallel to the wall and y is the normal distance to the wall.



Figure 3.4: Linear velocity distribution at wall, from Versteeg and Malalasekera [105].

To implement this method required the determination of velocities parallel to the surface at points located a small distance normal to the nodal locations of the body surface. Figure 3.5 illustrates the locations of these points, which were located at position vectors λn from every node, where n is the unit normal vector and λ is the distance offset from that node which is a factor of the distance of node from the body reference point ($\lambda = k || \mathbf{r} - \mathbf{r}_{ic} ||$), where k is a multiplying coefficient based on the body radius R.



Figure 3.5: Position of additional points for calculation of wall shear stress. Upper: Single Point. Lower: Two Points.

Using this normal vector and equation (3.9) the velocity parallel to the surface at these points could be calculated, from which the shear stress T could be determined if the velocity at the surface was assumed to be zero. This is typically presented in non-dimensional format as the time averaged mean coefficient of friction \bar{C}_F , given by

$$\bar{C}_F = \frac{2T}{\rho U_\infty^2}.\tag{3.15}$$

It rapidly became clear however that only using one point per node did not provide the quality of results desired, figure 3.6. This was likely caused by a feature of the DIVEX code which requires the location of the nascent vortices that represent the surface to be marginally offset from the actual surface as these are of finite size. To overcome this a second point per node was located at the outer edge of the vortex core; the location of the first additional point was kept constant. A distribution of \bar{C}_F could therefore be determined based on the distance between these two points and the corresponding difference in velocity parallel to the surface. As can be seen from figure 3.6 which compares these two results, this two-point method provides a distribution of \bar{C}_F which qualitatively is far closer to the profile obtained experimentally by Achenbach for a circular cylinder [113]. Where θ is the angle clockwise from the windward horizontal (typically the stagnation point). By then quantitively comparing the present distribution of \bar{C}_F obtained from two points with that of Achenbach at $Re = 100 \times 10^3$ a parameter study as to the value of k could be undertaken. This determined that a value of k = 0.0015, corresponding to a distance 0.075% of cylinder diameter, was optimum. A comparison of this profile of \bar{C}_F with that obtained by Achenbach [113] and those of a previous three-dimensional numerical analysis by Travin [114] which uses wall functions can be seen in figure 3.7.



Figure 3.6: Comparison of coefficient of friction distributions calculated by the present solver with one point per node and two points per node with experimental data of Achenbach [113]. Variation of \bar{C}_F with θ for a circular cylinder.

Figure 3.7 illustrates that the present results show excellent agreement with both prior experimental and numerical distributions over the portion of the boundary layer where the flow is laminar, $\theta \leq 60^{\circ}$ and $\theta \geq 300^{\circ}$. Once the flow becomes turbulent however, while the results are still qualitatively comparable, quantitively the level of agreement is not as close. This is attributable to the two-dimensionality of the present code and more specifically to the lack of a turbulence model; the introduction of which would be very desirable within future work. That said, more complicated studies such as the three-dimensional DES study of Travin [114] which includes a turbulence model and wall functions still does not capture all the features of this non dimensional wall shear stress. Although a difference in Reynolds number must also be accounted for. This does however predict the separation region, $70 \leq \theta \leq 85^{\circ}$, better than the present method. This though is achieved at considerable cost where due to the very small mesh size required for this resolution ($\simeq 5 \times 10^5$ cells) a substantially smaller timestep than the present model is required, and thus far greater run-time to model the same time interval.



Figure 3.7: Comparison of coefficient of friction distribution calculated by present solver with that obtained experimentally by Achenbach [113] at $Re = 1 \times 10^5$ and numerically by Travin [114] at $Re = 1.4 \times 10^5$. Variation of \bar{C}_F with θ .

3.2.2 Local Modelling of Rivulet

As was outlined in section 2.8 and at the outset of the present chapter, capturing the effect that addition of an artificial rivulet to a circular cylinder has on the body response is very significant in the validation of the present implementation of DIVEX within the wider RWIV context, as both experimental [5, 38, 40, 67] and numerical data [75, 80] is available. Given the importance of accurate modelling of these artificial rivulets to this problem, a method to better capture the geometry of these was introduced. This specified locally refined higher definition regions where individual vortices were positioned exactly, rather than assuming that these lay equi-distant on a straight line between two nodes. In addition to achieving a closer representation of a given shape these high definition regions also allowed greater concentration of resources through local refinement of parameters.



Figure 3.8: Definition of high definition region and reference points.

The method implemented is illustrated in figure 3.8. This is a modified version of the method developed by Lin [6], the major difference being that individual vortex locations were specified in a local coordinate system (x_{hd}, y_{hd}) in terms of the position vector \mathbf{r}_{hd} to an additional defined reference point, here chosen to be the mid-point of the base of the rivulet, rather than in global co-ordinate system. By defining the local coordinate system (x_{hd}, y_{hd}) in terms of the position vector from the reference point on the main body to the reference point within this system \mathbf{r}_{mb} , both static or oscillating rigid rivulets can be examined by the variation of a single parameter, the direction of this vector $\hat{\mathbf{r}}_{mb}$. The magnitude of which remains constant. This is possible because the rivulets considered are rigid (of fixed geometry) for consistency with previous literature [5, 38]. However as the focus of this chapter is to validate the present code, with the exception of section 3.5.6, only static artificial rivulets are considered as this is the case for which data is available [75, 80].

3.2.3 Allowing for Changes in Geometry

Originally designed for use with solid geometries [6, 7], the shape of a given body in DIVEX is prescribed at the outset. This geometry then remains fixed for all time, although the bodies themselves can undergo translation or rotation. This is reflected in the underlying governing equations, previously outlined in section 3.1.1, which assume the body in question to be solid. Therefore to ensure conservation of total circulation in the system considered when vorticity enters the flow at a body surface, an equal and opposite decrease is found within the solid body itself [6, 98, 99].

Due to the requirement for a degree of deformability of the modelled body within the final coupled solver, alterations were required to allow the body to undergo limited changes in geometry. However with the addition of local deformations, the underlying mathematics on which the present code is based are no longer true in the strictest sense. Surface accelerations due to deformation lead to a vorticity flux [115] which are not accounted for within the present code but which are accounted for in truly deformable codes such as that proposed by Lundgren and Koumoutsakos [115]. Within the present system however, these deformations are of the order of tenths of a millimetre on a cable several hundred millimetres in diameter, therefore only represent a one in one thousand magnitude change. Given this and that construction of a truly deformable solver would have involved a complete rewriting of the inherited code, implementing the changes necessary to account for this were not seen as feasible and were not the focus of the present work. This would however be a possible next step in the evolution of the final coupled solver created. Therefore while the process undertaken was not mathematically rigorous, controls were put in place to limit potential errors and checks were performed to quantify that the magnitudes of these were acceptable with the present context. These will be discussed in due course.

The method implemented to allow deformations to occur over a given timestep followed a three step process. First, the locations of these new nodes were calculated. In the final coupled solver these are based upon the output of the thin film solver of the following chapter, but for present purposes could be specified by a given input deck. Second, any vorticity now present within the body due to variation of nodal location was re-absorbed as if the body underwent a rigid translation or rotation. Finally a new nodal vorticity distribution for the body surface is calculated based upon the updated geometry. These vortices convecting and diffusing according to the original DIVEX code. The vast majority of vorticity is therefore accounted for, the exception being that directly due to surface deformation.

To quantify this effect, a circle with 360 nodes (359 panels) was modelled. After an initial start up period, each node was then subjected to a radial movement such that a sine wave of maximum height 0.15% of the original cylinder radius was superimposed on the original distribution over a period of 2000 timesteps. To illustrate which the calculated distribution of normalised radius (R/R_0) as the start, middle and end of this process are shown in figure 3.9. In which R_0 represents the initial value of the radius at each location. As can be seen from figure 3.10 both the instantaneous, $\Delta\Gamma$, and total circulation, Γ , were not significantly affected over this period and as such this assumption was deemed acceptable for the present situation.

Three additional limitations were placed on the present code. First to ensure nodes did not amalgamate toward the same location, motion was constrained to occur in the radial direction only. Second to ensure that the change of any one panel length (the distance between two nodes) was insignificant, constraints were placed on maximum nodal motion per timestep and total motion of a given node.



Figure 3.9: Temporal evolution of normalised radius distribution with timestep calculated by the present solver for deformation test case. Variation of R/R_0 with timestep.



Figure 3.10: Change in instantaneous and total circulation with timestep for deformable geometry. Variation in $\Delta\Gamma$ and Γ with timestep.



Figure 3.11: Change in normalised body area with timestep for deformable geometry. Variation in $1 - (A/A_0)$ with timestep.

This ensures a greater degree of stability as nodes can not rapidly displace to infinity. Finally as can be seen from figure 3.11 which shows the total variation in normalised body area $(1 - \frac{A}{A_0})$ with timestep, the total body area must be conserved. Here using the same convention as for the radius, the present area being given by A and the original value being given by A_0 . Closer examination of figure 3.11 highlights a link between the change in total area and the circulation, although this is not fully understood.



Figure 3.12: Comparison in time to solve 10000 timesteps for solid and deformable geometries.

While these conditions are artificial and the assumption that local deformations of the boundary does not significantly affect the vorticity within the flow is limited, it has been shown to be acceptable within the present context. Furthermore given the physical conditions present during RWIV and other assumptions made during the construction of the final coupled solver such as twodimensionality and for a fully coated surface (section 4.3) this is not the major assumption made. What can be said however is that by having to reformulate the geometry matrix used to calculate surface vorticity at every timestep in comparison with a non deforming geometry, there is an increase in time taken to solve for every timestep of $\simeq 20\%$, figure 3.12.

3.3 Circular Cylinder Basics

Having implemented the necessary changes within the DIVEX code, analysis could now be undertaken and results determined. As a datum and to allow convergence studies of the various parameters inherent within the DIVEX code, initial runs were performed on a smooth, static circular cylinder. These determined that to ensure a sufficiently detailed portrayal of the surface geometry and an accurate resolution of the vorticity distribution 359 panels each containing 7 nascent vortices should be used.

Through discussion with previous authors of the DIVEX code [116, 117], ranges for vortex parameters, such as decay constants, specific to the DIVEX code were obtained based on prior experience. Optimum values for each of these were then ascertained for the present geometry through studies such as figure 3.13. These values were typically found toward the limit which provided higher resolution, although with the exception of non-dimensional timestep $\Delta t^* = \frac{\Delta t U_{\infty}}{D}$, specific values are not discussed. The latter based upon the results displayed in figure 3.13 was fixed at $\Delta t^* = 0.01$.



Figure 3.13: Convergence of time averaged mean drag coefficient toward value obtained by ESDU 80025 [61] with reduction in non-dimensional timestep. Variation of \bar{C}_D with Δt^* at $Re = 20 \times 10^3$

From experimental and 'in-situ' analysis RWIV has been found to primarily

occur within the range, $50 \times 10^3 < Re < 150 \times 10^3$ [19, 33] (section 2.2). As outlined in section 2.1.1 however, over these sub-critical Reynolds numbers the aerodynamic coefficients \bar{C}_L and \bar{C}_D show negligible variation with Re [61]. This work will therefore focus on two cases of Reynolds number under smooth flow conditions at $Re = 20 \times 10^3$ and 100×10^3 , which are at the boundaries of the TrSL3 regime, as outlined in section 2.1.1. These specific values were also chosen to allow direct comparison with previous work. The former, which is used in the present chapter represents an approximate mean between the values of Re = 10×10^3 and 26×10^3 used in the numerical and experimental studies of Liu [80] and Matsumoto [81] that study the effect of adding artificial rivulets to a circular cylinder within a RWIV context. It should be noted that while this value is marginally outwith the Reynolds number range for RWIV previously established it was one of the authors who forwarded this range against whose data the present solver is to be validated. Given this, that when undertaken $Re\simeq 20\times 10^3$ was the only value at which both prior numerical and experimental analysis had been undertaken and that, as shown, the variation of aerodynamic parameters within the TrSL3 regime are small, the choice of this value was considered acceptable. The latter value of $Re = 100 \times 10^3$ is used in the following chapter to match that used by Lemaitre [9] for a previous study into thin film evolution.

As illustrated in table 3.1 the aerodynamic coefficients and the Strouhal number (2.3) predicted by the present solver are in excellent agreement with a selection of previous experimental and numerical studies [18, 118–120] at similar *Re.* While by virtue of the consistency of these values over the range of Reynolds number examined, table 3.1 also highlights that the present solver also captures the invariance of these parameters over the TrSL3 regime. The fluctuating lift coefficient C'_L , has been used here to quantify the lift force, as the time averaged mean lift coefficient for a circular cylinder should be zero. Given that this was determined to be $\bar{C}_L = 0.00035$ and 0.00042 for the two cases of Reynolds number investigated, it can be said that this was also ascertained within the present

	C'_L	\bar{C}_D	St
Present $Re = 20 \times 10^3$	0.519	1.254	0.211
Present $Re = 100 \times 10^3$	0.527	1.2624	0.210
Zdravkovich [18]	0.55	$\simeq 1.2$	$\simeq 0.2$
Norberg $[118]$	$\simeq 0.5$	-	$\simeq 0.2$
Breuer [119]	-	1.1-1.3	0.2-0.22
Cox [120]	-	1.1-1.2	0.23-0.25

Table 3.1: Comparison of fluctuating lift coefficient, time averaged mean drag coefficient and Strouhal number determined by present solver for plain circular cylinder with previous literature [18, 118–120].

numerical implementation. Likewise the fluctuating drag coefficients determined by the present solver, $C'_D = 0.057$ and 0.058, also agree well with the commonly accepted experimental value of 0.04 - 0.05 as given in Zdravkovich [18]. This is approximately an order of magnitude smaller than C'_L , and composes only 4.5% of the total drag force in the present case, which also agrees well with previous experimental values [18]. It should also be noted that while all calculations are impulsively started, after an initial period the flow settles into an approximately regular oscillatory pattern, figure 3.14, and the effects of this impulsive start become negligible. To minimise the effect of this when calculating time averaged quantities such as \bar{C}_L and \bar{C}_D , time dependant data from this initial regime $t^* < 60$ is not included.

Closer examination of the spectral analysis undertaken, shown here for the $Re = 20 \times 10^3$ case in figure 3.15, highlights a single peak within the Power Spectral Density (PSD). This corresponds to the Karman vortex shedding frequency for a circular cylinder. The Strouhal number of which, St = 0.211, agrees well with the commonly accepted value of $\simeq 0.2$ at this Reynolds number, table 3.1, and is certainly within the inherent experimental scatter of $\pm 10\%$ for this flow regime illustrated in figure 2.5.



Figure 3.14: Non-dimensional time history of lift and drag coefficients. Variation of C_L and C_D with t^* at $Re = 20 \times 10^3$.

The minor over-prediction of \bar{C}_D and C'_D in the present code is predominantly the result of a slight over-prediction of the suction pressure in the body wake which results due to the trailing vortices forming marginally too close to the rear of the body, figure 3.16. This is consistent with previous studies on different bluff body geometries and appears to be an inherent limitation of the present code [7, 106, 107]. With this exception however the calculated pressure profile, illustrated in figure 3.17, presently determined is in excellent agreement with both previous experimental [61] and computational results [121, 122]. Indeed at many locations providing a better representation of the experimental results than these numerical studies. The distribution of pressure being presented as a nondimensional coefficient, C_P (3.16) from which a time averaged mean coefficient of pressure can be determined \bar{C}_P .

$$C_P = \frac{2P}{\rho D U_\infty^2}.\tag{3.16}$$

This excellent agreement in the pressure profile is also apparent when the four parameters representative of the mean \bar{C}_P distribution of the present results are compared with those obtained for a smooth cylinder based on the empirical formulae defined in ESDU 80025 [61] over many experimental studies, table 3.2.



Figure 3.15: Power Spectral Density of plain circular cylinder at $Re = 20 \times 10^3$ indicating St peak .

Also defined in ESDU 80025 [61] and shown in figure 3.18, these four parameters illustrated in figure 3.18 are C_{pm} the minimum value \bar{C}_P attains on the cylinder surface and θ_m is the angle at which this occurs, C_{pb} the mean base pressure and θ_b is the angle at which this starts. It should be noted that while the values of C_{pm} , θ_b and θ_m presently determined and given in table 3.2 were for the upper surface of the cylinder, the results from both surfaces were very similar. The C_{pb} reported however was taken as the mean value of \bar{C}_P between the values of θ_b on the upper and lower halves of the cylinder respectively.

Parameter	Present	ESDU 80025 [61]
C_{pb}	-1.213	-1.2
$ heta_b$	86°	83.31°
C_{pm}	-1.327	-1.296
$ heta_m$	71°	70.12°

Table 3.2: Comparison of C_{pb}, C_{pm}, θ_b and θ_m , with ESDU 80025 [61].

Considering the fluctuating component of pressure C'_P , the predicted results were again found to be similar to those from previous experimental analysis [118, 123] as illustrated by figure 3.19. As was true for values in table 3.2 only the



Figure 3.16: Instantaneous picture of vortex particle location. Blue points represent clockwise circulation, red anti-clockwise circulation and green those representing the surface.



Figure 3.17: Comparison of pressure distribution around a plain circular cylinder determined by the present solver with previous experimental, ESDU 80025 [61], and the numerical results, Yokuda [121] and Dong [122]. Variation of \bar{C}_P with θ .

results from the upper surface of the cylinder are displayed, although those from the lower surface again produce very similar results. Figure 3.19 shows that the present results are in excellent agreement at angles $\theta \leq 75^{\circ}$ where the flow is laminar. However due to the lack of a turbulence model, the larger fluctuations caused by the closer formation of the vortex and three dimensional effects, the correspondence around the separation point and within the wake of the bluff body while of the correct magnitude does vary slightly from experiment, particularly within the range $110 < \theta < 145^{\circ}$. That said, using this distribution of C'_P and the inflection point of the \bar{C}_P profile to determine the mean separation point for the



Figure 3.18: Typical pressure distribution for circular cylinder and identification of pressure distribution parameters, C_{pb}, C_{pm}, θ_b and θ_m , from ESDU 80025 [61].

present solver θ_s as proposed by Zdravkovich [18], the value determined, $\theta_s = 83^{\circ}$, is in very good agreement with previous results [18, 61, 70] as outlined in section 2.4.1. In conjuction with the wall shear stress profile predicted, figure 3.7, the distribution of shear and pressure on the body surface obtained by the present solver can therefore be said to be validated for the datum circular cylinder.



Figure 3.19: Comparison of fluctuating pressure coefficient with previous experimental analysis of Nishimura [123] and Norberg [118]. Variation of C'_P with θ .

As a final means of validation, tests were carried out on the velocity distribution in the wake of the cylinder. Figure 3.20 displays the normalised time averaged streamwise velocity component u/U_{∞} along the centreline of cylinder, figure 3.21. As can be seen the present results are consistent with previous experimental [124, 125] and LES results [126]. There is a slight disparity very close to the body itself which again results from the vortices forming marginally too close to the rear of the body, the inherent two-dimensionality and the lack of turbulence model previously discussed. That said, the rest of the distribution is in excellent agreement.



Figure 3.20: Comparison of the normalised time averaged streamwise velocity component along the centreline of the cylinder with previous numerical analysis of Kassera [126] and experimental analysis of Cantwell [124] and Ong [125]. Variation of u/U_{∞} with x/D.



Figure 3.21: Schematic of centreline.

Given the excellent agreement between the present results and those previously obtained experimentally and numerically for all the parameters examined herein, both fluctuating and mean, the present code is therefore considered to be validated in determining the aerodynamic parameters of interest for a static circular cylinder.

3.4 Forced Oscillation VIV

Having validated the present solver for a static circular cylinder, one more benchmark test case was sought such that the response of the solver under body motion could also be validated; although this will not be examined explicitly within the final coupled solver. VIV was chosen, due to the quantity of previous literature available [15, 49–51], the previous history of the DIVEX code with this phenomenon [106, 107] and its close relationship with RWIV, section 2.3.1. While of the wide range of aspects of VIV studied within the literature [15, 49–51], the 'lock-in' region previously illustrated in figure 2.9 was chosen to be the particular focus of the present validation study for three reasons:

- The ease of implementation should a forced response be imparted rather than allowing a free response to develop. By applying such a forced sinusoidal oscillation a greater level of control is also exerted, as through specification of the normalised amplitude (a/D) and the frequency of vibration (here given in terms of a reduced velocity U_R (2.4)) oscillation is completely defined.
- A parametric study could therefore be undertaken by variation of these two forcing parameters parameters a/D and U_R .
- Behaviour within the 'lock-in' region is significantly different to behaviour outwith this region. System response can therefore be assessed by examination of the frequency response and the variation of aerodynamic coefficients.

For brevity only the across-wind degree of freedom is examined within the present work, although studies in the stream-wise and rotational degrees of freedom found similar results [127]. Likewise, in direct agreement with the findings of Sarpkaya [50], the results showed little variation with Reynolds number in the subcritical region studied, $20 \times 10^3 < Re < 100 \times 10^3$. As such only the $Re = 20 \times 10^3$ case is explicitly reported. The specific amplitude ratios and

frequencies of oscillation, given in terms of the reduced velocity, used in this parameter study are given in table 3.3.

Parameter	Values		
Normalised Amplitude, a/D	1%, 10%, 25%, 50%		
Reduced Velocity, U_R	1 - 8 (steps 0.5)		

Table 3.3: Parameter values used in VIV validation study.

By undertaking spectral analysis of the response at different frequencies of oscillation, three distinct regimes could be highlighted. 'Below lock-in' where the flow is dominated by the effect of the body oscillation. 'Lock-in' where the vortex shedding frequency f_s is synchronised to the body motion frequency f_b and a single response can be detected at this frequency. And 'above lock-in' where the flow begins to approach quasi-static conditions and the influence of body oscillation decreases. A typical spectral response from each regime can be seen in figure 3.22. During the 'below lock-in' regime two distinct peaks can be seen at the vortex shedding and body frequencies, while during 'lock-in' a single peak at the body frequency can be seen. Likewise a single peak at the shedding frequency can be seen in the 'above lock-in' regime. Figure 3.22 also illustrates the relative strength of each of these peaks. The body forcing frequency in the 'below lock-in' regime having considerably greater influence than the vortex shedding frequency; the magnitude of which agrees well with that of the vortex shedding peak in the 'above lock-in' regime. Notably a distinct increase can also be seen in the single peak during 'lock-in' over the other two shedding peaks as the frequency of Karman vortex shedding has synchronised with the frequency of body oscillation.

Through normalisation of the vortex shedding frequency by the body oscillation frequency (f_s/f_b) figure 3.23 illustrates both that 'lock-in' occurs around the expected value for a circular cylinder, $U_R \simeq 1/St \simeq 4.75$, and that the range of 'lock-in' increases with greater amplitude of vibration. The latter primarily



Figure 3.22: Spectral analysis below during and above 'lock-in' at a/D = 0.1. Top Left: 'Below lock-in' at $U_R = 2.5$. Top Right: During 'lock-in' at $U_R = 5$. Bottom: 'Above lock-in' at $U_R = 7.5$.



Figure 3.23: Comparison of effect normalised amplitude ratio, has on lock-in region with cross-wind motion. Variation of f_s/f_b with U_R for various a/D.

resulting due to the increased energy introduced into the system by the forced vibration. These findings are both in excellent agreement with previous studies [50, 106].

Therefore the existence, extent and variation of the 'lock-in' region with vibration amplitude ratio (a/D) was successfully predicted. As was the variation in frequency response both within and outwith the 'lock-in' region for across-wind forced oscillation. One final piece of validation was required; namely quantifiable verification of the effect of motion on aerodynamic response.



Figure 3.24: Comparison of variation of drag coefficient with reduced velocity of present results with previous experimental studies given by Carberry [128]. Variation of \bar{C}_D with U_R .

Figure 3.24 compares the present results with those from experimental studies reviewed by Carberry [128] for the time averaged mean drag coefficient over the reduced velocity range from just below to just above 'lock-in'. As this figure illustrates the results of the present solver agree well both qualitatively and quantitatively with previous studies, in that both the magnitude and local increase of \bar{C}_D within 'lock-in' are successfully predicted. Likewise the reduced velocity range over which 'lock-in' occurs and the prediction of a local maximum at $U_r \simeq 4.75$ are successfully captured. The predictions of form and magnitude of the latter also compare well with previous studies [128]. Although not explicitly shown C'_L likewise displayed a local maximum within the 'lock-in' region, while in both cases increased a/D ratios resulted in greater values of both aerodynamic coefficients. These findings further validating the present results with previous literature [50]. Finally as table 3.4 illustrates the present results also compare well with previous numerical studies, here given by the two and three dimensional LES studies of Tutar [129] for an amplitude of vibration $a/D \simeq 0.1$ at $Re \simeq 20 \times 10^3$. The present results can therefore be said to be in good agreement with previous numerical and experimental studies. As such the present numerical solver can thus be said to be further validated for a plain circular cylinder, this time for forced across-wind oscillation.

Study	St	C_{pb}	\bar{C}_D	C_L'
Present Results	0.199	-1.55	1.24	0.62
2D LES Tutar 2000	0.177	-1.55	1.32	0.63
3D LES Tutar 2000	0.191	-1.37	1.26	0.71

Table 3.4: Comparison of aerodynamic parameter with the numerical studies ofTutar [129].

3.5 Addition of Artificial Rivulets

Having validated the present solver for a plain circular cylinder under both static and dynamic conditions, the effect that addition of an artificial rivulet has on response can now be studied. This allows comparison with wind-tunnel [5, 38, 39, 43, 67] and previous numerical studies [75, 80] that have examined this particular problem and which were outlined in sections 2.6.1 and 2.8 respectively. This allows validation of the present solver within a RWIV context and provides independent confirmation of these previous numerical studies. Further studies are then undertaken which examine the effect of artificial rivulet location, form and size on the aerodynamic response. These may provide a clearer understanding of the role played by such aspects in RWIV.

As previously specified, the basic geometry considered consists of a horizontal circular cylinder aligned perpendicular to smooth flow at a subcritical Reynolds number typical for RWIV of $Re = 20 \times 10^3$. Unless otherwise specified the artificial rivulet used is of trapezoidal form, base width 0.07D and height 0.03D, whose angle clockwise from the windward horizontal (which in this case is also mean stagnation point of the incident flow) at centre is given by θ , figure 3.25. This particular form and size of artificial rivulet were chosen to best represent those used in previous wind-tunnel and numerical studies, the form and size of which vary significantly as illustrated in table 3.5, and to match those used in the numerical study by Liu [80]. Specific dimensions of the latter were provided by Matsumoto [82] as these were not explicitly reported.



Figure 3.25: Geometry of base artificial rivulet form and size, and the angle from windward horizontal θ where this is located.

Throughout the following analyses three rivulet configurations were used. Each was defined by a single angular parameter θ and was chosen to represent the rivulet configuration present during one of the RWIV excitation mechanisms proposed by Verwiebe [71] and discussed in section 2.4.2. Shown in figure 3.26 these three configurations were labelled single, multiple symmetric and multiple antisymmetric and corresponded to mechanisms 2.2, 1 and 2.1 respectively. For clarity the difference between the latter two cases being that the lower rivulet (that present on the lower surface) is at an equal angle to the upper rivulet from incident flow in the symmetric case and from its diametrically opposed point in

¹Portion of larger circle of radius 50 mm offset 45mm from body surface.

²Dimensions obtained through personal communication [82].

Paper	Form	Base Width	Height
Present	Trapezoidal	0.07D	0.03D
Matsumoto 1995 $[38]$	Rectangular	0.15D	0.024D
Flamand 1995 [39]	$Elliptical^1$	0.22D	0.03D
Bosdogianni 1996 [67]	Elliptical	0.13D	0.05D
Gu 2002 [40]	Elliptical	0.12D	0.045D
Zhan 2008 [74]	Square	0.043D	0.043D
Gu 2008 [36]	Elliptical	0.1D	0.06D
Li 2006 [75]	Arc Section	0.1D	0.033D
Li 2006 [75]	Arc Section	0.1D	0.05D
Liu 2007 [80]	Trapezoidal ²	0.07D	0.03D

 Table 3.5:
 Sizes and forms of previously used artificial rivulets, experimental and numerical.

the antisymmetric case, figure 3.26.



Figure 3.26: Three cases of rivulet geometry, where the incident flow is from the left.

3.5.1 Initial Validation

Examining how rivulet configuration, location, form and size affect the overall response will provide the majority of the validation of the present solver within a RWIV context. However before these aspects could be investigated, initial validation against specific numerical [75, 80] and experimental data [61, 81] was required. Two studies were therefore undertaken.

The first study compares the pressure distribution presently obtained for a single rivulet at $\theta = 40^{\circ}$ with that determined by Li [75]. As can be seen from figure 3.27 the presence of the artificial rivulet causes a 'jump' in C_P at this location and a departure from the mean coefficient of pressure distribution obtained for a plain circular cylinder. This results from the rapid change in geometry at the upstream edge of the rivulet and the local separation effect this causes. Both the magnitude and the location of this 'jump' are in excellent agreement with the two-dimensional LES predictions of Li [75]. As is the overall quantitative and qualitative profile of the distribution itself. There are however slight discrepancies between the two distributions in the region immediately behind the artificial rivulet. These can be attributed to the lack of a specific turbulence model in the present solver and a slight difference in form of rivulet used, a point discussed in greater depth in section 3.5.5. While the slight difference in the wake region C_{pb} can be attributed to a disparity in Reynolds number between these studies. This is illustrated by inclusion of the distribution of \overline{C}_P for a plain cylinder at $Re = 20 \times 10^3$ which was obtained from the empirical formulae in ESDU 80025 [61] which are based on many experimental results over a range of Re. These minor discrepancies aside however, the level of this agreement allows for considerable confidence in the pressure distribution presently determined and aids in the validation of the solver itself.

The second study uses the experimental work of Matsumoto [81] which examined how the circumferential location of a singular rectangular rivulet, base width 0.072D, and height 0.032D affects \bar{C}_L in smooth flow conditions at $Re = 26 \times 10^3$. Comparing these experimental wind-tunnel results with those from the present numerical solver shows that these are both qualitatively and quantitatively inline, figure 3.28. This excellent agreement further validates the present numerical solver within a RWIV context and instils a great deal of confidence for further studies. Specifics of this case will be elaborated upon in the following section before these further studies are undertaken.



Figure 3.27: Comparison of mean pressure coefficient with angle clockwise from the windward horizontal of present solver with the previous numerical study of Li [75] in presence of single artificial rivulet at $\theta = 40^{\circ}$ and the plain cylinder ESDU 80025 [61]. Variation of \bar{C}_P with θ .

3.5.2 Single Rivulet - Effect of Location

As the upper rivulet is thought to play the greatest role in RWIV (section 2.4.1), the principal study undertaken examines the variation of aerodynamic response of the body with the angle of artificial rivulet from the incident flow (windward horizontal) for the single rivulet configuration. Figure 3.29, is an expanded version of figure 3.28 and illustrates that the present results are in good comparison with both the plain cylinder case and the results of Matsumoto [81]. Four distinct flow regimes can be identified. These can be summarised as:

 $\theta \leq 40^{\circ}$ - here the rivulet has little effect on the mean aerodynamic loading of the body as it is too close to the stagnation point (at the windward horizontal) to form significant shear layers. Furthermore, while there is a 'jump' in \bar{C}_P in the region of the rivulet, here shown at $\theta = 20^{\circ}$ in figure 3.30, this is significantly smaller than the $\theta = 40^{\circ}$ case, shown in figure 3.27.

Likewise the pressure recovery upstream of the rivulet and the difference between the artificial rivulet and the plain cylinder profiles of \bar{C}_P distribution are also reduced. Furthermore as can be seen from figure 3.31, which displays the normalised time averaged mean and RMS velocities $(u/U_{\infty} \text{ and } u'/U_{\infty})$, while



Figure 3.28: Comparison of mean lift coefficient with rivulet position clockwise from the windward horizontal with previous experimental study of Matsumoto [81] for a single artificial rivulet. Variation of \bar{C}_L with θ .

a rivulet in this location causes the flow to separate it quickly re-attaches to the upper surface before separating at practically the same location as the lower surface where no rivulet is present, $\theta_s \simeq 83^\circ$. In both cases separation being defined by the inflection point method of Zdravkovich [18]. As such both the wake and shedding patterns remain largely unaffected and the flow pattern remains approximately symmetrical. As a result of this, both \bar{C}_L and \bar{C}_D are practically unaltered from the the plain cylinder case, figure 3.29.

 $40^{\circ} < \theta < 60^{\circ}$ - the presence of the rivulet in the $\theta = 50^{\circ}$ location, which will henceforth be taken as representative of this regime causes a very different response. As was true for the previous $\theta = 20^{\circ}$ case, due to the rapid change in geometry the flow separates from the cylinder surface marginally upstream of the rivulet. This causes the flow to increase in velocity creating a very large suction pressure directly above and downstream of the rivulet, figure 3.32, far larger than in the previous case before flowing parallel to the surface in the near wake of the rivulet. Evidence of which is given by the large negative pressure at the location of the rivulet and the minimal pressure recovery downstream of this illustrated in figure 3.33. This 'entrained' flow passes downstream of the separation point for the plain cylinder case at $\theta_s = 83^{\circ}$, which can be seen on the lower surface,



Figure 3.29: Comparison of mean lift and drag coefficient with angle from the windward horizontal in presence of single artificial rivulet with results from plain circular cylinder. Variation of \bar{C}_L and \bar{C}_D with θ .

before undergoing a separation of sorts at $\theta \simeq 110^{\circ}$. Again calculated using the inflection point method, $\frac{\delta^2 C_P}{\delta \theta^2} = 0$, proposed by Zdravkovich [18]. This can be more clearly seen in figure 3.34, which plots the velocity vectors at a two specific points in the shedding sequence. Although this is not a true separation point in the strictest sense as the flow has already left the surface, this 'saddle separation point' as it shall now be referred acts in a similar manner. The flow on the upper surface therefore behaves in a manner more akin to a classic turbulent boundary layer, remaining attached to angles of $\theta \simeq 110^{\circ}$. Unlike the lower surface where, as previously discussed, separation occurs at $\theta \simeq 80^{\circ}$ in good correspondence with undisturbed flow at this Re [18, 61, 70].

This process is further confirmed by an examination of the normalised mean RMS velocities, figure 3.32. These show very little fluctuation in the 'entrainment' region directly downstream of the rivulet before exhibiting large fluctuations immediately downstream of the 'saddle separation point'. The latter resulting from the 'flapping' of shear layers, due to vortex shedding, on the upper surface undergoing a much larger rotation than similar shear layer 'flapping' on the lower surface. The difference in the two extremes of which are more clearly illustrated in figure 3.34. Where the flow at the point of minimum lift generation (shown



Figure 3.30: Mean coefficient of pressure distribution for a single artificial rivulet at $\theta = 20^{\circ}$. Variation of \bar{C}_P with θ .

in the upper diagram) separates at this saddle separation point and is deflected upward due to the closer location of the vortex on rear face. This causes a wider wake which is rotated slightly toward the lower surface. In the lower diagram of figure 3.34, which displays the point of maximum lift generation, the vortex on rear face forms slightly further from the body and as such the saddle separation occurs at a larger angle from stagnation. The resulting wake is therefore narrower and is rotated more towards the lower surface than in the minimum lift generation case. That said because of the rotation and wake narrowing in both instances there is a net lift to the side of the rivulet ($\bar{C}_L > 0$) and a marginal reduction in $\overline{C_D}$ over that of the plain cylinder datum case, figure 3.29.

While the exact mechanism presently determined differs slightly from that proposed by Liu [80], both instances highlight this 'saddle separation point' in the flow behind the rivulet and both predict the separation point of the rivuletladen upper surface to be leeward of the plain lower surface. This results in the generation of an asymmetric flow profile with a narrower wake on the upper surface on which the rivulet is present than on the plain lower surface, figure 3.32.

Although experimental flow visualisation data for this problem is not available for direct comparison, limited numerical data is. A comparison of the present nor-



Figure 3.31: Normalised velocity contours for single artificial rivulet at $\theta = 20^{\circ}$. Upper: Mean velocity. Lower: RMS velocity.



Figure 3.32: Normalised velocity contours for single artificial rivulet at $\theta = 50^{\circ}$. Upper: Mean velocity. Lower: RMS velocity.



Figure 3.33: Mean coefficient of pressure distribution for a single artificial rivulet at $\theta = 50^{\circ}$. Variation of \bar{C}_P with θ .

malised mean velocity field (figure 3.32) with the mean velocity vectors obtained by Liu and Matsumoto [82] (figure 3.35) highlights that these are in excellent agreement. Both display the entrainment directly downstream of the rivulet and the later separation this causes. While the rotation and subsequent narrowing of the wake, the large suction pressure and the large 'flapping' effect discussed are visible in both cases. It should be noted that as it is the symmetric rivulet case at $\theta = 50^{\circ}$ which was under examination in figure 3.35, this highlights these features on both the upper and lower surfaces as it. To date this work is as yet unpublished and was made available through direct communication with one of the authors [82]. That said the flow patterns discussed show good agreement by visual comparison. More specific data against which to validate the flow patterns determined by the present solver in the presence of an artificial rivulet was however, not available and this excellent qualitative visual agreement is the best that can be offered at this time.

 $60^{\circ} < \theta < 100^{\circ}$ - here while the flow on the lower surface (without an artificial rivulet) separates as if a plain cylinder, the flow on upper surface (with the rivulet) separates permanently marginally windward of the rivulet location, here illustrated for the $\theta = 75^{\circ}$ in the velocity field plots of figure 3.36. This



Figure 3.34: Velocity vector plot at two points in the shedding sequence for single artificial rivulet at $\theta = 50^{\circ}$. Upper: Point of minimum lift generation. Lower: Point of maximum lift generation.



Figure 3.35: Time averaged mean velocity vectors for the symmetric rivulet case at $\theta = 50^{\circ}$ from a previous but as yet unpublished numerical study using LES. Reproduced with express permission of Prof. M. Matsumoto [82].

causes the creation of a significantly wider wake from this rivulet-laden surface than from the plain surface which effectively 'rotates' the mean wake in this direction and causes a negative lift force on the body (to the opposite side of the rivulet). The increase in wake width also causes a significant increase in \bar{C}_D over that obtained from a plain cylinder, figure 3.29. Examination of the distribution of \bar{C}_P , figure 3.37, highlights that while in this regime there is once again a substantial 'jump' at the rivulet location, here the upstream effect is significantly greater than either of the two previous regimes. Consequently there is a much larger discrepancy from the \bar{C}_P profile obtained from a plain cylinder. The flow therefore separates from the surface marginally sooner to account for the presence of the rivulet, a finding that compares well with the results of Li [75].

 $\theta \ge 100^{\circ}$ - the rivulet is now sufficiently within the wake that the effect this has on both the distribution of \bar{C}_P and on the flow pattern are negligible, shown here for the $\theta = 150^{\circ}$ case in figures 3.38 and 3.39 respectively. Therefore no significant 'jump' can be detected in the \bar{C}_P profile and the departure from the results of plain cylinder is negligible. Given this and the largely unaffected flow


Figure 3.36: Normalised velocity contours for single artificial rivulet at $\theta = 75^{\circ}$. Upper: Mean velocity. Lower: RMS velocity.



Figure 3.37: Mean coefficient of pressure distribution for a single artificial rivulet at $\theta = 75^{\circ}$. Variation of \bar{C}_P with θ .



Figure 3.38: Mean coefficient of pressure distribution for a single artificial rivulet at $\theta = 150^{\circ}$. Variation of \bar{C}_P with θ .

field, the time averaged aerodynamic coefficients (\bar{C}_L and \bar{C}_D) are approximately the same as for the plain circular cylinder and those from the $\theta \leq 40^{\circ}$ regime which as previously discussed also had negligible effect on the value of these, figure 3.29.

The detection of these four flow regimes compares well with the previous results [80, 81] as do the location and extent of these, further validating the present solver. That said, the explanations presently given as to the underlying cause of these are reported in far greater depth than previously reported. Further vali-



Figure 3.39: Normalised velocity contours for single artificial rivulet at $\theta = 150^{\circ}$. Upper: Mean velocity. Lower: RMS velocity.

dation of these results and this solver are obtained if the mean initial separation points θ_s on the rivulet-laden upper surface, defined using the inflection point method of Zdravkovich [18], are compared with those determined numerically by Liu [80]. Illustrated in figure 3.40, excellent agreement is found upto rivulets located at $\theta \simeq 90^{\circ}$ whereupon a difference in initial separation points becomes apparent. The latter is due to a difference in the mean separation point in the absence of rivulets (i.e. for the plain cylinder) determined by both solvers. For the present code this is calculated as $\bar{\theta}_s = 83^{\circ} \pm 3^{\circ}$, whereas Liu [80] determined this as $\bar{\theta}_s = 74^{\circ} \pm 2^{\circ}$. Figure 3.40 highlights this disparity by also displaying the separation points on the plain lower surface in both cases. In so doing this also illustrates the difference in separation points between the upper and the lower surface caused by the presence of the artificial rivulet.



Figure 3.40: Comparison of the separation points predicted on the upper and lower surface by the present solver with that predicted by Liu [80]. Variation of θ_s with θ .

Comparing the Strouhal number ascertained from the present study with that obtained experimentally by Matsumoto [81] over the range of rivulet locations examined further validates the present solver by again showing good agreement between the two data sets, figure 3.41. As expected given the similarity to the plain cylinder case, a single clear peak in the PSD could be found at approximately the Karman vortex shedding frequency of the plain cylinder in both the $\theta \leq 40^{\circ}$ and $\theta \ge 100^{\circ}$ regimes. A single peak could also be identified within the flow regime at $60^{\circ} < \theta < 100^{\circ}$ although this was at a slightly lower frequency than the plain cylinder case. Within the flow regime at $40^{\circ} < \theta < 60^{\circ}$ however the magnitude of the PSD was substantially reduced and a broadband frequency response was obtained, figure 3.42. From which it could be determined that Karman vortex shedding was suppressed within this regime. Reiterating the findings of Matsumoto [81] and Liu [80] and further validating the present study.

0.211



Figure 3.41: Comparison of the present study with that of Matsumoto [81] for the variation in Strouhal number with rivulet location. Variation of St with θ .



Figure 3.42: Spectral analysis within and outwith $40^{\circ} < \theta < 60^{\circ}$ regime. Left: Within regime, $\theta = 50^{\circ}$. Right: Outwith regime, $\theta = 20^{\circ}$.

Given that Karman vortex shedding is suppressed in this regime, the flow is therefore free to respond in a different manner. Taken in conjuction with the large negative lift curve slope $\frac{\partial \bar{C}_L}{\partial \theta}$ present at the same rivulet location (figure 3.29) suggests a 'galloping' type instability could ensue. Through fitting of sixth order polynomials to the \bar{C}_L and \bar{C}_D curves over the range of angles which included the large negative lift curve slope (40° < θ < 80°), a plot for the Glauert-Den Hartog criterion (2.6) with rivulet position from the windward horizontal θ was obtained, figure 3.43. Closer examination of which reveals that the criterion is satisfied within the range 53° < θ < 65° and a galloping instability is indeed possible. This is in excellent agreement with the previous work of Matsumoto [81], Bosdogianni [67], Zhan [74] and others, who as outlined in sections 2.6.1, 2.7 and 2.8 predict that a single D.O.F. galloping instability could take place when a single static artificial rivulet is located at approximately 55° from the incident flow.



Figure 3.43: Variation of $\frac{\partial \bar{C}_L}{\partial \theta} + \bar{C}_D$ with θ for single artificial rivulet in negative lift slope range.

3.5.3 Multiple Symmetric Rivulets - Effect of Location

The multiple symmetric rivulet configuration examined is more akin to excitation mechanism 1 outlined by Verwiebe [71], discussed in section 2.4.2, which would be expected to give stream-wise oscillations of RWIV. Given that the artificial rivulets in this configuration are located equal angles clockwise and anti-clockwise from the stagnation point of incident flow (in this case the windward horizontal) the effective cross-section is therefore always symmetric if this point is assumed to be at fixed location. As such prior experience suggested that $\overline{C_L}$ should be effectively zero throughout the range of rivulet angle examined and thus identical to the plain cylinder. This was indeed found with a perfectly symmetric geometry under smooth flow conditions, figure 3.44.



Figure 3.44: Comparison of mean lift and drag coefficient with angle from the windward horizontal in presence of multiple symmetric artificial rivulets with results from plain circular cylinder. Variation of \bar{C}_L and \bar{C}_D with θ .

Closer examination of the flow and the previous results of Matsumoto [85], however suggested that there was a possible instability at $\theta \simeq 50^{\circ}$ caused by the suppression of the Karman vortex, figure 3.45, as was previously found for the single rivulet configuration. Spectral analysis confirmed that at this location, and only in very close proximity to this location, a single dominant frequency could no longer be determined in the PSD. This is illustrated in figure 3.45 which displays a comparison of the Strouhal number of the present results with the experimental and numerical results determined by Matsumoto [82]. In addition to highlighting the suppression of Karman vortex shedding at $\theta \simeq 50^{\circ}$, figure 3.45 also illustrates the excellent agreement between the present results and previous experimental and numerical studies. This further validates the present solver and increases confidence in its usage within a RWIV context. The suppression of Karman vortex shedding raised the possibility of a 'galloping' type instability at the location as was found for the single upper rivulet configuration of the previous section. However as a perfectly symmetric geometry had been examined, no mean lift force was determined at any rivulet location and therefore the Glauert-Den Hartog criterion (2.6) was not satisfied.



Figure 3.45: Comparison of present and previous work for multiple symmetric rivulets. Left: Variation of \bar{C}_L with θ , from Matsumoto [82]. Right: Variation of St with θ , data obtained from Matsumoto [82].

A closer inspection of this region was therefore warranted. As such a number of possible methods of very slight asymmetry were introduced into the system; the level of which was similar to that which might be expected to occur in the wind-tunnel. Using the same size and form of artificial rivulet (figure 3.25), the first method of introducing asymmetry fixed the rivulet on one surface at $\theta = 50^{\circ}$ while the rivulet on the other surface was fixed at a different angle from windward horizontal between $45^{\circ} < \theta < 55^{\circ}$. Increments of $\theta = 1^{\circ}$ of which were examined for both combinations of 'moving' upper/fixed lower and fixed upper/'moving' lower rivulets, where 'moving' refers to a different fixed location. The results of this study are shown in figure 3.46. This highlights two points, first that the results are approximately mirror images of one another giving confidence in the consistency of these. Second at this particular location while C_D is not significantly affected by a slight asymmetry, C_L is very susceptible to slight variations in the geometry. Indeed should a variation of 1° be present, the resulting C_L is of similar magnitude to that discovered experimentally by Matsumoto, [85], figure 3.45.

A similar result was also obtained from the second method of introducing



Figure 3.46: Comparison of time averaged mean lift coefficient with slight imperfections in flow conditions. Left: Variation in location of one of the two rivulets. (Upper 'moving' meaning that the lower was fixed at $\theta = 50^{\circ}$ while this was fixed at the θ specified, and vice-versa. Right: Variation in angle at which free stream meets cylinder.

asymmetry into the system. Here both rivulets were fixed at their original locations $\theta = 50^{\circ}$, while the angle at which the free-stream flow meets the cylinder and hence the stagnation point of incident flow was varied by $\pm 3^{\circ}$ from the windward horizontal. As can be seen from figure 3.46, while the \bar{C}_D is once again not significantly affected, \bar{C}_L is very susceptible to slight variations. With a 1° offset again producing a \bar{C}_L of similar magnitude to that discovered experimentally by Matsumoto, [85], figure 3.45. The consistency of the results about the horizontal axis is again cause for confidence.

The indication from these two studies is therefore that the flow is very sensitive to slight imperfections if rivulets are located at $\theta \simeq 50^{\circ}$. These asymmetries could arise from multiple sources and as such are more likely to be introduced experimentally.

The same susceptibility to slight imperfections in flow configuration is not universal however. Three other locations $\theta = 20^{\circ}$, 70° and 150° chosen to corresponding to the representative cases for the three flow regimes identified for the single rivulet case, were also examined and are illustrated in figure 3.47. This highlights that while a rivulet at $\theta = 70^{\circ}$ has some effect on \bar{C}_L this is significantly smaller than for a rivulet located at $\theta = 50^{\circ}$. Figure 3.47 also shows that there is almost no variation in \bar{C}_L when rivulets are located at either $\theta = 20^{\circ}$ or 150°. This is therefore effectively identical to the plain cylinder case and provides



Figure 3.47: Comparison of time averaged mean lift coefficient with angle at which free stream meets cylinder for various symmetric rivulet locations. Variation of \bar{C}_L with θ .

further evidence that in these two flow regimes rivulet location has little overall impact on the flow response. Although figure 3.47 highlighted that this local sensitivity is not universal by introducing asymmetry through variation of the angle at which the free-stream flow angle affects \bar{C}_L , fixing the rivulet on one surface while offsetting the rivulet on the other surface displayed similar results in each rivulet location.

Returning to figure 3.44, the existence of the same four flow regimes in the multiple symmetric rivulet configuration as were found for the single rivulet configuration is further confirmed by closer examination of the time averaged mean coefficient of drag. For this configuration at angles $\theta \leq 40^{\circ}$ and $\theta \geq 100^{\circ}$ the rivulets were once again not sufficiently exposed to the free stream to affect the overall response, resulting in a mean drag comparable to a plain cylinder,

As was true for the single rivulet case, the $40^{\circ} < \theta < 60^{\circ}$ and $60^{\circ} < \theta < 100^{\circ}$ flow regimes bring a reduction and an increase in \bar{C}_D respectively. This was caused by the same mechanisms identified for the single upper rivulet configuration; specifically the narrower wake caused by the later final separation of the 'saddle separation point' in the first instance and the wider wake caused by the flow separating permanently at the rivulet in the latter. The corresponding increments in magnitude of \bar{C}_D over the single rivulet configuration are approximately proportional to the increase of that configuration over the plain cylinder; reflecting the influence of a rivulet on both the upper and lower surfaces of the cylinder rather than just the upper surface.

3.5.4 Multiple Antisymmetric Rivulets - Effect of Location

Emulating the type 2.1 excitation mechanism proposed by Verwiebe [71] and outlined in section 2.4.2, the multiple antisymmetric rivulet configuration was examined over the entire range of θ and not just close to the lateral meridians as this mechanism proposes. The results shown in figure 3.48 are consistent with those of both previous configurations and as expected demonstrate a symmetrical distribution about $\theta = 90^{\circ}$ axis. This is further emphasised in the distribution of \bar{C}_D which highlights this symmetry but with a magnitude comparable to the single rivulet configuration as a result of only one artificial rivulet being on the windward face.



Figure 3.48: Comparison of mean lift and drag coefficient with angle from the windward horizontal in presence of multiple antisymmetric artificial rivulets with results from plain circular cylinder. Variation of \bar{C}_L and \bar{C}_D with θ .

The \bar{C}_L profile shown in figure 3.48 however demonstrates greater complexity. \bar{C}_L was found to be approximately zero at $\theta = 0,90,180^\circ$ as the body is symmetric in these artificial rivulet locations. As the flow regimes for the multiple antisymmetric configuration corresponds to either the $\theta \leq 40^{\circ}$ or $\theta \geq 100^{\circ}$ flow regimes identified for the single rivulet case, depending upon the point of view, this has little effect on former and latter of these points. The same is not true for the requirement of zero lift at $\theta = 90^{\circ}$. The accomplishment of which requires a decrease in the angular range of the $60^{\circ} < \theta < 100^{\circ}$ flow regime identified region in the single rivulet configuration such that $\bar{C}_L = 0$ at this location. This is achieved by the forward 'shift' of the zero lift intercept. The final distribution is therefore identically symmetric about $\theta = 90^{\circ}$ but is reversed and inverted to take account of the artificial rivulets positions on the opposite surfaces of the body. The demonstration of which increases confidence in both the consistency and accuracy of the model and the conclusions drawn.

Finally given that asymmetry is present at intermediate θ , use of spectral analysis and the Glauert-Den Hartog criterion (2.6), as was done previously for two previous configurations, indicates two possible regions where a single D.O.F. galloping instability may occur, $53^{\circ} < \theta < 65^{\circ}$ and $115^{\circ} < \theta < 126^{\circ}$. The former of which directly agrees with the single rivulet configuration while the latter arises due to the reversal of lift direction, which accompanies the artificial rivulets appearance on the lower surface of the cylinder. These are again accompanied by an absence of dominant frequency in the spectral analysis in both cases due to the suppression of Karman vortex shedding.

3.5.5 Form and Size of Rivulet

Bosdogianni [67] states that "... to a large extent it is simply the presence of a protuberance that changes the smooth and symmetrical shape of the cable to a non-symmetrical one and not its magnitude or shape that develops galloping." The results of the previous section have indeed shown that in the case of static artificial rivulets of trapezoidal form, rivulet location does indeed have a major impact on the overall aerodynamic forces on the body. However to further cor-

roborate this assertion that the form and size of these static artificial rivulets do not significantly affect the outcome, both of these aspects were investigated independently. The main effects of varying both of these were very similar for each of the three configurations, single, multiple symmetric and multiple antisymmetric previously examined. Therefore only the data pertaining to the single rivulet data is explicitly presented.

Rivulet Form

Three additional rivulet forms were investigated; these were of triangular, rectangular and elliptic profile respectively, figure 3.49. The former two were chosen to represent the two extremes of the bluff body artificial rivulets used in previous wind-tunnel work as shown in table 3.5, while the latter elliptic form was selected to best exemplify the rivulet which was expected to physically form during RWIV. It also resembles some of the previous studies highlighted in table 3.5. To minimise any possible effects of size when studying rivulet form, the base width (0.07D) and height (0.03D) in all three forms of additional artificial rivulet examined were the same as for the trapezoidal rivulet previously used.



Figure 3.49: Variation of artificial rivulet form, constant base width of 0.07D and height of 0.03D.

By comparing how the time averaged mean coefficient of lift and drag varied with rivulet location, figures 3.50 and 3.51, for each of the three artificial rivulet forms presently examined with the trapezoidal form previously used, two points become apparent. First, the same four distinct flow regimes as previously determined for the trapezoidal single artificial rivulet case can be identified and the role of the rivulet within each remains constant, i.e. within both the $\theta \leq 40^{\circ}$ and $\theta \geq 100^{\circ}$ regimes the rivulet has little impact. Second, the variations in \bar{C}_L and \overline{C}_D due to rivulet form are significantly smaller than those due to rivulet location. Therefore in the sense that rivulet location plays a considerably larger role than rivulet form, the present results confirm Bosdogianni's assertion [67].



Figure 3.50: Comparison of time averaged mean coefficient of lift with rivulet position from the windward horizontal for various rivulet forms. Variation of \bar{C}_L with θ .



Figure 3.51: Comparison of time averaged mean coefficient of drag with rivulet position from the windward horizontal for various rivulet forms. Variation of \bar{C}_D with θ .

Upon closer inspection, the small variations in magnitude of C_L and C_D caused by a difference in artificial rivulet form may be attributed to differences in the severity of the upstream rivulet angle and the 'smoothness' of the overall form and the corresponding effect these have on the condition of the flow past the rivulet itself. The elliptical and rectangular artificial rivulets thus provide the extremities of the forms considered. The former therefore shows the largest variation in the $40^{\circ} < \theta < 60^{\circ}$ flow regime with the largest increase in \bar{C}_L (figure 3.50) and largest decrease in \bar{C}_D (figure 3.51) as the flow is better conditioned, leading to a narrower wake. Likewise in the $60^{\circ} < \theta < 100^{\circ}$ regime this form shows the smallest decrease in \bar{C}_L and increase in \bar{C}_D . Correspondingly the rectangular form exhibits the opposite effect, causing the widest wake and thus the largest \bar{C}_D and negative \bar{C}_L in the $60^{\circ} < \theta < 100^{\circ}$ regime and the smallest lift and drag reduction in the $40^{\circ} < \theta < 60^{\circ}$ regime. However in comparison with the effect of rivulet location these differences are small.

Rivulet Size

Three additional sizes of rivulet height and width were examined. While all the rivulet forms were examined, the patterns determined were similar in each case and only those for the elliptical rivulet form are explicitly reported. This particular form was selected as representative due to its anticipated resemblance to the rivulet that was expected to form under natural conditions. To avoid becoming significantly different from the dimensions used experimentally (table 3.5) the variation in artificial rivulet size with respect to the base case of width 0.07D and height 0.03D was restricted to only a few percent of cylinder diameter.

Inspection of the effect that rivulet width has within the range 0.03D to 0.1D, illustrated in figure 3.52, highlights that this parameter has negligible effect on any of the time averaged or fluctuating aerodynamic coefficients at any location θ . With the difference between the rivulets of greatest or smallest width within the inherent margin of error of the present DVM code in each instance. The variation in \bar{C}_D likewise showed similar results. Indeed should the leading (windward) edge of the rivulet be used to specify location θ_{le} rather than the centre of the rivulet θ as used elsewhere (figure 3.25), even the minor variation in the point where the flow separates from the surface to pass over the rivulet θ_s ceases. The latter



Figure 3.52: Comparison of time averaged mean lift coefficient with rivulet position from the windward horizontal for various rivulet width. Variation of \bar{C}_L with θ .

occurring at an approximately constant distance from the leading edge of the rivulet.

The central height of the elliptical rivulet was varied within the range 0.015Dto 0.038D in keeping with sizes used in previous wind-tunnel studies, table 3.5. Figures 3.53 and 3.54 highlight that increasing the height of the rivulet shifts the onset angle of the second and third flow regimes, previously identified as $40^\circ < \theta <$ 60° and $60^{\circ} < \theta < 100^{\circ}$, marginally windward within the height range studied. While reducing height had an opposite effect, marginally delaying the onset of these flow regimes. Within a given flow regime there is also a corresponding variation in time averaged mean aerodynamic coefficients, with greater height typically bringing about a larger difference from the \bar{C}_L and \bar{C}_D values obtained from the plain cylinder case. This effect however is less significant than the transition between the different flow regimes. These findings are in excellent agreement with the wind-tunnel study undertaken by Yamaguchi [12] and the earlier work by James [130] for a cylindrical rivulet and the graphs for a single span-wise protrusion as given in ESDU 80025 [61]. This has implications during the growth of the rivulet, as at different points in this process the varying height of the rivulet could cause the circumferential locations of most interest, $40^{\circ} <$

 $\theta < 60^{\circ}$ and $60^{\circ} < \theta < 100^{\circ}$, to move. If the central height of the artificial rivulet was reduced yet further it became impossible to determine that the rivulet was present. Figures 3.53 and 3.54 show this at $\simeq 0.007D$ where no significant variation in \bar{C}_L and \bar{C}_D could be detected.



Figure 3.53: Comparison of time averaged mean coefficient of lift with rivulet position from the windward horizontal for various rivulet heights. Variation of \bar{C}_L with θ .



Figure 3.54: Comparison of time averaged mean coefficient of drag with rivulet position from the windward horizontal for various rivulet heights. Variation of \bar{C}_D with θ .

Therefore for the case of a fixed artificial rivulet the present results corroborate Bosdogianni's [67] assertion that the location of this rivulet plays the greatest role in affecting the overall aerodynamic response. The studies undertaken also demonstrated that while rivulet form and size effect both the time averaged mean aerodynamic coefficients and the limits of the four flow regimes, in comparison with the effect of rivulet location these are small. That said although there was no previous data against which to directly validate these results and therefore to corroborate these findings, the consistency in the magnitude and distributions of \bar{C}_L and \bar{C}_D gives confidence in both the results and the solver.

3.5.6 Oscillating Rivulet

By choosing to locally model the artificial rivulets in greater detail via the method outlined in section 3.2.2 the means by which to prescribe the oscillation of these was also implemented. Simulations were therefore undertaken for a rigid artificial rivulet (of fixed geometry) undergoing sinusoidal oscillation of prescribed fixed amplitude and frequency on the surface of the cylinder. However it quickly became apparent that this was a very complicated problem with several competing features. Furthermore the paper of Gu [36], which examined the angle θ at which a separate artificial rivulet attached to the cylinder found equilibrium for a given wind speed U if allowed to move circumferentially, was the first to study oscillating artificial rivulets either numerically or experimentally within the wind-tunnel. Therefore given this lack of data against which to verify potential results, the time required to fully interpreting such a complex problem and that the focus of the present thesis is on the construction of a coupled solver (the aerodynamic component of which has been validated elsewhere in this chapter) only a brief study was undertaken.

Each of the three rivulet configurations previously identified, figure 3.26, were prescribed to undergo one of two oscillations; rigid body or rivulet only motion. The distinction being that in the former the cylinder also rotated. In an attempt to delineate the problem, mean rivulet location, amplitude and frequency of oscillation were examined independently over a range of values representative of the RWIV phenomenon. However due to interaction between these parameters and other external influences, such as the large variation in \bar{C}_L and \bar{C}_D with static rivulet location (the four flow regimes previously identified), definitive conclusions could not be reached. That said, it does appear that all three parameters were significant in the final results. However establishing this and determining the exact roles played by each of the parameters is a major body of work in its own right that would be a logical extension of the present work.



Figure 3.55: Variation of coefficient of pressure on cylinder surface as symmetric rivulets oscillate. Left: 'Jump' when rivulet in flow. Right: No "jump" when rivulet in wake.

Within the context of the present work two points did emerge. First, rigid body motion compelled a greater effect on overall response, as expected, due to the rotation of the cylinder itself. Second, the local 'jumps' in the pressure field found for a static rivulet were found to move with the artificial rivulet as the flow is tripped by the rivulet before separating, figure 3.55. The exception being when the rivulet is within the wake, $100^{\circ} < \theta < 260^{\circ}$, where as was found for a static artificial rivulet no significant 'jump' could be detected. Whether this separated flow subsequently re-attaches to the cylinder appears to vary with time, location of rivulet and the frequency and range of oscillation. Therefore determining between the effects each of these have on body response is again outwith the scope of the present study. It should however feature in any future examination focused on oscillating artificial rivulets such as the logical extension previously outlined.

3.6 Summary

In conclusion it is worth reiterating several of the key points addressed within this chapter. Initially the theory behind the DVM and the reasons for its choice as the basis for the aerodynamic solver for the present application were introduced. An overview of the salient points of the DIVEX code which uses this method was then given. Modelling updates implemented on the original code developed by Vezza [98], Lin [6, 99] and Taylor [7], including determining a distribution of shear stress and the reasons for each of these were then presented.

To validate the modified code within a RWIV context the remainder of the chapter focused on applying this solver to a related problem for which existing experimental [81, 82] and numerical [75, 80] data is available; specifically the effect that addition of an artificial rivulet has on the aerodynamic response. To achieve, this studies with a plain, static, circular cylinder and for a similar cylinder undergoing forced oscillations in the across-wind direction were first undertaken. The outcomes of which showed that the present results compared well with both previous experimental and numerical results at the sub-critical Reynolds numbers of interest $20 \times 10^3 < Re < 100 \times 10^3$ therefore validating the present solver for these problems. Parameters examined included; time averaged mean and fluctuating aerodynamic coefficients, pressure distributions, velocity distributions and frequency analysis, each of which was found to demonstrate excellent agreement.

The effect that addition of an artificial rivulet had on the overall body response was then examined. This found that while the size and form of a static rivulet did affect the overall response, the magnitude of this was small in comparison to the effect of rivulet location, in agreement with the work of Bosdogianni [67]. With respect to the effect of static rivulet location four distinct flow regimes were identified and discussed for three different rivulet configurations; a new interpretation for one these regimes being given for the first time. Where data was available these results were found to be in direct agreement with previous wind-tunnel [61, 81] and numerical studies [75, 80] for both time averaged and fluctuating aerodynamic parameters, pressure distributions, spectral response and in the identification of a possible single D.O.F galloping instability at certain rivulet angles. $53^{\circ} < \theta < 65^{\circ}$ for the single upper rivulet case examined.

Given these points, the excellent agreement found between the present work and previously undertaken analysis in all the studies undertaken, and the consistency of the present results themselves, the present code was therefore taken be validated. Either when considering a plain circular cylinder that was static or undergoing forced oscillation, or one where static artificial rivulets are added. As such the aerodynamic solver created was considered suitable for use in a wider RWIV context and specifically for use within a coupled solver to determine rivulet formation and evolution which is the focus of this thesis. The creation of which requires a solver to determine the evolutionary profile of a thin film of fluid given a specified aerodynamic loading to be coupled to the present solver. The construction and validation of such a thin film solver is the focus of the following chapter.

Chapter 4 Thin Film Solver -Pseudo-Spectral Method

Having developed a solver capable of the prediction of the aerodynamic loading on a body given its geometry in the previous chapter, the focus of present chapter was to construct and validate, within a RWIV context, a numerical solver capable of determining the evolution of a thin film given the aerodynamic loading on this. These two solvers could then be combined to create a final coupled solver capable of tracking the formation and evolution of rivulets under an external aerodynamic field these influence.

To achieve this an equation governing the evolution of a thin film of fluid on the outer surface of a circular cylinder subject to the effects of gravity, surface tension, pressure and shear was determined. A brief overview of related thin film flows are then given before the pseudo-spectral method and the reasons for its selection to numerically simulate this equation are outlined. Details of the particular implementation used are then discussed.

The resulting numerical solver created is then validated with respect to previous [8] and newly determined analytical solutions and the numerical studies of Lemaitre [9] for several combinations of gravity, surface tension, pressure and shear. Various further studies, within an RWIV context, including how the distribution and magnitude of aerodynamic loading (the pressure and shear which result from the external flow field) influence the rivulet's temporal evolution were then undertaken and new results discussed. A final determination as to the validity and suitability of the numerical solver created for problems in RWIV is then made.

4.1 Thin Film Approximation

Thin film flows are those in which the the thickness H in one spatial direction is much smaller than the characteristic length scale L in the other directions. The flow therefore predominantly occurs in the direction of one of these longer dimensions under external forces such as gravity, surface tension gradients, pressure or shear stresses. Thin film flows are therefore common in many settings and thus have been the subject of a large body of literature. Within the present context these flows typically consist of an expanse of liquid (in this instance water) bounded by a solid substrate on one surface and exposed to another fluid, usually a gas (air), on the other 'free' surface [8, 9, 131, 132].

The approach to be taken exploits the existence of the small aspect ratio $\epsilon = H/L$ in this one spatial direction to expand the continuity (3.2) and Navier stokes equations (3.3) in perturbation series, in powers of ϵ . By assuming that the problem is 'long-scale' and therefore that the "variations along the film are much more gradual than those normal to it, and in which variations are slow in time" [131] these equations are greatly simplified. The resulting thin film approximations (4.10) and (4.11), when combined with appropriate boundary conditions at the two surfaces, allow an evolution equation to be developed for a specified initial film profile.

Review articles such as that by Oron [131] highlight the large number of potential aspects of thin films which can be studied and the large number of application areas these could potentially encompass, including lubrication and shallow-water theory. Given the present application however of particular interest is a thin film of fluid of uniform fluid properties (surface tension γ and dynamic viscosity μ) on the outer surface of a circular cylinder. In that respect the works of Moffat [133], Reisfeld and Bankoff [8] and Duffy and Wilson [134, 135] are the most directly applicable. Although these typically concern a rotating circular cylinder they also provide a number of analytical solutions for a stationary cylinder. The latter of these authors, Duffy and Wilson, should be further referenced [136] as they also collaborated with the author during this project.

Moffat [133] derived a steady two-dimensional leading-order solution for a gradually varying flow on both the internal and external surfaces of a circular cylinder under the influences of gravity, surface tension and body rotation. Duffy and Wilson [134] extended this analysis to include solutions that become unbounded where the thin film analysis fails and in collaboration with Hunt [135] used the pseudo-spectral method to obtain highly accurate numerical solutions for the steady Stokes equations.

For a thin film present on the outer surface of a static cylinder, Black [132] examined application of a uniform shear stress, determining both steady and transient analytical solutions. Reisfeld and Bankoff [8] examined the effect of temperature under a variety of conditions and under a variety of loading combinations, including gravity and surface tension; determining analytical solutions in each instance. These authors also provided numerical confirmation of these results, using a pseudo-spectral method solver similar to that of Hunt [135]. To more closely resemble RWIV, Lemaitre [9] obtained an evolution equation under the additional effects of external aerodynamic loading, by both pressure and shear stress, in addition to gravity and surface tension for a cylinder undergoing oscillatory motion. As outlined in section 2.8 these authors then proceeded to obtain numerical results for a static cylinder, for a fixed distribution of pressure and shear determined from the distributions of \bar{C}_P and \bar{C}_F for a dry circular cylinder, i.e. one without a thin film of fluid present. This was again achieved using a pseudo-spectral method solver.

4.2 The Pseudo-Spectral Method

As highlighted in the previous section, prior studies into thin film flows on cylindrical surfaces have typically used pseudo-spectral methods to obtain numerical results [8, 9, 135] for a given evolution equation. The reasons for this are generally four-fold, however before outlining these, the differences between how spectral methods and other more commonly used methods obtain derivatives should first be summarised.

Finite difference, finite element and finite volume methods approximate derivatives locally, using values obtained by a low order function at the nearest of a large number of closely spaced grid points. Such derivatives are therefore a property of the local function and are designed to be exact for local polynomial functions of low order [137]. Spectral methods in contrast are global in nature and use a truncated series to approximate the overall function, v(x), which can then be differentiated exactly, rather than obtaining an exact local function which can be differentiated approximately. This series is typically given as

$$v_N(x) = \sum_{k=0}^N a_k \psi_k(x),$$
 (4.1)

where a_k are the expansion coefficients to be determined for a given set of trial functions ψ_k at a given number of nodes N. These trial functions should be orthogonal and given that the problem under investigation is periodic over the interval $0^\circ \leq \theta < 360^\circ$, trigonometric functions were chosen, as recommended by previous literature [137–139]. Returning however to why a spectral method was chosen, the typical four reasons for its usage are:

- 1: As the number of nodes N increases the errors typically decay at an exponential rather than polynomial rate, particularly for analytic functions such as trigonometric functions [137–139].
- 2: This in turn reduces the number of nodes required thus allowing very time and memory efficient calculations [139, 140].

- 3: "The method is virtually free of both dissipative and dispersive errors" [137]. This is particularly relevant for the present high Re flow as it ensures that numerical dissipation does not overwhelm the low physical dissipation and that large gradients do not become wave-trains due to dispersion [137, 140].
- 4: The approach can be used for problems where both solutions and variable coefficients are non-smooth [137, 138].

Given that the geometry of the present problem should remain smooth and approximately circular, and that the DVM solver to which it is to be combined uses evenly distributed nodes, the present problem also lacks the traditional difficulties encountered by spectral methods. These include irregular domains, strong shocks and variable resolution requirements. Given these points and prior success in similar problems [8, 9, 135], a spectral method solver was therefore thought to represent the best choice.

Having chosen to use trigonometric trial functions ψ_k , one of three techniques had to be chosen for determining the expansion coefficients, a_k in (4.1). These Tau, Galerkin or Collocation (pseudo-spectral) methods vary in how the residual $R_N(x)$ between the actual function and its approximation is minimised

$$R_N(x) = v(x) - v_N(x).$$
 (4.2)

The Tau and Galerkin methods find "variable coefficients and nonlinearities ... difficult to handle" [137] due to the way in which this residual is calculated. The pseudo-spectral method however only requires that the boundary conditions are satisfied (which all three methods require) and that the residual is zero, $R_N(x) = 0$, at each of the calculation nodes. Given that the evolution equation (4.24) which will be derived in the following section has such variable coefficients and its successful usage in related problems [8, 9, 135] the pseudo-spectral method was chosen. The original function can therefore be represented as

$$v(x_i) = \sum_{k=0}^{N} a_k \psi_k(x_i), \qquad i = 0, \dots, N$$
 (4.3)

which given that a Fourier series is used and thus that the trial functions are trigonometric can be rewritten as

$$v(x_i) = \sum_{k=-K}^{k=K} a_k e^{jkx}, \qquad i = 0, \dots, N$$
(4.4)

where this expansion contains N = 2K + 1 complex expansion coefficients and where j is the imaginary unit, $j^2 = -1$. The associated calculation nodes should then be equally spaced [137–140] over the interval, in good agreement with the previous aerodynamic solver. For the present range the location of these can be written as

$$x_i = \frac{2\pi i}{N}, \qquad i = 0, \dots, N$$
 (4.5)

and therefore using the discrete orthogonality relation for complex exponential functions,

$$\sum_{i=1}^{N} e^{j(k-l)\frac{2\pi i}{N}} = \begin{cases} N & if \ k-l = mN, \ m = 0 \pm 1, \pm 2, \dots \\ 0 & otherwise \end{cases}$$
(4.6)

we can obtain the expansion coefficients by multiplying both sides of (4.4) by e^{-jlx} and summing from i = 1 to N,

$$a_k = \frac{1}{N} \sum_{i=i}^N v(x_i) e^{-jkx_i}. \qquad k = -K, \dots, K.$$
(4.7)

Any derivative of a function can then be obtained by a three step process. First a forward Fast Fourier Transform (FFT) is applied to the prescribed data. The resulting Fourier coefficients a_k are then multiplied by a factor according to the rules outlined in table 4.1 once for the first derivative, twice for the second derivative and so on. The actual derivative values at these locations $v'(x_i)$ are then obtained by finally undergoing a reverse FFT. Further details of the method and a more complete mathematical overview can be obtained from the spectral method literature previously mentioned [138–140], although that by Fornberg [137] is particularly relevant for the numerical implementation used.

$$\frac{a_{k0} \quad a_{k1} \quad \dots \quad a_{k(N/2-1)} \quad a_{k(N/2)} \quad a_{k(N/2+1)} \quad \dots \quad a_{k(N/2-2)} \quad a_{k(N_1)}}{0 \quad \pi j \quad \dots \quad (N/2-1)\pi j \quad 0 \quad -(N/2+1)\pi j \quad \dots \quad -2\pi j \quad -\pi j}$$

Table 4.1: Multiplication factors for Fourier coefficients a_k based on Fornberg [137].

4.3 Mathematical Model

Two-dimensional, unsteady flow of a thin film of incompressible viscous fluid with uniform dynamic viscosity $\hat{\mu}$ and density $\hat{\rho}$ on the outer surface of a stationary horizontal circular cylinder of radius R will be considered, figure 4.1. To avoid confusion with the external fluid, the symbol being used herein when referring to a fluid property of the thin film. Suppose the film has a 'free' surface in contact with the atmosphere, such that this film is subject to a prescribed pressure, $P = P(\theta, t)$, and a prescribed shear, $T = T(\theta, t)$, exerted by the external aerodynamic field, which are functions of time t and the clockwise angle from the windward (left-hand) horizontal, θ where $0 \leq \theta < 360^{\circ}$. Where the pressure (above freestream) and the shear are given by



Figure 4.1: Notation used for thin film on external surface of horizontal circular cylinder.

As the cylinder is assumed horizontal and is assumed to be perpendicular to

the external flow which causes the pressure and shear loading. In this representation, the angles of inclination and yaw (figure 2.8) are therefore, $\alpha = 0$ and $\beta = 0$ respectively. This restricts all loading to act purely within the two-dimensional system defined and is in line with the previous analytical and numerical studies into aspects of RWIV which were outlined in sections 2.7 and 2.8. Should an inclined cylinder be considered then the effective component of gravity g_{eff} would be reduced, however this would also introduce uncertainties regarding the effective cylinder cross-section and the resulting aerodynamic loading.

A condition for zero fluid flux is set. While this does not preclude water axially flowing down the cable as would occur in a truly three dimensional representation at a given inclination or yaw, for the present two-dimensional representation this ensures that the amount of fluid within the thin film remains constant with time. Specifically the amount prescribed at problem inception. Given that the cylinder considered is horizontal and perpendicular to the free-stream flow, if such axial thin film flow occurs it is likely to be negligible. In addition all points on the surface of the cylinder are assumed to remain coated at all times, where if thickness is given as h then $h(\theta, t) > 0$. This eliminates problems related to contact lines and angles. As these approximations are commonly used in other thin film problems [8, 9, 132] under similar circumstances they were taken as being valid for the present situation.

4.3.1 Model Description

The film is taken to be thin in that its aspect ratio ϵ (defined by $\epsilon = H/R$, where H denotes a typical film thickness) satisfies $\epsilon \ll 1$.

The velocity and pressure of the thin film are given by $\hat{\mathbf{u}}$ and \hat{p} , respectively. Initially we refer the description to polar coordinates r, θ , z with the z axis along the axis of the cylinder; then the surface of the cylinder is given by r = R. We denote the film thickness by $h = h(\theta, t)$ (unknown *a priori*); then the free surface of the film is given by r = R + h. Near any station, $\theta = \text{constant}$, however an alternative local Cartesian coordinate system can be referred to. This system Oxyz has direction Ox tangential to the cylinder (increasing in the direction of increasing θ , so that $x = R\theta + \text{constant}$) and Oy along the outward normal to the cylinder, with y defined by y = r - R, so that the cylinder is at y = 0 and the free surface is at y = h. In the latter coordinate system the governing continuity (3.2) and Navier-Stokes equations (3.3) with only a gravitational body term g, at leading order in ϵ , can be written as

$$\hat{u}_x + \hat{v}_y = 0, \tag{4.9}$$

$$0 = -\hat{p}_x - \hat{\rho}g\cos\theta + \hat{\mu}\hat{u}_{yy},\tag{4.10}$$

$$0 = -\hat{p}_y, \tag{4.11}$$

where subscripts denote differentiation, and the orthogonal components of velocity and gravity are written in terms of the global co-ordinates (i, j), figure 4.1,

$$\hat{\mathbf{u}} = \hat{u}\mathbf{i} + \hat{v}\mathbf{j}, \qquad \mathbf{g} = -g(\mathbf{i}\cos\theta + \mathbf{j}\sin\theta).$$
 (4.12)

In equations (4.10) and (4.11) the inertia terms have been neglected; this is valid provided that the Reynolds number of the film $\hat{R}e$ (defined by $\hat{R}e = \hat{U}R/\hat{\nu}$, where $\hat{\nu} = \hat{\mu}/\hat{\rho}$ and \hat{U} are the kinematic viscosity and a typical velocity of the film respectively) is such that $\epsilon^2 \hat{R}e \ll 1$. Also since the film is thin ($\epsilon \ll 1$), terms such as \hat{u}_{xx} and \hat{v}_{yy} in the momentum balances are negligible, as is a contribution $\hat{\rho}g\sin\theta$ in (4.11).

Equations (4.9)-(4.11) are subject to the no-slip and no-penetration conditions on the cylinder surface,

$$\hat{u} = \hat{v} = 0 \quad \text{on} \quad y = 0,$$
 (4.13)

and, at the free surface, to the kinematic condition

$$\hat{v} = h_t + \hat{u}h_x \quad \text{on} \quad y = h, \tag{4.14}$$

the tangential stress condition

$$\hat{\mu}\hat{u}_y = T \quad \text{on} \quad y = h, \tag{4.15}$$

and the normal stress condition

$$\hat{p} = \hat{\gamma}\kappa + P \quad \text{on} \quad y = h,$$
(4.16)

where the coefficient of surface tension γ is assumed to be constant. As such, Marangoni effects due to gradients in surface tension can be ignored [8, 9, 131]. The mean curvature of the free surface κ can therefore be given to first order by

$$\kappa = \frac{1}{R} - \frac{1}{R^2} (h + h_{\theta\theta}) \tag{4.17}$$

a more complete derivation of which is given in appendix C, then the azimuthal volume flux of fluid in the film is given by

$$Q = \int_0^h \hat{u} \,\mathrm{d}y. \tag{4.18}$$

Using this and (4.9) we may replace (4.14) by the local conservation law

$$h_t + Q_x = 0. (4.19)$$

4.3.2 Evolution Equation

The final goal of this mathematical model is to determine an evolution equation for $h(\theta, t)$. To achieve this we first integrate the y component of the Navier-Stokes equation (4.11) subject to normal stress condition at the free boundary (4.16). From this we obtain

$$\hat{p} = \hat{\gamma}\kappa + P \tag{4.20}$$

which is very similar to (4.16). This illustrates that pressure at a particular location θ is independent in the normal direction y. By then integrating the x component of the Navier-Stokes equation (4.10) with respect to y subject to the no-slip and no-penetration boundary conditions on the cylinder surface (4.13)and tangential stress condition at the free surface (4.15) we obtain

$$\hat{u} = -\frac{1}{2\hat{\mu}} \left(\hat{\rho}g \cos \theta + \hat{p}_x \right) \left(2hy - y^2 \right) + \frac{Ty}{\hat{\mu}}.$$
(4.21)

The azimuthal volume flux can be determined from (4.18) as

$$Q = -\frac{1}{3\hat{\mu}} \left(\hat{\rho}g \cos \theta + \hat{p}_x \right) h^3 + \frac{Th^2}{2\hat{\mu}}.$$
 (4.22)

While by expanding (4.20) using the mean curvature of the free surface to first order (4.17) we obtain

$$\hat{p}_x = \frac{P_\theta}{R} - \frac{\kappa}{R^3} (h + h_{\theta\theta})_\theta \tag{4.23}$$

where the 1/R term of (4.17) cancels as an 'over-pressure' as it is constant over the surface. Finally, substituting azimuthal volume flux (4.22) into the local conservation law (4.19) and using (4.23) we obtain the evolution equation for $h(\theta, t)$

$$h_t + \left(\frac{Th^2}{2\hat{\mu}R}\right)_{\theta} - \left[\frac{h^3}{3\hat{\mu}R}\left(\hat{\rho}g\cos\theta - \frac{\hat{\gamma}}{R^3}(h+h_{\theta\theta})_{\theta} + \frac{P_{\theta}}{R}\right)\right]_{\theta} = 0.$$
(4.24)

This agrees with the corresponding equation given by Lemaitre [9] for flow over a stationary cylinder, i.e. in the absence of motion, and to the earlier equation of Reisfeld [8] for the case without aerodynamic loading, i.e. in the absence of pressure and shear forces. When fully expanded (4.24) becomes

$$h_{t} = \frac{1}{\hat{\mu}R} \Big[\hat{\rho}g \cos\theta h_{\theta}h^{2} - \frac{1}{3}\hat{\rho}g \sin\theta h^{3} - \frac{\hat{\gamma}h^{3}}{3R^{3}}(h_{\theta\theta} + h_{\theta\theta\theta\theta}) - \frac{\hat{\gamma}h^{2}}{R^{3}}(h_{\theta}^{2} + h_{\theta}h_{\theta\theta\theta}) + \frac{1}{R}(P_{\theta}h_{\theta}h^{2} + \frac{1}{3}P_{\theta\theta}h^{3}) - (Th_{\theta}h + \frac{1}{2}T_{\theta}h^{2}) \Big].$$
(4.25)

This equation is to be solved subject to an initial condition of the form $h(\theta, 0) = h_0(\theta)$, where $h_0(\theta)$ is the initial thickness of the film. For definiteness in the present work an initially uniform film, $h_0 = \text{constant}$, is chosen and the film is allowed to evolve according to (4.24) to see if rivulets develop.

4.4 Implementation of Present Code

As the evolution equation (4.24) is a fourth order, non-linear, non-constant coefficient partial differential equation, it cannot, in general, be solved analytically. Therefore, a pseudo-spectral (or collocation) method solver using an N point Fourier spectral mode in space and a fourth order Adams-Bashforth time marching algorithm was constructed. The reasons for choosing this spectral method were highlighted in section 4.2 but were particularly due to the periodic, continuous nature of the problem over the interval $0 \leq \theta < 360^{\circ}$ and the rapid rate of convergence it provides to the solution, given the presumed smoothness of the final result [137–140].

Meanwhile to simplify the problem, discretisation was only undertaken in the spatial domain with a 'packaged' ODE solver used to advance the resulting system of ODE's in time. This Method of Lines approach is the most common method of time-stepping [137] and allows the ODE solver to be developed and analysed separately from the spatial discretisation method. The fourth order Adams Bashforth Method chosen calculates the value at a particular node at the next time step $t + \Delta t$, based on the value at that node at the present time step t and derivatives at previous times steps $t - \Delta nt$,

$$y_{t+\Delta t} = y_t + \frac{y_{step}}{24} \Big(55y'_t - 55y'_{t-\Delta t} + 37y'_{t-2\Delta t} - 9y'_{t-3\Delta t} \Big).$$
(4.26)

Wherein y_{step} is the step-size, set as 0.5 in the present instance, and ' designates the first derivative. This explicit linear multi-step method was chosen over an implicit method such as the Runge Kutta method for a number of reasons. Firstly the Adams-Bashforth method is considerably quicker than implicit methods of the same order as it requires only one derivative calculation per timestep ($\approx 35\%$ quicker in comparisons with the fourth order Runge Kutta method for the present case). Secondly it is consistent to $O(\Delta t^5)$ locally and $O(\Delta t^4)$ globally while being conditionally stable for a given timestep size ($\alpha \Delta t \leq 0.3$) and is thus convergent [141]. Finally this was also the method used by Lemaitre [9]. That said however this method is not self-starting and a fourth order Runge Kutta method based on [142] was still required to calculate the first three timesteps.

The code itself was written in Fortran 77 for consistency with the previous aerodynamic solver [6, 7, 117], while to ensure sufficient accuracy of the results, double precision data storage was used. The differentiation routine was based closely on that proposed by Fornberg [137] while the FFT used within the three step procedure outlined in section 4.2 was the one-dimensional Fast Version (radix 8,4,2) FFT code made freely available by Ooura [143]. This particular version being chosen due to its open source nature and increased performance [143] over other free codes, such as that presented in Numerical Recipes [142].

4.4.1 Parameter Selection

Efficient usage of the FFT requires the number of nodes N to be a power of two [137]. Through a computational convergence study of the problem to be outlined in section 4.5.1, where the problem will be examined in a RWIV context, an equi-spaced distribution of N = 128 nodes was found to provide the optimum compromise between run-time and stability, and spatial resolution for the present code. A partial summary of this convergence study is given in table 4.2. This shows that the CPU time required to solve the problem for a given number of timesteps, in this instance 1×10^5 , is approximately proportional to the number of nodes used in the calculation $(t \propto N)$. The solution was also found to resolve to a greater final time for fewer nodes although a limit was reached and no simple correlation was found. These final times at which a resolved solution could be obtained τ_{max} , where a higher value indicates a more stable solution, are also given in table 4.2 for a timestep of $\Delta t = 1.0 \times 10^{-6}$ s. These use a reduced time $\tau = gh_0^2 t/3\nu R$, as defined by Reisfeld and Bankoff [8] in which the present problem becomes singular as $\tau \to 0.5$ due to increase in film thickness at the lowest point $h(270^\circ, \tau)$. As such while a faster more stable solution can be found

with fewer nodes this reduces overall spatial resolution. Examination of figure 4.2, which plots the normalised film thickness, h/h_0 with angle θ at $\tau = 0.44$, highlights that there is no visible difference between the film profile for N = 128 and 256 nodes, there is however a small discrepancy between these and the profile of N = 64 nodes due to insufficient resolution. This reinforces the selection of 128 nodes.

Number of nodes, N	32	64	128	256	512	1024
Time to solve, t, s	3.108	4.632	9.445	18.137	36.222	75.652
Final resolved time τ_{max}	0.487	0.485	0.479	0.448	0.227	-

 Table 4.2: Summary of the computational convergence study



Figure 4.2: Difference in normalised film thickness profile from 64, 128 and 256 nodes. Variation of h/h_0 with θ .

A similar study for timestep size for a given number of nodes was also undertaken and is reported in table 4.3. This revealed a timestep $\Delta t = 0.5 \times 10^{-6}$ s to be optimum for 128 nodes. Closer examination of table 4.3 also reveals that while a timestep of $\Delta t \leq 5.0 \times 10^{-5}$ s is required to calculate a solution at values less than $\Delta t 5.0 \times 10^{-7}$ s the differences were minimal and only the time to solve was significantly increased. From these studies, the number of nodes and timestep were fixed throughout this chapter at N = 128 and $\Delta t = 0.5 \times 10^{-6}$ s respec-

tively. These values compare well with those used by Lemaitre [9] of N = 128and $\Delta t = 1.0 \times 10^{-6}$ s respectively.

Timestep, Δt , $\times 10^{-5}s$	5	1	0.5	0.1	0.05	0.01
Time to solve, t, s	0.564	0.936	1.408	5.156	9.781	46.475
Final resolved time, τ	0.368	0.432	0.465	0.479	0.485	0.486

Table 4.3: Convergence study for optimum timestep size.

Standard values for gravity and the properties at an air-water interface at 20°C were selected for use throughout the present study and are listed in table 4.4. Other parameter values, such as wind speed of the external fluid U (which corresponds to the free-stream flow velocity U_{∞} of the DVM solver) were chosen to represent typical values for RWIV, as outlined in section 2.2, and to ensure that the ratio of initial film thickness to cylinder radius, $h_0/R = 6.3 \times 10^{-3}$. The latter was such that these were consistent with the value used in previous experimental and computational studies by Flamand [39] and Lemaitre [9]. With these parameter values, the Reynolds number implemented was a subcritical value typical for RWIV of $Re \simeq 100 \times 10^3$, [19, 33] where here $Re = UD/\nu$ concerns the external fluid, in this instance air.

The distributions of pressure P and shear T due to the external aerodynamic field were assumed to be constant with time and as such are only a function of angle, i.e. $P = P(\theta)$ and $T = T(\theta)$. These distributions were based upon the time averaged pressure and friction coefficients, \bar{C}_P and \bar{C}_F , for the external aerodynamic field around a dry cylinder, values for which were determined experimentally at a Reynolds number similar to that under investigation here of $Re = 100 \times 10^3$ by Achenbach [113]. The assumption that the rivulets will not affect the distributions of aerodynamic loading is limited. However as within the coupled solver of the following chapter these distributions will vary both temporally and with the evolution of the body geometry (including rivulets) this assumption can thus be seen as an interim stage. It also matches the assumption
Parameter	Value	
Cylinder Radius, R	0.08 m	
Initial film thickness, h_0	$5 \times 10^{-4} \mathrm{m}$	
Gravity, g	$9.806\mathrm{m/s^2}$	
Density of water, $\hat{\rho}$	$1000 \mathrm{kg/m^3}$	
Dynamic viscosity of water, $\hat{\mu}$	$1.002 \times 10^{-3} \mathrm{Ns/m}$	
Surface tension of water, $\hat{\gamma}$	$72 imes 10^{-3} \mathrm{N/m}$	
Density of air, ρ	$1.19\mathrm{kg/m^3}$	
Dynamic viscosity of air, μ	$1.82 \times 10^{-5} \mathrm{Ns/m}$	
Wind speed, U	$11.0\mathrm{m/s}$	

Table 4.4: Values of the standard parameters used in the numerical calculation.

made in the work of Lemaitre [9] which will be used extensively in verifying the present solver in a RWIV context.

One of the main limitations of the pseudo-spectral method is its susceptibility to aliasing of high frequencies, especially in the non-linear terms present in the evolution equation (4.24). To avoid this problem, and to maintain a high convergence rate (for the pseudo-spectral method this is greater for analytic functions than for non-analytic functions [138]), a truncated Fourier series representation of the aerodynamic coefficients \bar{C}_P and \bar{C}_F of the form

$$\bar{C}_P, \bar{C}_F = \frac{a_0}{2} + \sum_{k=1}^n \left[a_k \cos(k\theta) + b_k \sin(k\theta) \right],$$
 (4.27)

was used in the analysis, where a_k and b_k are Fourier coefficients.

To ensure that the number of terms in equation (4.27) was sufficient to reproduce the variation of the aerodynamic coefficients around the cylinder accurately, a brief study of the number of terms, n, required was undertaken. The results for both coefficients were similar but the error due to series truncation for the pressure coefficient C_P was constantly higher. As such this was used as representative and a plot of this truncation error is given in figure 4.3. This reveals



Figure 4.3: Variation of the coefficient of determination R^2 and percentage error at $\theta = 70^{\circ}$ with the number of terms, n, in the Fourier series truncation of the pressure coefficient \bar{C}_P .

that the coefficient of determination R^2 (4.28), which is not to be confused with the cylinder radius R, was high, $R^2 > 0.985$, throughout the range of number of terms investigated.

$$R^{2} = 1 - \frac{\sum_{i=1}^{N} (c_{i} - cm_{i})^{2}}{\sum_{k=1}^{N} (c_{i} - \bar{c})^{2}}.$$
(4.28)

Where within this definition c_i is the value to be modelled, cm_i is the value actually modelled and \bar{c} is the mean of the values to be modelled at N nodes. While the percentage error between the truncated series and experimentally determined values at a specific value of θ for the majority of the range ($0 \leq \theta < 360^{\circ}$) investigated was small. This however, was not true at either of the two sharp peaks in the \bar{C}_P profile at $\theta \simeq 70^{\circ}$ and $\theta \simeq 290^{\circ}$, where the largest errors occurred (figures 4.3 and 4.4). At these points the percentage error only began to plateau at a minimum value after $n \geq 20$ terms. As such the first twenty terms were chosen to represent the two coefficients. A comparison of the final truncated Fourier series representations of \bar{C}_P and \bar{C}_F^* (using n = 20 in (4.27)) with the original experimental coefficients of Achenbach [113] is shown in figure 4.4. It should be noted the friction coefficient shown, $\bar{C_F}^* = \bar{C}_F / \max(\bar{C}_F)$, is a normalised version of the actual coefficient, \bar{C}_F such that these can be shown at similar scale.



Figure 4.4: Comparison of modelled aerodynamic coefficients profiles used in the numerical calculations with those measured experimentally by Achenbach [113] at Re = 100×10^3 . Variation of \bar{C}_P and \bar{C}_F^* with θ .

4.5 Initial Validation

Initial validation of the numerical solver created was undertaken through comparison with analytical solutions for two combinations of loading. These were:

- 1: Gravity and surface tension loading only $(T \equiv 0 \text{ and } P \equiv 0)$,
- 2: Only a constant shear loading $(T(\theta) \equiv \text{constant}, \text{ with } P \equiv 0, \gamma = 0 \text{ and } g = 0).$

4.5.1 Gravity and Surface Tension Loading

Here validation was undertaken by comparison of the present results with the analytical solution of Reisfeld and Bankoff [8] who investigated the heating or cooling of a thin viscous film of fluid on the outer surface of a circular cylinder, subject to the forces of gravity and surface tension. Under isothermal conditions this corresponds directly to the present study when no aerodynamic loading due to the external flow is considered (i.e. when pressure $P \equiv 0$ and shear $T \equiv 0$). In this case, the evolution equation (4.24) thus represents a direct competition between the effects of gravity and surface tension on the film. The ratio between these is given by the Bond number, $Bo = \rho g R^3 / h_0 \gamma$. For very large values, $Bo \rightarrow \infty$, Reisfeld and Bankoff [8] determined an analytical solution in terms of a reduced time $\tau = g h_0^2 t / 3\nu R$ for film thickness around the cylinder subject to an initial condition of $h(\theta, 0) = 1$. This can be given in the present notation as

$$h(\theta,\tau) = \begin{cases} (1+2\tau)^{-1/2} & \text{for} \quad \theta = 90^{\circ}, \\ (1-2\tau)^{-1/2} & \text{for} \quad \theta = 270^{\circ}, \\ (\cos\theta_0/\cos\theta)^{1/3} & \text{for} \quad 0^{\circ} \le \theta < 360^{\circ}, \theta \ne 90^{\circ}, 270^{\circ}, \end{cases}$$
(4.29)

where $\theta_0 = \theta_0(\theta, \tau)$ is determined from

$$W(g(\theta), \sin 75^{\circ}) - W(g(\theta_0), \sin 75^{\circ}) + 2(3)^{1/4} (\cos \theta_0)^{2/3} \tau = 0, \qquad (4.30)$$

in which $W(\phi, k)$ is an incomplete elliptic integral of the first kind defined by

$$W(\phi, k) = \int_{0}^{\phi} \frac{dx}{\sqrt{1 - k^2 \sin^2 x}},$$
(4.31)

and $g(\chi)$ satisfies

$$\cos g(\chi) = \frac{\sqrt{3} - 1 + (\cos \chi)^{2/3}}{\sqrt{3} + 1 - (\cos \chi)^{2/3}}.$$
(4.32)

To ensure a valid comparison between the present work and the analytical solution a study was undertaken to determine practical values for the numerical representation of $Bo \to \infty$. To achieve this, values of cylinder radius R and initial film thickness h_0 were varied but in such a manner as to ensure the ratio of $h_0/R = 6.3 \times 10^{-3}$ was maintained. All other parameters were kept at the original values as previously listed in table 4.4. The results of this study are shown in figure 4.5 which plots the final resolved reduced time τ_{max} against \hat{Bo} . As was discussed in section 4.4.1, equation (4.29) predicts that the film thickness at the lowest point, $h(270^\circ, \tau)$, becomes singular as $\tau \to 0.5$. Therefore $\tau_{max} \to 0.5^-$ as $Bo \to \infty$, as such figure 4.5 illustrates that a value of $Bo \ge 1 \times 10^7$ is sufficient to represent this condition in practice.



Figure 4.5: Variation of final resolved time τ_{max} with Bond number of the thin film \hat{Bo} .

Using the parameters chosen to study RWIV in table 4.4, results in $\hat{Bo} = 1.4 \times 10^5$. This is below the lower end of the range established for the exact solution (4.29) to be applicable. Therefore for the purposes of validation of the numerical model, the initial thickness of film h_0 and radius of the cylinder R were each increased by an order of magnitude to 0.8 m and 0.005 m, respectively. These values give a Bond number of 1.4×10^7 , which in sufficient to numerically represent the analytical solution. To validate the pseudo-spectral solver constructed, the computed results predicted for this Bond number were compared with the analytical solution given in (4.29) - (4.32). Figure 4.6 shows the evolution of the normalised film thickness h/h_0 at the uppermost and lowest points of the cylinder, $\theta = 90^\circ$ and 270°, with reduced time τ . The results show excellent agreement up to $\tau \simeq 0.45$ where the numerical calculations become increasingly less accurate and more unstable due to the singularity that arises at $\theta = 270^\circ$ as $\tau \to 0.5$.

To ensure this validation was undertaken over the entire cylinder surface and not just at two specific points ($\theta = 90^{\circ}$ and 270°), the present results were also compared to the analytical solution (4.29) - (4.32) for all θ at specific instants of reduced time. Figure 4.7 shows two such comparisons, specifically at $\tau = 0.2$ and 0.4, which are typical of the excellent agreement for all values of $\tau \leq 0.45$. This



figure also clearly demonstrates the growth of the film on the lower surface.

Figure 4.6: Comparison of the analytical solution of Reisfeld and Bankoff [8] given in (4.29) - (4.32) with computed numerical results for normalised film thickness for temporal evolution at top ($\theta = 90^{\circ}$) and bottom ($\theta = 270^{\circ}$) of the cylinder at $\hat{Bo} = 1.4 \times 10^7$. Variation of h/h_0 with τ at $\theta = 90^{\circ}$ and 270° .



Figure 4.7: Comparison of the analytical solution of Reisfeld and Bankoff [8] given in (4.29) - (4.32) with computed numerical results for spatial distribution of normalised film thickness on cylinder at $\hat{Bo} = 1.4 \times 10^7$. Variation of h/h_0 with θ at $\tau = 0.2$ and 0.4.

4.5.2 Constant Shear Loading Only

Having validated the solver for both loads (gravity and surface tension) which were not caused by the external aerodynamic field, a further study was required which validated the solver for a load which was caused by the aerodynamic field. To achieve this however, first an analytical solution must be derived. To simplify this the case of a purely shear-driven flow was selected where the only loading on the thin film was a constant shear T. Details of the derivation of this analytical solution, along with a note on the limits of its applicability, are presented in appendix D.

This shows that the (implicit) solution for $h(\theta, t)$ satisfying the initial condition $h(\theta, 0) = h_0(\theta)$ is given by

$$h = h_0 \left(\theta - \frac{Th}{\hat{\mu}R} t \right). \tag{4.33}$$

The solution to which at any time is dependent upon the initial profile of film thickness. If the initial profile is taken to be uniform, i.e. $h_0(\theta) = \text{constant}$, then the distribution of the film remains uniform for all time. This evolutionary response is correctly predicted by the present solver, but is rather uninteresting. As such a simple non uniform initial profile was also examined, specifically the trigonometric function, $h_0(\theta) = H(1 - a \cos \theta)$, where H > 0 and |a| < 1. Using this initial profile an implicit equation which determines h as a function of θ and t can be derived as

$$\frac{h}{H} = 1 - a \cos\left(\theta - \frac{Th}{\hat{\mu}R}t\right). \tag{4.34}$$

This solution becomes invalid if the wavelike profile of the free surface 'breaks' at some instant, that is, if $\partial h/\partial \theta$ becomes infinite. Using the parameters from table 4.4 together with an initial film thickness of $H = 5.0 \times 10^{-4}$ m, a peak amplitude a = 0.1 and unit shear stress $T = 1 \text{ N/m}^2$, then (D.13) predicts that the free surface breaks at time

$$t_{\rm s} = \frac{\hat{\mu}R}{nHaT} = \frac{1.002 \times 10^{-3} \cdot 0.08}{1 \cdot 0.5 \times 10^{-3} \cdot 0.1 \cdot 1} = 1.6032 \,\rm{s}, \tag{4.35}$$

and at position

$$\theta_{\rm s} = \frac{3\pi}{2n} + \frac{1}{na} \; (\bmod 2\pi) \simeq 123^{\circ}.$$
(4.36)

At which point the pseudo-spectral method solver will also fail. The variables H and a were chosen to best represent the experimental results of Cosentino [19, 25]

and the parameters already established in table 4.4; while T was chosen as unity for simplicity.

As can be seen in figure 4.8, which shows a comparison between the present results and the theoretical profile (4.33) at this 'breaking' time ($t_s = 1.6032 \,\mathrm{s}$) both the value at which the solution failed and which this occurred $\theta_s \simeq 123^\circ$ were accurately predicted by the present numerical solver. While a closer examination of the overall form shows that this also is in excellent agreement with the analytical solution. Indeed this was found for all times $t \leq 1.6032 \,\mathrm{s}$. Figure 4.8 highlights this via a comparison between the analytical and the presently determined profiles at this 'breaking' time and a typical earlier instant, $t = 1 \,\mathrm{s}$, which was chosen as representive for all times up to this 'breaking' time.



Figure 4.8: Comparison of analytical solution for constant shear (4.34) with computed numerical results for spatial distribution of normalised film thickness on cylinder at t = 1 s and t = 1.6032 s. Variation of h/H with θ .

As was true for the previous validation study, excellent agreement between the present results and the analytical solution was found for both specific instants in time, figure 4.8, and for the temporal evolution at specific angular locations. The latter can be seen in figure 4.9 which shows the temporal evolution of normalised film thickness at two specific points at the top and bottom of the circular cylinder, $\theta = 90^{\circ}$ and $\theta = 270^{\circ}$, which were taken as representative for the entire cylinder. In addition to illustrating the excellent temporal agreement between the analytical solution and the present results figure 4.9 also shows that maximum thickness of the film also agrees with the analytical solution (D.13). The latter is identical to the prescribed value 0.1 and is achieved twice at each location as the 'bulge' of fluid accumulates at it rotates around the cylinder and evolves.



Figure 4.9: Comparison of analytical solution for constant shear (4.34) with computed numerical results for normalised film thickness for temporal evolution at top ($\theta = 90^{\circ}$) and bottom ($\theta = 270^{\circ}$) of the cylinder. Variation of h/H with θ .

The quality of the agreement between the present results and the analytical solutions both temporally and spatially is excellent for both validation studies undertaken. The pseudo-spectral method solver created can therefore be said to be justified within the present context and as such can be used with confidence for problems more directly related to RWIV.

4.6 Test Cases

Given the nature of the governing thin film evolution equation (4.24) and following the lead of Lemaitre [9] four different combinations of surface tension γ , gravity g, pressure P and shear T were considered. These were:

- 1: Gravity and surface tension $(T \equiv 0 \text{ and } P \equiv 0)$,
- 2: Shear and surface tension $(P \equiv 0 \text{ and } g = 0)$,

- 3: Pressure and surface tension $(T \equiv 0 \text{ and } g = 0)$,
- 4: Full loading $(P, T, g \text{ and } \gamma \neq 0)$.

In addition, further studies regarding specific aspects pertinent to the overall result were undertaken. These include how a variation in profile or magnitude of the distributions of \bar{C}_P or \bar{C}_F , or a change in Reynolds number, affects the evolutionary response of the film.

4.6.1 Gravity and Surface Tension Loading

In the validation study of section 4.5.1 the combination of gravity and surface tension loading was examined for values of initial film thickness and cylinder radius to ensure representation of an infinite Bond number. These were not the values of $h_0 = 5 \times 10^{-4}$ m and R = 0.08 m which were outlined as typical for RWIV in table 4.4. It is the latter values which are presently studied and for which figure 4.10 illustrates the temporal evolution of film thickness h as a function of θ . This provides a clear representation of the film's evolution and the rate at which this occurs, and is the first time such a temporal representation has been reported. As can be seen from figure 4.10 the film 'thins' on the upper surface of the cylinder ($0^{\circ} < \theta < 90^{\circ}$) while a 'spike' in the film thickness is formed on the lower surface at the lowest point, $\theta = 270^{\circ}$. This continues to accumulate fluid and increase in thickness until the lubrication approximation is violated and the theoretical method used herein is no longer valid. This is consistent with the analytical solution (4.29), the computational work of Reisfeld and Bankoff [8] and the previous validation study undertaken in section 4.5.1.

Relative Magnitudes of Gravity and Surface Tension

To examine how the relative magnitudes of the loadings which result from gravity and surface tension affect the evolutionary response a study was undertaken in which the magnitude of the gravity number $G = gh_0^3/3\nu^2$, which measures



Figure 4.10: Numerical prediction of temporal evolution of film thickness in real time, under gravity and surface tension effects only. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

the ratio of gravitational to viscous forces was varied. To achieve this two additional fluids, benzene and mercury with approximately equal Bond number but distinctly different gravity number, were examined in addition to the datum fluid of water which is used elsewhere within this thesis. The standard properties of these fluids at $20^{\circ}C$ are given in table 4.5.

Property	Water	Benzene	Mercury
Density, $\hat{\rho}$, kg/m ³	1000	880	13600
Dynamic viscosity, $\hat{\mu}$, Ns/m ²	1.002×10^{-3}	0.656×10^{-3}	1.55×10^{-3}
Surface tension, $\hat{\gamma}$, N/m	0.072	0.039	0.51
Bond Number \hat{Bo}	1.4×10^7	3.1×10^7	2.7×10^7
Gravity Number \hat{G}	4.1×10^5	$7.4 imes 10^5$	3.2×10^7

Table 4.5: Parameters and fluid properties of water, benzene and mercury at $20^{\circ}C$.

In a similar manner to the Bond number study of figure 4.5 five different cases of film thickness and cylinder radius were examined for each fluid. In each case only the initial film thickness and radius of the cylinder were varied and only such that the ratio between these was kept constant at $h_0/R = 6.3 \times 10^{-3}$. The results showed that provided the Bond number is greater than $\simeq 1 \times 10^5$, the lowest value studied, altering the relative effects of loading due to gravity or surface tension by varying G by upto two orders of magnitude does not significantly change the solution at the same value of reduced time τ ; rather this only alters the real time t taken to arrive at that solution. This is due to the dominance of gravitational loading over surface tension loading at these Gravity and Bond numbers. This dominance will be lost at lower values of Bond number and the pattern of these results is no longer expected to exist. However given the actual dimensions of film thickness and stay radius to be studied, these lower values are outwith the scope of the present work, and the reader is referred to Reisfeld and Bankoff [8] who provide a fuller discussion of such flows.



Figure 4.11: Comparison of normalised film thicknesses of water, benzene and mercury with angle θ at real time $t = 0.4 \times 10^{-3}$ s. Variation of h_0/h with θ for various \hat{G} at fixed t.

To demonstrate this timescale relation, figures 4.11 and 4.12 show the normalised thickness h/h_0 of the three fluids (water, benzene and mercury) at the same instant of real time t and reduced time τ , respectively. Here as was the case within the initial validation study of section 4.5.1, the values of cylinder radius R and initial film thickness h_0 were again increased to 0.8 m and 0.005 m respectively. This ensured that the Bond number was significantly greater than 1×10^5 , specifically $\hat{Bo} = 1.4 \times 10^7$ for the datum water case. The exact values of these non-dimensional parameters as they relate to each of the three fluid are also shown in table 4.5. The values of τ and t for figures 4.11 and 4.12 were chosen to represent a late reduced time value, $\tau = 0.4$, and the corresponding real time tfor water, by which point any differences would have expected to manifest. This choice was however arbritrary, as the same result was obtained at any value below $\tau \simeq 0.45$.



Figure 4.12: Comparison of normalised film thickness of water, benzene and mercury with angle θ at reduced time $\tau = 0.4$. Variation of h_0/h with θ for various \hat{G} at fixed τ .

Closer inspection of figure 4.11 reveals that as mercury has a significantly greater gravity number, $O(10^7)$, than that of the other two fluids, due to the dominance of gravitational effects this film thus evolves considerably faster. Whereas because the gravity numbers of benzene and water are more closely matched and lower than that of mercury, $O(10^5)$, these films develop at approximately the same rate, with water, which has the lowest value of G, being the slowest to evolve. In the limiting case of gravity and surface tension being varied in such a way that G remains constant the minimal variations at fixed time t, shown in figure 4.11, are eliminated altogether, and the fluids evolve at the same rate.

4.6.2 Shear and Surface Tension Loading

Next the temporal evolution of the thin film under the effect of shear T and surface tension γ were examined. Under this combination of loading, two distinct symmetrically placed rivulets form and continue to grow until the thin film approximation is violated, one each on the upper and lower surfaces. Figure 4.13 illustrates this evolution at early times. The location of these rivulets being just windward of the clockwise and anti-clockwise separation points for a dry cylinder at the same Re which is consistent with the previous experimental analysis of Bosdogianni [67] and the numerical results of Lemaitre [9]. These rivulets also occur at approximately the same angles at which the minimum values of the \bar{C}_F distribution occur, figure 4.4, which further agrees with [9, 67].



Figure 4.13: Numerical prediction of temporal evolution of film thickness in real time, under shear and surface tension effects only. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

Closer examination of the temporal evolution of the rivulets in figure 4.13 reveals that these are of approximately equal thickness, profile and growth rate. The minor discrepancies between these can be attributed to small asymmetries in the applied shear loading distribution which results from the faithful approximation of asymmetries in the aerodynamic coefficients determined experimentally by Achenbach [113]. This was further confirmed as given a perfectly symmetric distribution of \bar{C}_F the two rivulets were indeed found to be identical and located symmetrically with respect to the stagnation point at the windward horizontal.

When compared with the only other computational results [9] available for the same problem, the present data is found to be in excellent agreement as illustrated in figure 4.14. This displays a magnified image of both results at a specific instant in time, $t = 6.9 \times 10^{-3}$ s, the only time at which the results of Lemaitre [9] were available. The minor differences between the two solutions can be attributed to a variation in the distribution of \bar{C}_F used in the two schemes, as was true for the slight asymmetry.



Figure 4.14: Comparison of present numerical results with those from Lemaitre [9] for variation of film thickness ($100 \times \text{actual}$) at $t = 6.9 \times 10^{-3}$ s, under the effects of shear and surface tension, where the incident wind acts from the left.

To confirm this, the governing evolution equation from Lemaitre [9] was used in the present solver with the present distribution of \bar{C}_F . When these results are compared with those obtained using the present evolution equation (4.24), they were found to be identical over the entire time range under investigation. Figure 4.15 displays one such representative comparison at a 'late' time of t = 15.0×10^{-3} s, by which point any potential discrepancies between the two would have been expected to manifest. That said, this choice is arbritrary and later times were also examined with similar results. This demonstrates three points:

1: that the present solver is consistent with that of Lemaitre [9].

- 2: that both forms of the evolution equation are effectively equivalent.
- 3: that the differences displayed in figure 4.14 are attributable to differences in the distribution of \bar{C}_F .

As such the latter must play an important role in the profile of the rivulet which evolves, this will therefore be investigated further in section 4.6.3 when the distribution of \bar{C}_P is considered.



Figure 4.15: Comparison of normalised film thickness under the effects of shear and surface tension from present numerical solver for evolution equation (4.24) and equation (17) of Lemaitre [9], using identical \bar{C}_F distribution, at $t = 15.0 \times 10^{-3}$ s. Variation of h/h_0 with θ .

Effect of Shear Stress Magnitude

To investigate the effect that the magnitude of the shear loadings has on the rivulets which evolve, a study was undertaken in which the distribution of \bar{C}_F was kept constant but the applied shear stress was altered by varying the incident wind speed U. Several such values were investigated, although only three are reported here as these are representative of all the cases examined. Specifically these are the wind speed that has been used in the previous cases, U = 11 m/s, and two others U = 7.8 m/s and U = 13.5 m/s, which were chosen such that the applied shear in each case was 0.5 and 1.5 times that of the original (11 m/s) case

respectively. The choice of these upper and lower limits also ensured that these wind speeds were within the typical range identified for RWIV of $5 \text{ m/s} \leq U \leq 15 \text{ m/s}$ identified in section 2.2.



Figure 4.16: Comparison of normalised film thickness with angle from windward horizontal, for the effect of varying shear loading value through variation of the incident wind speed with constant distribution of \bar{C}_F at $t = 6.9 \times 10^{-3}$ s. Variation of h/h_0 with θ for various wind speeds U.

The results showed that when only the magnitude of shear T applied is altered the resulting film thickness evolution profile does not change in distribution only in magnitude, figure 4.16. This is due to the dominance of the effects of shear over the effect of surface tension for the specific parameters presently under investigation. Should different parameters be used such that the magnitudes of surface tension and shear loading are closer it is expected that such a 'scaling' effect would no longer hold. However as the parameters used herein are representative of RWIV, table 4.4, this is not the subject of the present study. That said as can be seen from figure 4.17 should surface tension loading be removed from either the present shear loading case or the pressure loading case to be examined in section 4.6.3, then the results at a given time are not significantly altered. This indicates that the effects of surface tension are negligible for the present studies. However this may not always be true especially within the coupled solver. As such surface tension is included in all studies to follow.



Figure 4.17: Comparison of normalised film thickness with angle from windward horizontal, if surface tension is or is not included for the shear and pressure loading cases at $t = 6.9 \times 10^{-3}$ s. Variation of h/h_0 with θ with and without γ .

In addition to illustrating the 'scaling' effect figure 4.16 also highlights that the angles θ at which these rivulets form on the upper and lower surfaces of the cylinder was independent of wind speed if the distribution of \bar{C}_F was unaltered. While the particular value of $t = 6.9 \times 10^{-3}$ s was chosen to correspond to previous comparison, the selection of this was arbitrary as this effect was seen throughout the evolutionary response. The only temporal variation being that the difference in magnitude between the U = 7.8 m/s and 13.5 m/s cases and the base 11 m/s case increases with time. While the assumption that the distribution of \bar{C}_F does not change with U, and hence Re, may be an idealisation, over the subcritical range of Reynolds number corresponding to the velocities studied, $82 \times 10^3 \leq$ $Re \leq 143 \times 10^3$, any variation in the distribution of \bar{C}_F is expected to be slight [113]. A point which will be discussed in greater depth in section 4.6.3, when the effect of Re on rivulet evolution is studied.

4.6.3 Pressure and Surface Tension Loading

Under only a combination of pressure and surface tension loading, the results show distinct similarities to those for the previous case of shear and surface tension loading, section 4.6.2. Here again rivulets symmetric with respect to the



Figure 4.18: Numerical prediction of temporal evolution of film thickness in real time, under pressure and surface tension effects only. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

incident flow (the stagnation point at the windward horizontal) form just windward of the separation points on both sides of a dry cylinder, figures 4.18 and 4.19, and continue to increase in thickness until the thin film approximation is violated. The slight asymmetries in these rivulets are again due to reproduction of asymmetries in the applied \bar{C}_P profile, figure 4.4, as with symmetric \bar{C}_P the temporal distribution of film thickness was again symmetric, as was true for shear loading. The locations of these rivulets are marginally windward of those formed in the shear and surface tension case, figure 4.19; closer examination of which highlights that the 'sizes' of the rivulets under the present pressure loading are the same order of magnitude as that of the previous shear loading condition.

Here the definition of rivulet 'size' has been used loosely, as the differences in thickness and width of the rivulets formed between the shear and pressure loading cases is noticeable. However the amount of additional fluid accumulated within each (based on the relative cross sectional areas) is approximately the same. Namely 1.25×10^{-3} mm² and 1.72×10^{-3} mm² for the shear and pressure cases at $t = 6.9 \times 10^{-3}$ s, respectively. The main reason for this variation in shape can be attributed to the differences in the distribution of the aerodynamic coefficients, \bar{C}_P and \bar{C}_F , which will be discussed further in section 4.6.3. Given these similar cross sectional areas and the similar rates of rivulet growth, which can be ascertained from figures 4.13 and 4.18 respectively, the indication is that the effects of pressure and shear are of approximately equal importance in rivulet formation and evolution for the present problem. This corresponds well with the findings of Lemaitre [9].



Figure 4.19: Comparison of present numerical results with those from Lemaitre [9] for variation of film thickness ($100 \times actual$) at $t = 6.9 \times 10^{-3}$ s, under the effects of shear and surface tension, with those under pressure and surface tension loading, where the incident wind acts from the left in both cases.



Figure 4.20: Comparison of present numerical results with those from Lemaitre [9] for variation of film thickness ($100 \times actual$) at $t = 6.9 \times 10^{-3}$ s, under the effects of pressure and surface tension where the incident wind acts from the left.

For purposes of consistency the present results were again compared to those



Figure 4.21: Comparison of normalised film thickness under the effects of pressure and surface tension from present numerical solver for evolution equation (4.24) and equation (17) of Lemaitre [9], using identical \bar{C}_P distribution at $t = 15.0 \times 10^{-3}$ s. Variation of h/h_0 with θ .

of Lemaitre [9], under the same loading conditions. The results at $t = 6.9 \times 10^{-3}$ s, which is indicative of the entire time studied, can be seen in figure 4.20. This highlights that the two are again in excellent agreement, with the minor differences between the two solutions again attributable to a variation in the distribution of \bar{C}_P used as was found for the previous shear loading case (figure 4.14). As when the results determined herein were compared with those when equation (17) of Lemaitre [9] was used with the present solver and with the present distribution of \bar{C}_P , these were found to be identical over the entire time range under investigation. Figure 4.21 illustrates an arbitrary representative case at the same time as the previous shear case comparison at $t = 15.0 \times 10^{-3}$ s. This again confirms the equivalence of the two governing equations and further validates the present solver within a RWIV context.

Effect of Pressure Distribution

To complement the work of section 4.6.2, where the effect of differing magnitudes of shear based on a constant distribution of \bar{C}_F were investigated, a study was undertaken to examine the opposite. Specifically how varying the distribution of



Figure 4.22: Comparison of four distributions mean pressure distributions, namely Achenbach [113], base ESDU [61], scaled ESDU and matched ESDU. Variation of \bar{C}_P with θ .

 \bar{C}_P while keeping a constant magnitude of pressure affects the evolution of the film profile. To achieve this the original distribution of time averaged mean pressure determined from Achenbach [113], henceforth referred to as $\bar{C}_{P(Achenbach)}$, was compared to the distribution of \bar{C}_P , for a Reynolds number of 100×10^3 , given by the formulae in ESDU 80025 [61] which are an empirical fit based on numerous experiments over a range of Re. The latter distribution henceforth being referred to as $\bar{C}_{P(baseESDU)}$. Figure 4.22 illustrates these two different distributions for \bar{C}_P as well as two additional distributions that were created such that the differences between $\bar{C}_{P(Achenbach)}$ and $\bar{C}_{P(baseESDU)}$ could be further investigated. The definitions and the reasons for choosing these, are as follows:

1: Scaled ESDU - This ensured that the 'maximum' value of \bar{C}_P (in this case a minimum at peaks $\theta \simeq 75^\circ$ and $\theta \simeq 285^\circ$) matched that of Achenbach [113] but in such a way that the same profile as the base ESDU distribution was maintained. To achieve this the profile of $\bar{C}_{P(baseESDU)}$ was multiplied by a scaling factor such that:

$$\bar{C}_{P(ScaledESDU)} = \frac{|\max C_{P(Achenbach)}|}{|\max \bar{C}_{P(baseESDU)}|} \times \bar{C}_{P(baseESDU)}$$

This distribution of \bar{C}_P was chosen because it provided an additional case

of the type studied in section 4.6.2, and the means to examine whether it is the maximum value contained within, or the profile of, the aerodynamic coefficient distribution \bar{C}_P or \bar{C}_F that has the greater effect on the evolutionary response.

2: Matched ESDU - This also ensured that the 'maximum' value of C_P matched that of Achenbach [113]. Here, however, rather than 'scaling' the distribution, the equations for C
{P(baseESDU)} were used as were, but with an artificially low minimum pressure coefficient C{pm} = -1.75, which matched that of C
_{P(Achenbach)}. As a result, C
_{P(MatchedESDU)} gives a closer representation of C
_{P(Achenbach)} based on the ESDU formulae [61], as the frequency content of the two distributions are closer. This distribution of C
_P was chosen as it provided a means of examining how the magnitudes of the coefficients a_k and b_k in the truncated Fourier series (4.27) and hence the frequency content affected the evolutionary response.

The results for normalised film thickness for these various distributions of \bar{C}_P for a given representative time corresponding with previous sections of $t = 6.9 \times 10^{-3}$ s are shown in figure 4.23, although again this choice was arbritrary. This demonstrates that three distinct differences can be distinguished in the evolutionary response profiles.

Firstly, due to the large variations in the value of C_P over a small range of θ that accompanies the onset of adverse pressure gradient leeward of the separation points at $\theta \simeq 75^{\circ}$ and $\theta \simeq 285^{\circ}$ in the Achenbach and Matched ESDU distributions, and the faithful representation of actual experimental data in the former case, the higher frequency components of the truncated Fourier series have significantly greater magnitudes than those of the base ESDU and scaled ESDU distributions. This is reflected in the increased magnitudes of Fourier coefficients a_k and b_k for large values of k in the Achenbach and Matched ESDU distributions. As a result of these higher frequency components the profiles of evolutionary film thickness for the Achenbach and Matched ESDU distributions of \bar{C}_P display a



Figure 4.23: Comparison of the effect that various distributions of \bar{C}_P have on the normalised film thickness profile, at $t = 6.9 \times 10^{-3}$ s, under pressure and surface tension loading. Variation of h/h_0 with θ .

noticeable wavelength, which corresponds to the highest Fourier component considered in the truncated series and is therefore a numerical feature rather than a fundamental property and is worse for pressure loading than shear loading as it is the derivative of pressure which is present in the evolution equation (4.24). However, the effect is greatly magnified by the normalisation in figure 4.23 and is not noticeable in figure 4.20 which also presents the film thickness for the Achenbach distribution (at 100 times actual thickness), where the dominant feature in which is the location of rivulets at approximately the separation points on the dry cylinder. Figure 4.23 also shows that such fluctuations also occur in profiles of evolutionary film thickness based on the base ESDU and scaled ESDU distributions, but that these are much less noticeable and the profiles of film thickness are therefore considerably smoother.

Secondly, although a rivulet can be detected under all four distributions of \bar{C}_P at approximately the separation points of the dry cylinder, there is little correspondence between the thickness or shape of these. Those determined from $\bar{C}_{P(Achenbach)}$ and $\bar{C}_{P(MatchedESDU)}$ are considerably thicker, (i.e. h/h_0 is larger) and narrower (i.e. small angular range of θ), while the rivulets determined from $\bar{C}_{P(baseESDU)}$ and $\bar{C}_{P(ScaledESDU)}$ are thinner (i.e. h/h_0 is smaller), wider (i.e. large

angular range of θ) and considerably more asymmetric about the thickest point, in that these display a shallow increase in thickness windward of maximum rivulet thickness and a sharp decrease leewards. Furthermore although the maximum value of \bar{C}_P was the same in the $\bar{C}_{P(Achenbach)}$, $\bar{C}_{P(MatchedESDU)}$ and $\bar{C}_{P(ScaledESDU)}$ cases, the evolutions show that this has little bearing on the maximum thickness of the film; this point is further emphasised by the differences in the thickness that result from the $\bar{C}_{P(Achenbach)}$ and $\bar{C}_{P(MatchedESDU)}$ distributions, since although these have identical magnitudes and similar profiles, the differences in magnitudes of the frequency content of these coefficients, particularly in the upper frequency range, result in a rivulet of greater thickness forming in the $\bar{C}_{P(Achenbach)}$ case. As such, the evolutionary responses with different distributions of \bar{C}_P produce different free surface profiles. Although not reported explicitly due to the absence of a suitable theoretical distribution of \bar{C}_F , a corresponding result was likewise found under different distributions of \bar{C}_F .

Finally, as was true for the combination of shear and surface tension loading, examined in section 4.6.2, the present two cases of constant profile of \bar{C}_P distributions but varying magnitude ($\bar{C}_{P(baseESDU)}$ and $\bar{C}_{P(ScaledESDU)}$) showed results that were 'scaled' images of each other throughout the temporal range studied. This again can be attributed to the dominance of the loading which results from the external aerodynamic field, in this instance pressure, over surface tension loading as was shown previously in figure 4.17.

Effect of Varying Reynolds Number

Using the typical ranges of wind speed $(5 \text{ m/s} \leq U \leq 15 \text{ m/s})$ and cable diameter $(100 \leq D \leq 250 \text{ mm})$ previously highlighted in section 2.2, a range of Reynolds number over which RWIV could occur can be determined. The maximum extent of this range, $33 \times 10^3 < Re < 250 \times 10^3$, is significantly larger than that determined experimentally by Cosentino [19] and Matsumoto [33] namely $50 \times 10^5 < Re < 150 \times 10^5$ (reported in section 2.2). In this range of Re however, the distribution of \bar{C}_P , determined from the formulae given in ESDU 80025 [61] is identical, taking the form of $\bar{C}_{P(baseESDU)}$ used in section 4.6.3. As such if a constant distribution of aerodynamic loading is assumed and only the wind speed is altered then the results at differing Reynolds numbers are once again found to be 'scaled' images of each other at any specific instant in time, as was found to be the case for \bar{C}_F (section 4.6.2) and \bar{C}_P (section 4.6.3).



Figure 4.24: Effect of varying Reynolds number through change of flow regime on the variation of normalised film thickness with angle from windward horizontal at $t = 6.9 \times 10^{-3}$ s, for shear and surface tension loading. Variation of h/h_0 with θ .



Figure 4.25: Effect of varying Reynolds number through change of flow regime on the variation of normalised film thickness with angle from windward horizontal at $t = 6.9 \times 10^{-3}$ s, for pressure and surface tension loading. Variation of h/h_0 with θ .

As outlined in section 2.1.1 turbulence of the incoming flow, surface roughness and changes in the thin film geometry could however all cause a significant reduction in the critical Reynolds number. As such the flow over the cylinder at the *Re* under investigation may not always occur in the sub-critical flow regime as previously assumed. To investigate what effect such a change in flow regime would have upon the evolutionary response at a given wind speed (here chosen as the base U = 11.0 m/s case) two additional distributions of \bar{C}_P and \bar{C}_F from different flow regimes were considered. These were again based upon experimental data obtained by Achenbach [113] and were taken at a Reynolds number just below critical of 240×10^3 , and at a super-critical Reynolds number of 3600×10^3 . The distributions of \bar{C}_P and \bar{C}_F being once again represented by a twenty term truncated Fourier series (4.27). The results show that for both the shear, figure 4.24, and pressure loading, figure 4.25, a variation in flow regime resulted in a variation in rivulet location and size, with the flow regime corresponding to a larger Reynolds number typically resulting in a thinner rivulet forming at a greater angle from the incident flow (the stagnation point at windward horizontal). The angle at which the rivulet formed in each instance agreeing well with the angle at which the minimum values of the \bar{C}_P and \bar{C}_F distributions were located in each case. Should symmetric distributions of \overline{C}_P or \overline{C}_F be used, then the resulting evolutionary profile, and the rivulets formed, are once again found to be symmetric with respect to the incident flow. However due to the faithful reproduction of slight asymmetries in the experimental data, the evolutionary profiles shown in figures 4.24 and 4.25 produced here are slightly asymmetric, with the rivulet on the lower surface of super-critical case displaying the largest deviation.

In practise a variation in Reynolds number would likely result in a change in both the incident wind speed and the distribution of \bar{C}_P and \bar{C}_F , although the latter will likely not be as large as shown here unless it results in a shift in flow regime. Therefore the two effects outlined here would probably act in combination, the extent of each contribution dependent upon the particular case under examination. What is clear however is that in all cases and in all flow regimes studied rivulets were found to form on both the upper and lower surfaces marginally windward of the separation points of a dry cylinder at the same Reynolds number.

4.6.4 Full Loading

The temporal evolution of film thickness for pressure, shear, surface tension and gravity all acting in unison is shown in figure 4.26. Similar to the previous two cases, under full loading conditions two distinct rivulets can be seen to form and continue to grow until the thin film approximation is violated, one each on the upper and lower surfaces. However, in this instance the symmetry about the axis of the incident wind of the previous cases, should a symmetric distribution of \bar{C}_P and \bar{C}_F be used, is lost due to the effect of gravity. Gravitational loading results in the rivulet that evolves on the lower surface under full loading conditions being thicker than the rivulet which evolves on the upper surface.



Figure 4.26: Numerical prediction of temporal evolution of film thickness in real time, under full loading conditions (gravity, shear, pressure and surface tension effects). Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

The difference in thickness of these two rivulets can be quantified by means of

the normalised thickness of the rivulet h/h_0 . As the specific time instant used for comparisons in the previous cases of $t = 6.9 \times 10^{-3}$ s, this was $h_{upper}/h_0 = 1.39$ for the upper rivulet and $h_{lower}/h_0 = 1.68$ for the lower rivulet respectively. Where h_{upper} and h_{lower} are the thickness of the upper and lower rivulet at the point of maximum thickness. Figure 4.27 demonstrates this temporal increase in thickness with time of both rivulets while also highlighting that the lower rivulet is growing faster than the upper rivulet due to the effect of gravity, as common experience would suggest. The latter is achieved by plotting the ratio between the thickness of the lower and upper rivulets (h_{lower}/h_{upper}). Furthermore, while the point of maximum thickness of the lower rivulet moves leeward from the point where it occurred under only pressure and surface tension loading (section 4.6.3), specifically from $\theta \simeq 288^{\circ}$ to $\theta \simeq 282^{\circ}$, the thinner upper rivulet moves marginally windward from $\theta \simeq 65^{\circ}$ to $\theta \simeq 63^{\circ}$. This simple result agrees with intuitive expectations as to the effects that gravity should have on the system in so doing increasing confidence in the model.



Figure 4.27: Early temporal evolution of normalised film thickness of upper and lower rivulets and ratio between these.

The present results were again compared to those of Lemaitre [9]. Figure 4.28 illustrates both evolutionary profiles at the same instant in time used previously, $t = 6.9 \times 10^{-3}$ s. This highlights the two results are again in excellent agreement, with the minor differences between the two solutions once again attributable to

variations in the distribution of \bar{C}_P and \bar{C}_F used. As when the present results were compared with those determined using the present solver and the present distributions of \bar{C}_P and \bar{C}_F but the governing equation of Lemaitre [9], these were again found to be identical over the entire time range under investigation. This reiterates previous findings as to the consistency of the solvers and the equivalence of the governing evolution equations which were illustrated in figures 4.15 and 4.21.



Figure 4.28: Comparison of present numerical results with those from Lemaitre [9] for variation of film thickness (100×actual) at $t = 6.9 \times 10^{-3}$ s, under the full loading conditions.

While a comparison has been made with the work of Lemaitre [9] for each of the loading combinations presently examined, due to the complexity of the distributions of \bar{C}_P and \bar{C}_F used, analytical solutions are not available to validate against. As such, the present data has been validated against the only other data available although this did come from another numerical solver [9]. While this is far from ideal, as the two numerical solutions have been obtained independently and the results have been generated by two separate solvers it allows a greater degree of confidence in both sets of results and the method used.

As a final means of comparison figure 4.29 displays the evolutionary profiles of the present solver for the rivulets formed under the three loading combinations examined here at $t = 6.9 \times 10^{-3}$ s; shear and surface tension, pressure and surface tension and full loading. This highlights the difference in rivulet form and locations discussed in one figure. Furthermore the slight asymmetry which results from faithful reproduction of experimental \bar{C}_P and \bar{C}_F are put into context when compared to the effect of gravitational loading, which results in a slight windward movement of the upper rivulet and a leeward movement and thickening of the lower rivulet respectively.



Figure 4.29: Comparison of variation of film thickness ($100 \times \text{actual}$) at $t = 6.9 \times 10^{-3}$ s of present numerical results for three loading cases, shear and surface tension, pressure and surface tension and full loading, where the incident wind acts from the left in all three cases.

4.6.5 Comparison with Aerodynamic Solver

As a link between the previous, present and following chapters, a final study was undertaken in which evolutionary film thickness profile which results from the time averaged mean distributions of \bar{C}_P and \bar{C}_F determined numerically by the DVM solver in chapter 3, was compared with those determined based on the distributions of \bar{C}_P and \bar{C}_F of a particular experiment and used elsewhere within this chapter [113]. Both sets of \bar{C}_P and \bar{C}_F were assumed to remain constant with time and both were based on a plain circular cylinder in the absence of rivulets. These are illustrated in figure 4.30, where the distribution shown \bar{C}_F^* is once again normalised. While the distributions of \bar{C}_P and \bar{C}_F determined by the DVM solver have previously been shown to be in good agreement with typical experimental results, figures 3.17 and 3.7, there are differences between these and the experimental distributions of Achenbach [113]. This is particularly true of the minimum pressure value C_{pm} and the area immediately behind separation within the distribution of \bar{C}_F . While the former appears to be a distinct feature of the Achenbach [113] distribution of \bar{C}_P and is not found for example within the ESDU results [61], the latter is due to three-dimensional effects and the lack of turbulence model in the present aerodynamic solver. This was previously discussed in greater depth in section 3.2.1. That said, however these differences aside the distributions of \bar{C}_P and \bar{C}_F are in good agreement.



Figure 4.30: Comparison of the distributions of aerodynamic loading based on Achenbach [113] with those determined by the DVM solver at $\text{Re} = 1 \times 10^5$. Variation of \bar{C}_P and $\bar{C_F}^*$ with θ .

Figure 4.31 displays a comparison between the evolutionary profile which results due to the distribution of \bar{C}_P and \bar{C}_F from Achenbach [113] and that calculated in the DVM solver at an arbitrary representative time chosen to correspond to previous examinations as $t = 6.9 \times 10^{-3}$ s, for each of the loading combinations previously examined; shear and surface tension, pressure and surface tension and full loading. While the evolutionary profiles predicted for the shear and surface tension loading case are in good agreement both quantitatively and qualitatively, figure 4.31, with only minor differences in rivulet thickness and location, there



Figure 4.31: Comparison of the evolutionary profile predicted by the distributions of \bar{C}_P and \bar{C}_F determined by the DVM solver with those which result from the distributions of \bar{C}_P and \bar{C}_F based on Achenbach [113]. Variation of normalised film thickness h/h_0 with angle clockwise from the windward horizontal θ at $t = 6.9 \times 10^{-3}$ s, under three different loading cases. Top: Shear and surface tension loading. Middle: Pressure and surface tension loading. Bottom: Full loading conditions.

are discrepancies within the other two loading cases.

As was discussed in section 4.6.3 this results due to the large spectral content present in the higher frequencies of the Achenbach [113] distribution of \bar{C}_P which is not present within the distributions based on ESDU 80025 [61] or that determined from the DVM solver. In practise the magnitude of this high frequency content is expected to be considerably reduced. Thus while differences between the evolutionary profiles do exist, that which results from the Achenbach [113] distribution of \bar{C}_P , fluctuates about that determined using the distribution of \bar{C}_P determined by the DVM solver. The latter almost acting as a smoothed response, i.e. without the noticeable wavelength. This is further confirmed if evolutionary profile which evolves based on the distribution of C_P determined by the DVM solver is compared with that resulting from the distribution of C_P based on ESDU 80025 [61], which was previously used in section 4.6.3 when different profiles of C_P were examined. As shown in figure 4.32, although only for a single representative time, again chosen as $t = 6.9 \times 10^{-3}$ s, there is now excellent agreement between the evolutionary profiles at all times. As such, previous differences in evolutionary profile can be attributed to variations in the spectral content of aerodynamic coefficient distribution. The level of agreement increases confidence in both the numerically determined distributions of both \bar{C}_P and \bar{C}_F and illustrates that the simplifications made in determining \bar{C}_F , within section 3.2.1, were valid for the present application.

Due to the presence of the high frequency terms within the \bar{C}_P distribution there are again discrepancies within the evolutionary profiles predicted under full loading conditions. However as was true for the combination of pressure and surface tension loading, the profile predicted from the distributions of \bar{C}_P and \bar{C}_F determined by the DVM solver again acts as a smoothed response when this is compared with the evolutionary response resulting from distribution of \bar{C}_P based on ESDU [61] and distribution of \bar{C}_F based on Achenbach [113], the results are in excellent agreement at all times examined, as illustrated in figure 4.33.



Figure 4.32: Comparison of the evolutionary profile predicted by the distribution of \bar{C}_P determined by the DVM solver with those which result from the distribution of \bar{C}_P based on ESDU 80025 [61]. Variation of normalised film thickness h/h_0 with angle clockwise from the windward horizontal θ at $t = 6.9 \times 10^{-3}$ s, under pressure and surface tension loading.



Figure 4.33: Comparison of the evolutionary profile predicted by the distributions of \bar{C}_P and \bar{C}_F determined by the DVM solver with those which result from the distribution of \bar{C}_P based on ESDU 80025 [61] and the distribution of \bar{C}_F based on Achenbach [113]. Variation of normalised film thickness h/h_0 with angle clockwise from the windward horizontal θ at $t = 6.9 \times 10^{-3}$ s, under full loading.

Given this level of agreement, for all three loading cases, the time averaged mean distributions of \bar{C}_P and \bar{C}_F obtained numerically for the plain circular cylinder case by the DVM solver can therefore be said to act as a valid input for the thin film solver for the final coupled solver. Combining these two solvers to form which is the initial focus of the following chapter.

4.7 Summary

In conclusion it is worth reiterating several of the key points addressed within this chapter. The rationale behind the thin film approximation was outlined and the most pertinent previous literature was presented. The theory behind the pseudo-spectral method was then introduced and the reasons for its choice as the basis for the thin film solver constructed for the present application were summarised. A fourth order non-linear, non-constant coefficient PDE was then derived as the governing evolution equation for a thin film on the outer surface of a stationary horizontal circular cylinder. The salient points of the pseudo-spectral code developed to solve this were then presented.

To validate this solver within the greater RWIV context the remainder of the chapter focused on applying this to related problems with existing results for varying combinations of gravity, pressure, shear and surface tension loading. Initial cases considered problems with analytical solutions, both pre-existing [8] and newly determined. The present results being found to be in excellent temporal and spatial agreement with these, while accurately predicting failure where appropriate.

Further studies examined the effect that loading due to the external aerodynamic field had on evolutionary response were then undertaken, with four combinations of gravity, pressure, shear and surface tension loading investigated. In each instance rivulets were found to form at approximately the separation points of a dry circular cylinder [67] and to continue to increase in thickness until the thin film approximation was violated, which agreed well with the pre-
vious work of Lemaitre [9]. A comparison of the evolutionary profiles likewise showed excellent agreement where this was reported, with the minor discrepancies between these being shown to result from marginally different distributions of aerodynamic loading. To ensure the complete response of the present work was available, evolutionary surfaces of thin film profile were also given in each case for the first time. Furthermore it was determined that shear and pressure loading are of approximately equal importance in the creation of rivulets and therefore both are required within the coupled model. While in the absence of gravity the results were found to be symmetric with respect to the incident flow.

The present work also examined several other aspects of loading for the first time, notably investigating how the distribution and magnitude of aerodynamic loading and Reynolds number influence the evolutionary response. These studies found that while a variation the magnitude of the aerodynamic loading caused 'scaled' evolutionary profiles for the parameters presently examined, the profile and in particular the frequency content of the distributions of \bar{C}_P and \bar{C}_F had a significantly greater effect on the size and form of the rivulets formed. Finally the distributions of \bar{C}_P and \bar{C}_F determined by the DVM solver were found to form rivulets where appropriate and to predict evolutionary responses which agreed well with experimental data of similar spectral content. This further validated both the present work and that of the previous chapter.

Given this and the excellent agreement between the present work and that previously examined, whether numerically or analytically, in all the studies undertaken here, the present code was therefore deemed validated and to be suitable for use within a wider RWIV context. More specifically, however, for use within a coupled solver to determine rivulet formation and evolution under external aerodynamic loading, the construction and validation of which is the initial focus of the following chapter. Cases studies of the final solver are then undertaken and results presented.

Chapter 5 Coupled Solver

The DVM and pseudo-spectral method solvers of the previous two chapters are combined in the present chapter to form a coupled solver capable of predicting rivulet formation and evolution subject to an external aerodynamic field these in turn influence. Details of the coupling process and convergence studies are presented before investigations of specific loading cases are discussed.

Experimental studies [19, 39, 43, 71, 74] which consider the evolution of 'natural' rivulets have concentrated on ascertaining the conditions under which RWIV occurs and not on the exact form of these rivulets. Likewise, this is the first time such a solver has been created. Therefore specific data against which to verify either the coupled solver, or the results it predicts is unavailable. That said, both individual solvers combined to create this coupled solver were independently validated for related problems in chapters 3 and 4 respectively. Thereby generating a significant level of confidence in the present solver. This was increased when the rivulets which form under various combinations of loading from a static aerodynamic field, calculated in the previous chapter, were compared with rivulets formed under the same loading condition based on a transient, coupled aerodynamic field which were determined herein. By doing this, the effect of each of these various loads on coupled rivulet formation and evolution are ascertained, and the present results are verified with those from the independent thin film solver. Two studies investigating how the magnitude of the initial film thickness and the angle of attack in plane affect the resulting rivulets are then presented. These draw together several of the aspects previously examined and act as a 'proof of concept' for the coupled solver. As a conclusion to this chapter and to the complete body of work presented within this thesis, the geometric conditions determined to be the worst for RWIV by Gu [43] and Zhan [74] are then investigated in a final study and the results discussed.

5.1 Construction of Coupled Solver

While the pseudo-spectral solver created in chapter 4 and the changes made to the DVM code in chapter 3 were undertaken in the knowledge that these would be combined, some additional refinements were still required to allow construction of the final coupled solver. A flowchart displaying the basic operation of which is shown in figure 5.1. The most significant of these updates are now discussed.



Figure 5.1: Simplified flowchart of final coupled solver.

5.1.1 Varying Nodal Location

The individual DVM and pseudo-spectral solvers each have particular requirements for the spatial distribution of calculation points. As such the number of nodes in these can be, and typically is, different. A consequence of which is that these will be located at different spatial locations, figure 5.2. Values of parameters such as film thickness h and the stresses due to aerodynamic loading, Pand T, which are passed between the two solvers, figure 5.1, must therefore be interpolated from one set of nodal locations to another.



Figure 5.2: The nodal locations of the aerodynamic and thin film solvers may be different as the numbers of nodes used in each may also be different.

There are various methods by which such interpolation can be achieved, here however a spline based approach is used to ensure continuity of derivatives. Splines are polynomial functions between pairs of points, "whose coefficients are determined 'slightly' non-locally ... to guarantee global smoothness in the interpolated function up to some order of derivative" [142]. In the absence of the surface tension, whose effects to date have proven small, the highest derivative of the governing evolution equation (4.24) is second order. An interpolated function that is continuous through this derivative would therefore seem the logical choice. As such, a cubic spline method which is continuous through the second derivative was chosen. In addition to fulfilling this requirement cubic splines are typically stabler and less prone to oscillation than other polynomial methods [142].

The cubic spline based interpolation process used is based closely on that outlined in Numerical Recipes [142] due to its widespread usage and proven level of accuracy. This operates in two stages; first a table is constructed which contains the values of the parameter to be interpolated at each of the original nodal locations. The second stage then calculates updated values at each new nodal location using cubic splines determined from these tables. To check the accuracy of the process implemented for the problem under investigation, a series of test cases were examined, although only one is explicitly reported. Here the thin film profile obtained using 256 nodes in the pseudo-spectral solver and 360 nodes in the DVM solver was compared with the profile obtained if both solvers used 360 nodes.



Figure 5.3: Comparison of the evolutionary profile for normalised film thickness profile, at $t = 15 \times 10^{-3}$ s, under pressure and surface tension loading based on actual values at 360 nodes, and those interpolated for 256 nodes. Variation of h/h_0 with θ .

As can be seen from figure 5.3 which shows the normalised film profile for a combination of pressure and surface tension loading at a time of $t = 15 \times 10^{-3}$ s, the distributions of film thickness for both nodal combinations were effectively equivalent. The choice of this loading combination and time however was arbritrary, as with a sufficient number of nodes this level of agreement held for the complete time range and all loading combinations studied. Given this level of agreement, the interpolation routine outlined here was considered validated thereby allowing different numbers of nodes in the DVM and pseudo-spectral components of the final coupled solver.

5.1.2 Smoothing Data

As discussed in section 3.1.2, one innate limitation of vortex methods, particularly those which employ an operator splitting scheme (3.11), is that "the statistical nature of the results requires the averaging or smoothing of the velocity and pressure distributions and integrated quantities (e.g. lift and drag forces whose instantaneous values often do exhibit unrealistically large variations)" [102]. Therefore while the mean distributions \bar{C}_P and \bar{C}_F calculated in section 3.3 were smooth, at a specific instant there may be significant noise within a given distribution, C_P or C_F . Consequently after interpolating the transient aerodynamic loading determined in the DVM solver such that it can be passed to the thin film solver, this was subjected to smoothing algorithm to reduce any such noise, figure 5.4.



Figure 5.4: Simplified flowchart of coupling between two solvers.

A simple five point triangular smooth (5.1) was applied. This uses weighted functions to ascertain a smoothed value q^* at a particular node j based on the original value at the node q_j and at its neighbouring points $q_{j-2}, q_{j-1}, q_{j+1}, q_{j+2}$. This method was chosen as it is well-suited to, and easy to implement for, equallyspaced nodes. It also has a low operation count and is effective in reducing high-frequency noise. Furthermore by using an odd number of points and symmetrically balanced coefficients it further ensures that the position of any peaks or other features are maintained.

$$q_j^* = \frac{1}{9} \times \left(q_{j-2} + 2q_{j-1} + 3q_j + 2q_{j+1} + q_{j+2} \right).$$
 (5.1)

The effect that this smoothing algorithm (smoothed profile) has on the actual data (actual profile) is shown in figure 5.5 which displays the distribution of C_P at a particular, representative, instant of film evolution $t = 15 \times 10^{-3}$ s. This was chosen to correspond to the previous study highlighted in figure 5.3. Although not explicitly shown this smoothing algorithm also caused similar reductions in the underlying noise within the distribution of C_F .



Figure 5.5: Comparison of smoothed coefficient of pressure distribution with the actual distribution obtained from the DVM solver with angle from the windward horizontal at $t = 15 \times 10^{-3}$ s. Variation of C_P with θ .

5.1.3 Fourier Series Representation of Data

As was previously discussed in section 4.4.1, the performance of the pseudospectral method is greatly improved given analytic data. Therefore in addition to interpolating and smoothing data when it was passed between the DVM and thin film solvers, figure 5.4, it was also reformulated such that it could be represented through a truncated Fourier series (4.27) as was previously the case when fixed distributions of C_P and C_F were considered in chapter 4 (figure 4.4). To ensure that these new representations accurately represented the original data the number of terms n within this truncation could be varied to satisfy a specified value of R^2 (4.28) at each timestep. However to ensure that the data was not overly simplified and to remove the possibility of 'aliasing', upper and lower limits of n = 100 and n = 10 terms were imposed. Increasing the specified value of R^2 therefore gave a better representation of the underlying distribution. However as R^2 increases, the stability of the overall solution decreases. As while the smoothing algorithm of the previous section removed some of the underlying noise, figure 5.5, some of the sharp peaks in the distributions of C_P and C_F are caused by the formation of the rivulets, as was shown previously when artificial rivulets were considered in section 3.5.2. A better fit therefore requires higher frequency components within C_P and C_F to model these sharp variations. These peaks however lead to very large, very rapidly changing derivatives, which given the governing evolution equation (4.24) significantly reduce the stability of the pseudo-spectral method.

A study was therefore undertaken to determine the value of R^2 to best capture these peaks in a stable manner. At values of $R^2 \ge 0.97$ however, there were insufficient timesteps of the DVM solver to allow the effects of the thin film evolution to be studied due to the very large peaks causing very large gradients in the evolution equation (4.24) and causing the solver to fail. A value of $R^2 = 0.96$ was therefore chosen as this provided the best representation of the data while still allowing a sufficient number of timesteps.

Fulfilling the imposed R^2 criteria typically required more terms for the distribution of pressure than for the distribution of shear, due to the increased size of peaks present within C_P . Therefore while it would have been possible to apply different R^2 criteria to the C_P and C_F distributions respectively, in keeping with the work undertaken in chapter 4 when the evolution of a thin film given constant pressure and shear loading was examined, an equal criteria was used for both distributions. By comparing the distributions of C_P and C_F modelled with the respective smoothed distributions and the distributions actually calculated by the DVM solver it was found that these agreed both quantitively and



Figure 5.6: Comparison of modelled coefficient of pressure distribution with smoothed distribution and actual distribution obtained from the DVM solver with angle from windward horizontal at $t = 15 \times 10^{-3}$ s. Variation of C_P with θ .

qualitatively except at very sharp peaks; which although captured were done so at a reduced magnitude. A representative example of the three distributions of C_P are shown in figure 5.6 at the time previously used to examine the effect of smoothing, figure 5.5.

To confirm that the modelled distributions of C_P and C_F gave an accurate representation of the actual distributions calculated by the DVM solver further studies were undertaken. The results for both C_P and C_F were similar but as the 'sharper' C_P distribution again represented the more difficult case, only these results are explicitly reported.

This first study undertaken tracked the number of terms n required to ensure the R^2 criteria was satisfied at every timestep. As can be seen from figure 5.7 while the number of terms varied between individual timesteps it typically fell within the range 12 < n < 20. This figure also illustrates that the coefficient of determination between the actual data (calculated by the DVM solver) and the smoothed data (once it had passed through the 5 point triangular smoothing algorithm) remained high, $R^2 > 0.995$, and approximately constant throughout the interval studied figure 5.7 proving that the smoothing algorithm did not affect the underlying distribution of C_P .



Figure 5.7: Comparison of R^2 between the actual distribution of C_P obtained by the DVM solver and that modelled, and between the distribution of C_P before and after smoothing. Variation of R^2 and number of terms n in truncated Fourier series representation required to achieve this with timestep.

The second study was undertaken to ensure that there were no significant differences between the actual, smoothed and modelled mean loading distributions (\bar{C}_P and \bar{C}_F), as while instantaneous differences are inherent the same mean distribution should be applied in all cases. To limit the effect of any initial transience present within the response and to ensure that this mean was based over a representative time interval a value of t = 0.5 s was chosen. Specifics regarding the details of this \bar{C}_P distribution are discussed in section 5.4.2, suffice to say that for present purposes the discrepancies illustrated in figure 5.8 were negligible for pressure loading, while shear loading was closer still. Therefore the mean the distributions modelled in the thin film solver, \bar{C}_P and \bar{C}_F , were the same as those actually calculated in the DVM solver.

The final study quantified the difference between the distribution actually determined by the DVM solver and that obtained after the smoothing algorithm was applied and between the actual distribution and that modelled, figure 5.4. To do this two common statistical measures were calculated. These were the mean \bar{q} and standard deviation σ_q of the difference between the smoothed q^* or modelled value q_m and the actual (DVM) value q at every point j over a given number



Figure 5.8: Comparison of mean coefficient of pressure distributions obtained by the DVM, the smoothed distribution and the distribution modelled. Variation of \bar{C}_P with θ .

of timesteps N. Where these are given in equation (5.2) in terms of difference between the smoothed value and the actual value but could equally be given in terms of difference between the modelled value and the actual value by replacing q^* by q_m . These values were then normalised using the maximum mean value for that particular loading distribution such that comparisons between pressure and shear could be performed. As can be seen from figure 5.9 which illustrates the normalised mean difference between the actual distribution calculated by the DVM solver and those modelled for both C_P and C_F , pressure loading was once again found to be the tougher case. The discussion to follow will again therefore focus on this, although the underlying trends are similar for shear loading.

$$\bar{q} = \frac{1}{N} \sum_{j=0}^{N} q_j - q_j^* \qquad \sigma_q = \sqrt{\frac{1}{N} \sum_{j=0}^{N} (q_j - q_j^*)^2}.$$
 (5.2)

As expected the agreement between the distribution actually calculated by the DVM solver and the distribution once this had been smoothed was better than that between the distributions actually calculated and that modelled for both measures, figure 5.10. This is because the differences between the former are the result of smoothing existing data whereas the discrepancies in the latter are due



Figure 5.9: Comparison of normalised mean difference between modelled and actual loading distributions obtained from the aerodynamic solver for shear and pressure loading conditions. Variation of \bar{q} with θ .

to a truncated Fourier series representation of the values themselves. That said the typical mean error between the actual and modelled distributions of C_P is still very small $\ll 1\%$, except in the regions $60^\circ < \theta < 90^\circ$ and $270^\circ < \theta <$ 300° , where pressure 'jumps' due to rivulet formation occur. While these 'jumps' are modelled they could not be fully captured due to extremely rapid changes involved, as was discussed in section 5.1.2. It is these differences which cause the increase in both the normalised mean and standard deviation. However given that these differences were necessary for stability and that the normalised σ_q is typically less than $\simeq 2\%$, this modelled distribution was considered acceptable. Furthermore the limitations of the present process are in-line with the 'meancycle' approach used by Cosentino [19] (section 2.4.4) which also smooths such pressures fluctuations, and is the only other paper to consider such transience in a RWIV context. That said better capturing these sharp peaks in a more stable manner is an area which should be considered for future revisions of the present code.

It should be re-iterated that when film thicknesses were passed from the pseudo-spectral to the DVM solver only interpolation was required due to the inherent smoothness of the data, section 4.4.1. Whereas due to the noise and rapid



Figure 5.10: Comparison of statistical differences between both the smoothed and modelled distributions of C_P and the original distribution obtained by the DVM. Upper: Mean difference between distributions. Lower: Standard deviation in difference between distributions.

variations present in the distributions of C_P and C_F when these were passed from the DVM to the pseudo-spectral solver, interpolation, smoothing and a Fourier series representation were all undertaken, figure 5.4.

5.1.4 Transient Aerodynamic Loading

Just as the DVM solver was previously updated such that it could model small variations in body geometry, section 3.2.3, so the pseudo-spectral solver was also updated such that it could model varying aerodynamic loading. Previously spatial derivatives of C_P and C_F were only calculated once at the outset of a run of

the pseudo-spectral solver as these distributions remained constant with time, now however as these change with time the pseudo-spectral solver was altered such that spatial derivatives could be calculated at each timestep new data was input. For the coupled solver this after these distributions had been interpolated, smoothed and represented as a truncated Fourier series at the end of each timestep of the DVM solver as illustrated in figure 5.4. As such, the updated solver took longer to run for a given number of timesteps.

To ascertain the magnitude of this increase in time to solve a given number of timesteps a study was undertaken. For simplicity this used the pseudo-spectral solver running independently but performing the differentiation of a constant aerodynamic loading distribution at every timestep rather than just once at outset. The C_P and C_L distributions used were the same twenty term Fourier series truncations (4.27) determined in section 4.4.1 which were based on the experimental data of Achenbach [113]. Each of the three possible combinations of aerodynamic loading were considered; shear only, pressure only and both pressure and shear. In each instance while there was an increase in time to solve, the evolutionary film thickness profile predicted was the same as that predicted for a single differentiation at outset under similar loading. The derivatives calculated at every timestep were also found to remain fixed. Both of which highlight the repeatability of the method used.

As can be seen from figure 5.11 and table 5.1, which illustrate each of these three loading combinations individually, the increase in time to solve per timestep for both shear only and pressure only loading is approximately constant and approximately the same in each case, $\simeq 235\%$ for 128 nodes. While when both shear and pressure loading were simultaneously applied this increase in time to solve per timestep was again constant, here however the difference being approximately double that of either the shear or pressure only loading cases, $\simeq 455\%$. This was a direct consequence of two distributions being read in and differentiated at each timestep rather than one. That said if no aerodynamic



Figure 5.11: Comparison of the time to solve a given number of timesteps for differentiation once at outset with the time to solve for differentiation at every step, for three different combinations of aerodynamic loading. Upper: Shear only. Middle: Pressure only. Lower: Shear and pressure.

loading was considered, i.e. only gravity and surface tension loading, then there was no increase in time to solve.

Number of	Increase in time to solve per timestep					
Timesteps	Shear Only, $\%$	Pressure Only, $\%$	Shear and Pressure, $\%$			
50×10^3	230.4	236.8	457.9			
100×10^3	226.8	236.3	457.0			
200×10^3	228.4	238.2	452.3			
300×10^3	231.9	241.0	455.6			
400×10^3	232.0	241.6	454.5			
500×10^3	232.2	242.9	464.6			

Table 5.1: Increase in time to solve per timestep for differentiation at every timestep rather than once at outset, under different combinations of aerodynamic loading; shear only, pressure only and shear and pressure.

In practice as the pseudo-spectral solver effectively undergoes a separate run every time it is called, figure 5.1, the coupled solver only requires one differentiation per DVM timestep, provided this is sufficiently small. The previous study therefore represents the worst case scenario and amplifies the increase in time to solve per timestep over that which would practically occur. Furthermore in comparison with the time to solve a timestep of the aerodynamic solver the increase caused by the present update is very small.

5.1.5 Area Conservation

The updated DVM solver was previously verified to ensure that total area modelled is conserved in section 3.2.3, while in validation of analytical surfaces the pseudo-spectral solver was likewise verified for conservation of fluid in section 4.5.1. However given the updates made here as part of the coupling process including interpolating film thicknesses between nodal locations, which although demonstrated to be negligible in section 5.1.1, may mean that total area is no longer conserved. Therefore to ensure that it was, an additional refinement was made to the final coupled solver.

Once the thickness of the film h has been calculated and interpolated onto the nodes of the DVM solver at the end of each run of the pseudo-spectral solver, figure 5.1, this distance is added to the cylinder radius R to determine an updated magnitude of the position vector of each body node $||\mathbf{r}||$. As the direction of the position vector of each node is fixed such that the location of any node can only be altered radially (thus ensuring that the distribution of nodes remains approximately constant and equal) the new body surface can be calculated using the unit vector of each node $\hat{\mathbf{r}} = \frac{\mathbf{r}}{||\mathbf{r}||}$ and the updated magnitude $||\mathbf{r}|| = R + h$, as shown in figure 5.12. The total area of the body can then be calculated, as the sum of a series of triangles from the cylinder centre (which is also the body reference node). This process is also illustrated schematically in figure 5.12.



Figure 5.12: Process of nodal location for area conservation. Left: Original body nodes and free surface nodes. Centre: Area calculation. Right: Updated free surface nodal location to conserve area.

The update implemented calculates the difference between the present area Aand the original area A_0 (for uniform thin film thickness) such that the magnitude of each nodal position vector is further offset, h_{eff} , by an additional equal amount to eliminate this. This offset is calculated via the assumption that any difference in area, $A - A_0$, can be represented in as rectangle of length $2\pi(R + \bar{h})$ and height h_{eff} , where \bar{h} is the mean thickness of the film for a given number of nodes N, $\bar{h} = \frac{1}{N} \sum_{j}^{N} h_{j}$. This additional thickness is therefore given as

$$h_{eff} = \frac{A - A_0}{2\pi (R + \bar{h})},$$
(5.3)

while its implementation into the coupled solver is illustrated schematically in figure 5.12. As can be seen from figure 5.13, although the magnitude of this additional offset does increase marginally with time it is still negligible in comparison with the magnitude of the original position vector $R + h_0$ and is therefore many orders of magnitude less than the original film thickness, $h_0 = 0.25 \times 10^{-3}$ m in the example given. Which was used in conjuction with a radius of R = 0.08 m to correspond to the parameters used in the final studies as outlined in section 5.3. Total area can therefore seen to be preserved within the coupled solver, figure 5.13.



Figure 5.13: Variation in normalised body area and position vector with timestep for deformable geometry.

5.2 Solver Parameter Selection

Before the coupled solver could be used, a series of convergence tests were required to ensure the accuracy of the final solution. Several important parameters such as the number of nodes N and timestep size Δt of the aerodynamic and thin film portions of the combined solver were examined. In each case using the values determined in sections 3.3 and 4.4.1 for the DVM and pseudo-spectral solvers running independently as the baseline.

The rivulets expected to evolve 'naturally' herein should represent an intermediate case between the plain circular cylinder and that with much larger artifical rivulets previously modelled in sections 3.3 and 3.5 respectively. Given that the DVM solver proved successful in both cases and that the pseudo-spectral solver effectively operates as a sub-routine of this, being called once per timestep as shown in figure 5.1, parameters specific to the DVM code such as those related to vortex merging were kept constant, as was the number of nodes within the aerodynamic portion of the coupled solver which was maintained at the previous value of N = 360. This ensured a distribution of one node per degree which was thought prudent to ensure that a distribution of aerodynamic loading could be determined over any rivulet which formed, particularly as the 'natural' rivulets expected to form here are predicted to be narrower than typical artificial rivulets from previous studies which were approximately 8°, table 3.5.

Having fixed the number of aerodynamic nodes, a study was undertaken to ascertain whether the timestep size should be altered from the $\Delta t^* = 0.01$ determined in section 3.3. Various values in the range $0.005 \leq \Delta t^* \leq 0.05$ were examined for both pressure and shear loading. As these were found to be very similar, only the results for pressure loading are explicitly reported. Figure 5.14 plots the normalised film thickness profile h/h_0 at $t = 15 \times 10^{-3}$ s for a representative selection of these. The choice of which was again arbitrary and was selected to match that used in figures 5.3, 5.5 and 5.6. As figure 5.14 illustrates, at values less than or equal to $\Delta t^* = 0.01$ the differences between these timestep sizes were found to be negligible; the original value of $\Delta t^* = 0.01$ was therefore used.

As the DVM solver is impulsively started an initial startup period is required for these effects to become negligible. However starting the coupled solver at different points in the shedding cycle means the aerodynamic loading to which



Figure 5.14: Comparison of the evolutionary profile for normalised film thickness profile, at $t = 15 \times 10^{-3}$ s, under only pressure loading for various non-dimensional timestep sizes Δt^* . Variation of h/h_0 with θ .

the film is initially subjected varies. Therefore to eliminate the latter and to minimise the effect of the former, a fixed number of steps was allowed to run in every case before the first coupled timestep was undertaken. Again the result determined in section 3.3 for the independent DVM solver was used, at which point the time was effectively reset to zero.

For the independent pseudo-spectral solver 128 nodes were found to be sufficient to determine the evolutionary profile of film thickness. However for the present purposes, this corresponds to approximately one pseudo-spectral node for every three DVM nodes. To ascertain whether this was sufficient for the present purpose, various values of 2^n nodes were examined for the same conditions as the previous timestep size investigation, i.e. the film thickness distribution at $t = 15 \times 10^{-3}$ s for pressure only loading; 2^n nodes being chosen for optimal performance of the FFT, section 4.4.1. As can be seen from figure 5.15 the distributions of film thickness with 256 nodes or greater were effectively equivalent. In light of the interpolation procedure implemented (section 5.1.1), which performs better with more nodes near the point to be interpolated and such that more than one node per degree was modelled, the number of thin film nodes was set as the higher value of 512. By comparing figures 5.14 and 5.15 further confidence as to the selection of these two parameter values was achieved due to the similarity of the two results shown.



Figure 5.15: Comparison of the evolutionary profile for normalised film thickness profile, at $t = 15 \times 10^{-3}$ s, under only pressure loading for various numbers of thin film nodes N. Variation of h/h_0 with θ .

Given that the number of pseudo-spectral nodes was increased, a similar study to that undertaken in table 4.3 for the timestep of the independent thin film solver was performed. This revealed that the timestep size of the original thin film solver should be, and subsequently was, reduced from $\Delta t = 0.5 \times 10^{-6}$ to $\Delta t = 0.5 \times 10^{-7}$ s to accommodate the increased number of nodes. Although the pseudo-spectral and DVM solvers operate in different time scales, these are linked by the relation $t^* = \frac{tU}{D}$. As the values of all these variables are known the number of timesteps run by the pseudo-spectral solver for every timestep of the DVM solver is automatically calculated. For typical values of U = 11 m/s and D = 0.16 m this equates to approximately 3000 steps of the pseudo-spectral solver.

To measure the impact that coupling the pseudo-spectral and DVM solvers and the additional changes implemented herein have had on overall performance, the time to solve a given problem for a given number of aerodynamic timesteps for the final coupled solver was compared to the original DIVEX solver. In keeping with parameter values determined within this section, the problem in question is



Figure 5.16: Variation in time to solve 5000 timesteps with DIVEX solver, Updated DVM solver, Updated DVM + Thin Film solver and coupled solver.

a basic circular geometry of 360 aerodynamic nodes for 5000 timesteps of $\Delta t^* = 0.01$ of the DVM solver. While 512 nodes and a timestep of $\Delta t = 0.5 \times 10^{-7}$ s was used in the thin film portion of the coupled solver. As can be seen from figure 5.16 and table 5.2, coupling the two solvers has caused an increase in time to solve this problem of approximately 14.4% over the updated DIVEX solver used in chapter 3 and consequently a 36.5% increase over the original DIVEX solver. However, bearing in mind that for these 5000 timesteps the solver effectively runs 1.5×10^7 thin film timesteps for 512 nodes, when the time to undertake 3000 such timesteps of the thin film solver at each timestep of the DVM solver is added to that obtained from the updated DIVEX solver (updated solver + thin film) then the cost of the coupling process itself is reduced to 3.4%. Given that the times represented in figure 5.16 represent the worst case when both pressure and shear loading are considered, then the time spent within the coupling process was considered acceptable within the overall context. As was the overall increase in time to achieve the final coupled solver.

Number of	Increase in time to solve per timestep of Coupled Solver over					
Timesteps	DIVEX, $\%$	Updated DVM, $\%$	Updated DVM + Thin Film, $\%$			
1000	31.8	10.5	-0.1			
2000	32.3	11.2	0.3			
3000	33.5	12.6	1.7			
4000	35.3	13.9	3			
5000	36.4	14.4	3.4			

Table 5.2: Increase in time to solve per timestep of coupled solvers over previous solvers, DIVEX, the Updated DVM solver and the Updated DVM + Thin Film solver.

5.3 Physical Parameter Selection

I

Within the previous two chapters the physical conditions studied were selected such that the solvers created could be validated against previous literature. Particular values being chosen to match previous studies [9, 80]. The coupled solver however is not so restricted. That said, given that the fluids under consideration were still air and water, with the exception of two changes, the values used were those given previously in table 4.4.

The first change was to model a Reynolds number of 100×10^3 . This was chosen as it represented the mid-point of the typical range for RWIV, $50 \times 10^3 < Re < 150 \times 10^3$, outlined in section 2.2. That said, given the Reynolds number independence in the TrSL3 flow regime discussed in section 2.1.1 and the values examined in the previous chapters, $Re = 20 \times 10^3$ and $\simeq 100 \times 10^3$ respectively, this was not expected to significantly affect the solution.

The second change was to halve the initial film thickness h_0 from 0.5×10^{-3} m to 0.25×10^{-3} m in order to better represent the physical film measured by Cosentino [19]. This found a rivulet of thickness approximately 0.55×10^{-3} m forming on a thin-film of $0.2 - 0.25 \times 10^{-3}$ m. As the value of cylinder radius was previously fixed at R = 0.08 m to match the full size cable used in this study,

section 4.4.1, this should minimise any effects of scale and allow a more direct comparison of variables such as rivulet thickness.

5.4 Results

To ascertain whether rivulets do indeed form using distributions of C_P and C_F based on the evolving cross-section geometry and to determine the differences between these results and those for fixed aerodynamic loading, specifically that of a dry cylinder, four loading combinations were examined. These differed by means of which loads were passed between the pseudo-spectral and DVM solvers and were chosen to match the combinations studied in sections 4.6.2 to 4.6.4 of the previous chapter. Specifically these were:

- 1: Shear and surface tension $(P \equiv 0 \text{ and } g = 0)$
- 2: Pressure and surface tension $(T \equiv 0 \text{ and } g = 0)$
- 3: Shear, pressure and surface tension (g = 0)
- 4: Full loading $(P, T, g \text{ and } \gamma \neq 0)$

By choosing these cases the individual effects of shear and pressure loading on coupled rivulet formation could be studied independently, before the combined effect with and without gravitational loading was considered. The former case representing more realistic physical loading conditions.

5.4.1 Shear and Surface Tension Loading

Figure 5.17 displays the temporal evolution of film thickness under the combination of shear and surface tension loading. From this three points become apparent. First, as was found previously when this combination of loading was examined for a fixed C_F distribution, section 4.6.2, two larger rivulets formed quickly at approximately the separation points of a dry circular cylinder, one each on the upper and lower surface. While the initial locations of these, $\simeq \theta = 83^{\circ}$ and



Figure 5.17: Numerical prediction of temporal evolution of film thickness in real time, under shear and surface tension loading. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

 $\simeq \theta = 277^{\circ}$, are marginally leeward of those which formed under the fixed C_F distribution based on Achenbach [113], they are in excellent agreement with the locations of rivulets which formed using the fixed C_F distribution based on the time averaged mean of the DVM code, section 4.6.5. Rivulet location herein being defined as the point of maximum film thickness clockwise from the windward horizontal θ . This difference can therefore be attributed to the use of a different distribution of C_F as was shown graphically in figure 4.31 for the fixed loading case.

Secondly the growth rate, maximum thickness and form of both the upper and lower of these larger rivulets are approximately equal, figure 5.18. Even the temporal movement leeward of the points of maximum thickness does so symmetrically with respect to the mean stagnation point of the incident flow, here the windward horizontal, figure 5.19. This leeward progression occurring as the shear load acts tangentially to the surface. As such, once rivulets form they are continually 'pushed' away from the stagnation point of the incident flow. This causes the point of maximum thickness to move from $\theta \simeq 83^{\circ}$ to $\theta \simeq 93^{\circ}$ on the upper surface and from $\theta \simeq 277^{\circ}$ to $\theta \simeq 267^{\circ}$ on the lower surface. The latter is shown graphically in figure 5.19 in terms of the angle anti-



Figure 5.18: Comparison of the temporal evolution of normalised film thickness of upper and lower rivulets under shear and surface tension loading. Variation of h/h_0 with t.



Figure 5.19: Comparison of temporal evolution of rivulet location of the upper and lower rivulets from the windward horizontal under shear and surface tension loading. Upper rivulet, clockwise from this point, lower rivulet anti-clockwise from this point. Variation of θ and θ_{ac} with t.

clockwise from the windward horizontal θ_{ac} such that the location of the lower rivulet can be directly compared with that of the upper rivulet. As the quantity of fluid within the film is fixed, this action causes the thickness of the film on the windward face to decrease with time and that in the rivulets and on the leeward face to consequently increase. Figure 5.20 highlights the former by illustrating the temporal evolution of normalised film thickness at the windward horizontal, $\theta = 0^{\circ}$. Therefore while there are small temporal variations in both the thickness and location of the upper and lower rivulets, even the evolution of two rivulets can be said to occur in a symmetric manner. The proof of this common sense result instilled confidence in both the solver and the results obtained.



Figure 5.20: Evolution of normalised film thickness with time of thin film at the windward horizontal ($\theta = 0^{\circ}$), under shear and surface tension loading. Variation of h/h_0 with t.

Finally and most importantly, as can be seen from figure 5.18 the maximum thickness of both these rivulets is now self limiting. This is in direct contrast to the results determined in section 4.6.2 when the same combination of loading was examined and where rivulet thickness continued to increase until the thin film approximation was violated. Unlike the previous study however which used a fixed distribution of C_F and that did not account rivulet formation, the distribution of C_F used herein does account for the growth of the rivulet and therefore changes with time. As a result the rivulets which develop under an external aerodynamic field they influence grow rapidly to a thickness of $\simeq 0.65 \times 10^{-3}$ m ($\simeq 0.65$ mm) before remaining of approximately constant thickness. This is a major finding and better reflects the real rivulets of limited thickness which have been found 'in-situ' under RWIV conditions and those which have been found experimentally in the wind-tunnel [45, 82, 93]. The maximum thickness of which, measured experimentally under full loading conditions by Cosentino [19] as $h \simeq 0.55 \times 10^{-3}$ m, is qualitatively in-line with that determined here.

Comparing the location and thickness of the present rivulets with those that evolve using a fixed distribution of C_F (specifically \bar{C}_F determined by the independent DVM solver for a plain cylinder) which was examined in section 4.6.5, instils further confidence by virtue of the similarity of these findings. Figure 5.21 illustrates this comparison for the upper rivulet, although due to the symmetry previously discussed, the findings for the lower rivulet were very similar. The results of figure 5.21 highlight that while both the location and the rate at which the rivulets evolve are comparable the maximum thickness is not. This is due to the maximum thickness of rivulet formed under a fixed distribution of C_F continuing to grow until it violates the thin film approximation. The excellent agreement between the growth rate and the location of this rivulet though are cause for confidence. Validation however cannot be undertaken as this is the first time coupled rivulet formation has been studied and no data with which to verify the present findings is available.

To determine what effect the formation of these rivulets had on the overall aerodynamic response, time averaged aerodynamic parameters for the present case were compared with those determined previously for a plane circular cylinder, section 3.3, and the symmetric configuration of artificial rivulets at $\theta = 90^{\circ}$, section 3.5.3. The latter being chosen to best represent the location of the rivulets found to form here which varied with time. As table 5.3 illustrates, the present results for both \bar{C}_L and C'_L are very similar to the plain circular cylinder, with no net lift predicted in both cases. There is however a marginal reduction in



Figure 5.21: Comparison of temporal evolution of upper rivulet location and normalised thickness formed under shear and surface tension loading between the present coupled solver and the previously examined un-coupled solver. Variation of θ and h/h_0 with t.

 \overline{C}_D . Which in addition to the result from pressure loading case will be discussed when this is examined in the following section. That said, spectral analysis of the present results revealed a single strong peak at an unchanged Strouhal number, which again agreed well with the plain cylinder case. This indicating that the frequency of Karman vortex shedding frequency was unaltered by the rivulets which form under this combination of loading.

	C'_L	\bar{C}_L	\bar{C}_D	St
Present Shear Loading	0.524	0.0013	1.153	0.208
Present Pressure Loading	0.512	-0.0071	1.104	0.206
Plain Cylinder	0.519	0.0004	1.254	0.211
Artificial Rivulet	0.509	0.012	1.441	0.198

Table 5.3: Comparison of C'_L , \bar{C}_L , \bar{C}_D and St of present shear loading and pressure loading results with those for the plain circular cylinder and symmetric artificial rivulet at $\theta = 90^{\circ}$ cases.

As the thickness of rivulets which form in the present case ($\simeq 0.65 \times 10^{-3} \text{ m}$) are significantly smaller than the artifical rivulets studied ($4.8 \times 10^{-3} \text{ m}$), and given that any change in the present surface profile is smooth (without sharp corners), it is perhaps unsurprising that the present results closely match those of the plain cylinder. Furthermore when the thickness of the present rivulets is normalised with respect to the cylinder diameter (0.004D) these are found to be below the limit of 0.007D found in section 3.5.5 at which the DVM solver can definitively detect the effect of an artificial rivulet. Therefore within the margins of error of the present solver the formation of the rivulets under this combination of loading has negligible effect on the overall aerodynamic response of the body.

5.4.2 Pressure and Surface Tension Loading

The temporal evolution of film thickness for a combination of pressure and surface tension loading can be seen in figure 5.22. This highlights many of the same features of the previous shear loading case. Most notably that two symmetric rivulets of self limiting thickness were found to form marginally windward of the separation points of the dry cylinder, at $\theta \simeq 73^{\circ}$ and 287° respectively, although two other rivulets were also found in the wake. That said however, the rivulets formed at approximately the separation points under pressure loading were once again found to be located just windward of those formed from shear loading as was determined for evolution due to fixed distributions of C_P and C_F , figure 4.19.

Studies into the location, growth rate and thickness of the two rivulets at approximately the separation points of the dry circular cylinder confirmed that those present on the upper and lower surfaces were once again symmetric images of one another. Figure 5.23 illustrates the latter two points by comparing the temporal evolution of the upper and lower rivulets' maximum thickness. By comparing this with the temporal evolution of the maximum thickness of the rivulets which form under shear loading, figure 5.18, it can be established that while the magnitude of the rivulets' self limiting thickness are similar, $\simeq 0.72 \times 10^{-3}$ m from pressure loading as opposed to $\simeq 0.65 \times 10^{-3}$ m from shear loading, the rates at which these evolve are not. The maximum thickness of the rivulets which evolve due



Figure 5.22: Numerical prediction of temporal evolution of film thickness in real time, under pressure and surface tension loading. Variation of h with θ and t. Where θ is measured clockwise from the windward horizontal.

to pressure loading from the external aerodynamic field developing considerably faster than the maximum thickness of those forming due to the shear loading this causes. This is in good agreement with the results of the previous chapter when fixed distributions of C_P and C_F were used, figure 4.19. As are the sharper peaks and noise present in figure 5.22 due to larger pressure derivative terms within the evolution equation (4.24) previously discussed.



Figure 5.23: Comparison of the temporal evolution of normalised film thickness of upper and lower rivulets under pressure and surface tension loading only. Variation of h/h_0 with t.

Unlike the shear loading case however while the upper and lower rivulets do form at approximately the same angle from the windward horizontal, clockwise and anti-clockwise respectively, the location of maximum film thickness does not vary significantly with time; remaining approximately constant at $\theta \simeq 73^{\circ}$. Where because pressure acts normal to the surface, despite some local movement of fluid to allow for the changes in film thickness, there is not the same net flow from the windward to the leeward face that was found in the shear case, figure 5.20.

By comparing the location and thickness of the rivulets formed using the present coupled solver with those determined using a fixed distribution of C_P (specifically \bar{C}_P determined by the independent DVM solver for a plain cylinder) as was previously done for shear loading, the present findings and findings therein were further confirmed. Most notably, while the location and the initial growth rate of the rivulets which evolve are similar, the former being fixed in both cases, the maximum thicknesses are distinctly different. Where the rivulets formed under a fixed pressure distribution continued to increase in thickness until the thin film approximation was violated while those formed under a pressure distribution that varied accounting for the rivulet growth maintained an almost constant thickness once formed. Figure 5.24 illustrates this comparison for the upper rivulet, although a comparison of the lower rivulet is very similar because of the inherent symmetry. The magnitude of this self limiting rivulet thickness is again comparable with those determined by Cosentino [19] for the same cable dimensions.

As previously shown in table 5.3, \bar{C}_L and C'_L determined in the presence of these self limiting rivulets under this combination of loading were comparable to those obtained from a plain circular cylinder. \bar{C}_D is however once again reduced in comparison. This is attributed to the lower back pressure C_{pb} determined in the present distribution of \bar{C}_P in comparison with that determined by the DVM solver for a plain cylinder, figure 5.25. As although there are considerable variations and 'jumps' in instantaneous C_P the time averaged mean (\bar{C}_P) is similar to that



Figure 5.24: Comparison of temporal evolution of upper rivulet location and normalised thickness formed under pressure and surface tension loading between the present coupled solver and the previously examined un-coupled solver. Variation of θ and h/h_0 with t.



Figure 5.25: Comparison of the distributions of time averaged mean coefficient of pressure determined under pressure and surface tension loading by the coupled solver loading with that determined for a plain circular cylinder by the independent DVM solver. Variation of \bar{C}_P with θ .

of a circular cylinder. The reduction in C_{pm} is attributable to the formation of rivulets in this location. While the formation of a second smaller rivulet, as can be seen in 5.22, likewise causes a second smaller minimum of \bar{C}_P whose location corresponds to this rivulet. From the data generated from the present solver this is thought to arise due to the flow re-attaching to the body after the first rivulet before separating once again to form the second rivulet, although without experimental data to determining whether this a real or numerical effect is impossible and as such this is a tentative interpretation. It does however appear to be this minimum which causes the smaller C_{pb} and thus \bar{C}_D . What is definite is that \bar{C}_P is symmetric with respect to the incident flow. This further confirms previous results concerning the symmetric formation of rivulets under pressure loading, figure 5.25. A similar result is found for the shear case although this is not explicitly shown.



Figure 5.26: PSD for pressure and surface tension loading, indicating St peak.

Through examination of the PSD, figure 5.26, it can be determined that for the combination of pressure and surface tension loading the Strouhal number and therefore that the Karman vortex shedding frequency is unaltered. As such, the results for C'_L , \bar{C}_L , \bar{C}_D and St are all in excellent agreement with the previous shear loading. Given that the normalised thickness of the self limiting rivulets formed here, 0.0045D, are again below the limit of 0.007D found in section 3.5.5 for which the DVM solver can definitively detect the effect of an artificial rivulet, this is unsurprising. The combination of pressure and surface tension loading, like the combination of shear and surface tension loading previously examined, was therefore found to have negligible effect on the body's response.

5.4.3 Combined Pressure and Shear Loading

Having studied the effects of both shear and pressure loading separately, a study for both loadings acting in combination with surface tension was undertaken. As the results themselves were found to be similar to the previous two independent cases only a brief summary is presently given.



Thickness of Film, h,(m)

Figure 5.27: Numerical prediction of temporal evolution of film thickness in real time, under shear and surface tension loading. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

As expected given the symmetry of the rivulets which formed in the individual loading cases, the rivulets which formed under a combination of pressure and shear were again found to be approximately symmetric with respect to the incoming flow, figure 5.27. While once again the rivulets formed were found to be of self limiting thickness; the maximum magnitudes of which were very similar to the pressure loading case, figure 5.28. The initial growth rate of these rivulets however was marginally quicker than the corresponding growth rate for the independent pressure case as shear loading now works in combination with this to form the rivulet and as such the rivulets under combined loading evolve more quickly. Likewise these rivulets were found to occur at an intermediate location between where the rivulets were found to form in the independent pressure and shear cases, figure 5.29. That said the exact locations of these rivulets at $\theta \simeq 77^{\circ}$ and $\theta \simeq 283^{\circ}$, were once again found to be closer to the locations of those formed under only pressure and surface tension loading. Likewise these locations of maximum rivulet thickness were also found to remain approximately constant with time, which is also in closer agreement with the pressure loading case than the shear loading case. Therefore while it can be said that both pressure and shear loading play a role in the evolution of these rivulets, it appears that pressure has greater influence.



Figure 5.28: Comparison of the temporal evolution of normalised film thickness of upper rivulet for the shear, the pressure and the combined shear and pressure loading combinations. Variation of h/h_0 with t.

It should be noted that while the upper rivulet was illustrated in figures 5.28 and 5.29, as the rivulets were symmetric the results of the lower rivulet were again very similar. The choice, as in the previous two loading cases examined, was arbitrary and the upper rivulet was presently selected to be consistent with these. Furthermore although not explicitly reported the present combination of pressure, shear and surface tension loading was also found to have negligible effect on overall response. The consistency between these three cases in conjunction
with the other points discussed is again cause for confidence in the solver and the results it determines.



Figure 5.29: Comparison of temporal evolution of upper rivulet location clockwise from the windward horizontal, for the shear, the pressure and the combined shear and pressure loading combinations. Variation of θ with t.

5.4.4 Full Loading

Given that the coupled solver is two-dimensional and that the governing evolution equation (4.24) was derived for a horizontal cylinder, the full loading case to be examined here essentially represents the physical loading on a horizontal cable perpendicular to the incoming flow, i.e. $\alpha = \beta = 0^{\circ}$. By comparing the evolutionary profile of film thickness for the present case, figure 5.30, with those from the previous three loading combinations, significant differences become apparent.

The most noticeable of these differences being that, as expected, the symmetry of the three previous cases examined is lost due to the effect of gravity. The 'noise' present when pressure loading was considered either individually or in combination with the shear loading is also considerably lessened. In combination with the lack of symmetry, this indicates that gravity has a stronger influence than either of the loadings due to the external aerodynamic field (pressure and shear) for the present conditions, although these do still play a role. Therefore while a distinct rivulet can be seen to form on the lower surface at approximately



Figure 5.30: Numerical prediction of temporal evolution of film thickness in real time, under full loading conditions. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

the lowest point on the cylinder, $\theta \simeq 277^{\circ}$, the temporal evolution of the upper surface is more complicated and necessitates more detailed review. These two surfaces are therefore discussed separately.

As can be seen from figure 5.31 the thickness of this lower rivulet, like those of the previous loading cases investigated, is self limiting. In this instance at approximately 0.68×10^{-3} m which is consistent with previous results and is quantitatively in-line with the upper rivulet measured experimentally by Cosentino [19]. A comparison with the experimental values for lower rivulet thickness cannot be made however, as to the author's knowledge at the time of writing no such measurement had been undertaken. Furthermore as this rivulet forms on the lower surface, due to the additional effect of gravity this evolves more quickly than any of the previous loading combinations examined, figure 5.31.

The location of this rivulet as defined by the point of maximum thickness has likewise moved leeward (towards the lowest point) from the combined shear and pressure case due to the effect of gravity. This is illustrated in table 5.4 which gives the time averaged mean position $\bar{\theta}_{ac}$ of the rivulets which form under each of the four loading cases considered, in terms of the angle anti-clockwise from the windward horizontal. Although using an anti-clockwise angle is a departure



Figure 5.31: Comparison of the temporal evolution of normalised film thickness of lower rivulet for the full, combined shear and pressure, shear only, pressure only loading cases. Variation of h/h_0 with t.

from convention, this format was chosen to highlight that in all cases the rivulet still formed on the windward side of the lowest point. While the mean of this angle, $\bar{\theta}$, is used to account for the temporal leeward progression of the rivulets formed under only shear and surface tension loading. By determining that the rivulet which forms on the lower surface evolves more quickly and has moved leeward from that formed under similar loading but in the absence of gravity, and through the consistency of maximum rivulet thickness determined in all four loading combinations examined, additional confidence is gained in the coupled solver and the results it predicts.

	Full	Pressure and Shear	Shear	Pressure
Angle Anti-Clockwise, $\bar{\theta}_{ac}$	83°	77°	88°	73°

Table 5.4: Comparison of the mean angle anti-clockwise from the windward horizontal $\bar{\theta}_{ac}$ at which the lower rivulet formed under full loading with the combinations of loading previously examined; combined pressure and shear, shear only and pressure only.

While the lower rivulet is easy to distinguish, the evolution of the thin film on the upper surface is considerably more complicated. A small rivulet does



Figure 5.32: Temporal evolution of film thickness at a location 67° clockwise from the windward horizontal under full loading conditions. Variation of h with t.

periodically form on the upper surface at approximately $\theta = 67^{\circ}$, as can be seen from figure 5.30, before moving away in a'rippling' motion due to the combination of aerodynamic and gravitational loading. The thickness and location of this rivulet therefore vary with time. Figure 5.32 illustrates this periodic formation of rivulets by tracking the variation in film thickness at $\theta = 67^{\circ}$, from which the period of formation can also be determined as approximately 0.23 s. Interestingly this is three times the period of Karman vortex shedding for this body. The same value that was found by Matsumoto [42] to play an important role in the HSV phenomenon summarised within section 2.3.3 of the literature review. As such the periodic appearance of this rivulet at an angular location which has already been determined to have a major impact on the flow via the artifical rivulets studies of section 3.5 and the experimental and in-situ data summarised in section 2.4.1 may prove to be a contributory factor to the underlying cause of the HSV phenomenon and explain why a frequency of $\frac{1}{3}f_s$ is so important to this. This however is only an initial observation and as the axial vortex previously identified [42] as being of central importance to this cannot be captured in the present work this conjectured link is very tentative. Further work to confirm any such link would require extensive experimental testing and is beyond the scope

of the present work.

As can be seen from figure 5.33 which is a magnified version of the area of interest of the overall film surface evolution, figure 5.30, the 'rippling' motion originates in two locations each moving in a different direction before joining the larger lower rivulet. Although smaller ripples develop behind the separation point of a dry cylinder and move in a leeward direction, as these are within the wake they have little effect on the overall flow. It is the rivulets which form in the region previously determined to be danger for RWIV $\theta = 67^{\circ}$ and which move in a windward direction that will therefore be discussed in more detail. Numerically determining such periodic motion within the wake however provides further confidence in the abilities of the solver to capture both rivulet formation and evolution and the RWIV phenomenon.



Angle Clockwise from Windward Horizontal, θ , (Degrees)

Figures 5.34 and 5.35 track the magnitude and location of the point of maximum thickness for each of these 'ripples' for the first second of evolution studied. These illustrate that after an initial transient period of approximately 0.2 s where the whole film is evolving that the formation and movement of these 'ripples' is approximately periodic. The extent of the initial transient period corresponding well with the time for the thickness of lower rivulet to self limit as was shown

Figure 5.33: Numerical prediction of temporal evolution of film thickness in real time, under full loading conditions of region near the upper rivulet. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

in figure 5.31. A new rivulet evolves approximately every 0.23 s at $\theta \simeq 67^{\circ}$, before beginning to move windward under the effect of gravity. These increase in thickness until $\theta \simeq 50^{\circ}$ where they begin to 'spread', decreasing in thickness and increasing in circumferential velocity until joining the lower rivulet approximately 0.6 s later. Given that the thickness of both the lower rivulet and the mean surface do not vary after the initial formation period, it can be surmised that there is no net movement of fluid during this rippling process. Rather individual points on the film surface rise and fall in a manner similar to ocean waves [144].



Figure 5.34: Temporal evolution of maximum thickness of individual ripples on the upper surface, under full loading conditions. Variation of h with t of ripples.



Figure 5.35: Temporal evolution of individual ripple location clockwise from the windward horizontal, under full loading conditions. Variation of θ with t of ripples.

For consistency with the previous loading cases and to allow a measure of verification the location of the lower rivulet, its rate of growth and its maximum thickness were compared with those of the lower rivulet formed using fixed distributions of C_P and C_F (specifically \overline{C}_P and \overline{C}_F determined by the independent DVM solver for a plain cylinder). As can be seen from figure 5.36 all three were once again in excellent agreement at early times of film evolution. After which the lower rivulet formed under a fixed distribution of pressure and shear continued to grow until the thin film approximation was violated, while those formed under pressure and shear distributions that varied accounting for the rivulet growth maintained an almost constant thickness and location once formed. The magnitude of this self limiting rivulet thickness is again comparable with those determined by Cosentino [19] for the same cable dimensions. Comparing the upper rivulet was more difficult given the periodic nature of those formed presently. What can be said however is that the angle of original formation $\theta \simeq 67^{\circ}$ agrees very well with that where the upper rivulet was found to form under fixed distribution of C_P and C_F in section 4.6.4. As such the present results can once again be said to be in-line with those of the previous chapter.



Figure 5.36: Comparison of temporal evolution of lower rivulet location and normalised thickness formed under full loading between the present coupled solver and the previously examined un-coupled solver. Variation of θ_{ac} and h/h_0 with t.

Finally the time averaged aerodynamic parameters for the present full loading

case were compared with those of the plane circular cylinder and the independent shear and pressure loading cases. As can be seen from table 5.5 while C'_L remains unaltered and there is a similar small reduction in C_D , there was an order of magnitude increase in the net lift force to $C_L = 0.035$. However given that this is still within the statistical margin of error of the present code and that the normalised thickness of the self limiting lower rivulet formed here, 0.0043D, is once more below the limit of 0.007D found in section 3.5.5 at which the DVM solver can definitively detect the effect of an artificial rivulet. Without experimental confirmation it cannot at present be be determined whether this is a real force due to asymmetry or just a feature of the present solver. However given the very small magnitude of this lift force and the corresponding effect that this would have on a physical body this would prove difficult to detect. That said, despite a reduction in magnitude of the single strouhal peak corresponding to Karman vortex shedding this was also found to remain unaltered at St = 0.209. Full loading like the three loading combinations previously examined was therefore also found to have negligible effect on the bodies response.

	C'_L	\bar{C}_L	\bar{C}_D	St
Present Full Conditions	0.535	0.035	1.186	0.209
Shear	0.524	0.0013	1.153	0.208
Pressure	0.512	-0.0071	1.104	0.206
Plain Cylinder	0.519	0.0004	1.254	0.211

Table 5.5: Comparison of C'_L , \bar{C}_L , \bar{C}_D and St of present full loading results with those from the independent shear and pressure cases and those of the plain circular cylinder.

5.5 Case Studies

As a conclusion to this thesis, final case studies were undertaken to establish the effect that specific parameters have on overall evolutionary response, and in particular on any rivulets which form. Two such studies examining initial film thickness h_0 and angle of attack in the plane normal to the cable axis γ_{θ} are presented here.

5.5.1 Initial Thickness of Thin Film

A range of initial film thicknesses between $0.1 \times 10^{-3} \text{ m} \leq h_0 \leq 0.35 \times 10^{-3} \text{ m}$ were investigated (0.1 - 0.35 mm) under full loading. These limits were chosen to be $0.1 \times 10^{-3} \text{ m}$ smaller and larger than the 'base carpet' value $h_0 = 0.2 - 0.25 \times 10^{-3} \text{ m}$ measured experimentally by Cosentino [19]. While for consistency the initial profile in each case was uniform, $h(\theta, 0) = h_0(\theta) = \text{constant}$.

As the results of figure 5.37 illustrate, while a distinct rivulet of self limiting thickness does form at approximately the same mean location for each of the initial film thicknesses investigated, $\theta \simeq 277^{\circ}$, the time taken to do so varies; with thicker films growing faster than thinner films. This is a direct result of the normal velocity of the free surface at a particular thickness \hat{u} , which can be derived by substituting the present thickness h into equation (4.21). In the absence of surface tension this is

$$\hat{u} = -\frac{1}{2\hat{\mu}} \left(\hat{\rho}g \cos \theta + \hat{p}_x \right) h^2 + \frac{Th}{\hat{\mu}}.$$
(5.4)

From which it can be seen that the normal velocity of the film and hence the growth rate of the terms corresponding to pressure and gravitational terms loading increases in proportion to the present free surface thickness squared (Pand $g \propto h^2$), whereas the shear term only does so in direct proportion to the free surface thickness ($T \propto h$). This in turn explains why the rivulets which form under pressure only loading grow faster than those due to shear only loading and why this growth rate increases when these act in combination and increases again when gravitational loading is also considered, figure 5.31. As the normal velocity increases with film thickness, equation (5.4) also clarifies why initially thicker films grows more quickly than thinner films and why the rate of h/h_0 for any film increases with time, figure 5.37. Where as the film becomes thicker the growth rate (the gradient of $\frac{dh}{dt}$) of that rivulet increases as can be seen in figure 5.38.



Figure 5.37: Temporal evolution of the normalised thickness of the lower rivulet for various initial thicknesses of film under full loading conditions. Variation of h/h_0 with t for varying h_0 .



Figure 5.38: Temporal evolution of the thickness of the lower rivulet for various initial thicknesses of film under full loading conditions. Variation of h with t for varying h_0 .

Interestingly the maximum self limiting thickness of the lower rivulet was almost independent of the initial thickness of film. Figure 5.38 illustrates this for the first 0.75 s of evolution which is sufficient for all bar the thinnest film, $h_0 = 0.1 \times 10^{-3}$ m, to reach this state, which although not explicitly shown does so at approximately t = 1.3 s. This value of $h \simeq 0.67 \times 10^{-3}$ m agrees well with those thicknesses obtained from the combinations of loading previously examined, indicating that for the loading conditions and parameter ranges presently examined that there is a maximum thickness of rivulet which can form before further growth is restricted by the coupled aerodynamic field. The exact reasons for the particular limit are unclear as is whether this is a real phenomenon or due to one of the assumptions made in the development of the present solver. That said however, examination of different cylinder radii did result in different maximum thicknesses in each case, this though was not the focus of the present investigation. Intuitively it makes sense that such a limit would exist, just as there is a limiting wind speed for the actual formation of rivulets within RWIV [19, 33, 39, 71]. That the present value qualitatively matches that found experimentally by Cosentino [19] for the same diameter of cable is encouraging.



Figure 5.39: Comparison of film thicknesses profiles which result from various initial thicknesses of film under full loading conditions at a time of 1.4 seconds. Variation of h with θ for varying h_0 at t = 1.4 s.

Because of the consistency of this self limiting thickness and the condition for zero fluid flux, i.e. conservation of thin film area modelled, the actual evolutionary profiles once this limit is achieved are significantly different for the various initial film thicknesses studied. While the lower rivulets all have the same maximum thickness those which evolve from thinner films (smaller h_0) have significantly smaller base widths than those which form from thicker fluids (larger h_0). Figure 5.39 illustrates this at t = 1.4 s by which time all lower rivulets have ceased to increase in thickness. This figure also highlights that a rivulet was found to form on the upper surface for all initial thicknesses of film considered. While the actual thickness and location of this varied as it moved windward to join the lower rivulet, the mean location at which this formed was approximately constant, matching the base case of $h_0 = 0.25 \times 10^{-3}$ m as $\theta \simeq 67^{\circ}$ in all cases, section 5.4.4. The mean thickness of this upper rivulet did not show such a simple relation with initial film thickness however. That said, in all the cases examined it was significantly smaller than the lower rivulet.

In comparison with the base case of $h_0 = 0.25 \times 10^{-3}$ m there were small variations in both \bar{C}_L , \bar{C}_D and C'_L . These however were within the margins of error inherent in the DVM solver as can be seen in table 5.6. Likewise spectral analysis determined a single strong peak at the Strouhal number in all cases, although there were variations in the specific magnitude of this. Therefore while it appears that the initial film thickness causes a variation in the exact geometry of the film which evolves it has little effect on the overall response of the body. As the thickness of the rivulets formed, 0.004D, is once more below the limit of 0.007D found in section 3.5.5 at which the DVM solver can definitively detect the effect of an artificial rivulet, this is unsurprising; the consistency of this result however is again cause for confidence in the solver and the model.

h_0	C'_L	\bar{C}_L	\bar{C}_D	St
$10\mathrm{mm}$	0.501	-0.029	1.082	0.214
$20\mathrm{mm}$	0.522	0.007	1.106	0.208
$25\mathrm{mm}$	0.535	0.035	1.186	0.209
$35\mathrm{mm}$	0.488	0.041	1.093	0.212

Table 5.6: Comparison of C'_L , \bar{C}_L , \bar{C}_D and St for varying values of initial film thickness, under full loading conditions.

That said, the present result possibly explains why a definitive range for the

amount of water required to obtain RWIV has not been determined, as was outlined in section 2.2. As although the maximum thickness of rivulet formed was found to be limited, the initial film thickness required to produce this was not. Therefore while an upper limit on the initial film thickness which could evolve was established a similar lower limit could not be determined as given a sufficient period of time rivulets were found to form for all values of film thickness presently studied. This however is contingent on the assumptions made herein, particularly that for zero net fluid flux.

5.5.2 Angle of Attack in Plane

The angle of attack in plane γ_{θ} , as outlined in appendix A, is a variable constructed such that the three-dimensionality of a yawed inclined cable can be represented in two dimensions, figure 5.40. If we assume the incident wind speed in plane U_{eff} and effective gravity g_{eff} remain fixed at the values for the full three-dimensional system (U and g respectively) as $\gamma_{\theta} = f(\alpha, \beta)$ this allows a range of angles of inclination and yaw to be examined by the variation of a single parameter. Using the typical limiting ranges for RWIV of $20^{\circ} < \alpha < 45^{\circ}$ and $20^{\circ} < \beta < 60^{\circ}$ outlined in section 2.2, a range of angles of attack in plane to be examined was established. Cases at 5° intervals of this $10^{\circ} < \gamma_{\theta} < 50^{\circ}$ range were thus investigated.



Figure 5.40: Schematic defining angle of attack in plane γ_{θ} and effective wind speed in plane U_{eff} .



Figure 5.41: Numerical prediction of temporal evolution of film thickness in real time, under full loading conditions at $\gamma_{\theta} = 20^{\circ}$. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

As was true for the full combination of loading acting on the base case of a horizontal cable perpendicular to the incoming flow ($\gamma_{\theta} = 0^{\circ}$), section 5.4.4, a large rivulet of self limiting thickness was found to form on the lower surface for all values of γ_{θ} studied. Likewise 'ripples' were found to develop on the upper surface before moving circumferentially to join the lower rivulet, although whether or not an upper rivulet formed, was found to be dependent upon the angle of attack in plane; a point discussed in due course. Figure 5.41 highlights the temporal evolution of one particular case, specifically $\gamma_{\theta} = 20^{\circ}$, where an upper rivulet did form. This figure also illustrates that in this instance the lower rivulet formed was considerably narrower than the base case ($\gamma_{\theta} = 0^{\circ}$) and that this oscillated periodically by approximately $\theta \pm 2^{\circ}$ from its mean location $\bar{\theta} = 265^{\circ}$. The variation in thickness and location of both the upper and lower rivulets with angle of attack in plane are now discussed in turn.

As can be seen from figure 5.42, which displays the results at intervals of $\gamma_{\theta} = 10^{\circ}$, the rate of growth and the self limiting thickness of the lower rivulet in each case are very similar to both the base case ($\alpha = \beta = \gamma_{\theta} = 0^{\circ}$) and to each other. It can therefore be said that when varied independently γ_{θ} has very little effect on either the thickness or the growth rate of the lower rivulet. It does



Figure 5.42: Comparison of temporal evolution of normalised film thickness for various angles of attack in plane. Variation of h/h_0 with t for different γ_{θ} .



Figure 5.43: Schematic of effect that varying angle of attack in plane γ_{θ} has on rivulet location.



Figure 5.44: Comparison of location of mean location of lower rivulet and range of oscillation of this with angles of attack in plane. Variation of $\bar{\theta}_{ac}$ with γ_{θ} , where $\bar{\theta}_{ac}$ is measured anti-clockwise from windward horizontal.

however influence where this rivulet forms. As with larger γ_{θ} the component of aerodynamic loading directly opposing gravitational loading increases. This acts to 'blow' the rivulet further up the leeward face figure 5.43. The mean location of the lower rivulet therefore varies with γ_{θ} , as can be seen from figure 5.44. This also illustrates the range of oscillation about this mean location, which reaches a maximum at the $\gamma_{\theta} = 20^{\circ}$ case previously shown, while at $\gamma_{\theta} > 35^{\circ}$ such small periodic oscillations were no longer detected. Both of which are observations which agree well with the experimental work of Wang et al. [68] on the location and oscillation of the lower rivulet, as was outlined in section 2.4.1. This detected a rivulet "which tends to be more influenced by the gravity force ... running in an approximate straight line" and which moved up the leeward face with increasing γ_{θ} , when the system is described in the present nomenclature. This is further evidence that the numerical solver created within this thesis captures fundamental characteristic features of the rivulets found to evolve during experimental study.



Angle Clockwise from Windward Horizontal, θ, (Degrees)



Ascertaining a discernable pattern amongst the upper rivulets proves more complicated. As can be seen for figure 5.45 which displays a magnified version of the upper surface for the first second of evolution of the $\gamma_{\theta} = 35^{\circ}$ case (akin to figure 5.33), once the angle of attack in plane reaches this value, no discernible upper rivulet could be detected although 'ripples' were still found. This may indicate a possible connection to the previous result, figure 5.44, as when no upper rivulet formed no oscillation of the lower rivulet could be detected. The present study could not establish such a link however. That said, the period of ripple formation was found to be constant at 0.23 ± 0.02 s in all cases which agrees well with the base case ($\gamma_{\theta} = 0^{\circ}$), as did the time interval between formation and these ripples joining the lower rivulet. Given that these appear to have a link to the vortex shedding, the $\frac{1}{3}f_s$ result previously highlighted, this is perhaps unsurprising.

As outlined in section 2.4.1 of the literature review the upper rivulet is thought to play a key role in RWIV mechanism. As such, the fact that these upper rivulets were only found to occur at $\gamma_{\theta} \leq 35^{\circ}$, under the conditions examined herein, was thought to be an important result. Quantitively this result is also in excellent agreement with the literature [5, 39, 43, 74]. As when the cable geometries under which RWIV has previously been detected experimentally are transformed into the present notation, table 5.7, oscillation was not found to occur at values significantly greater than that determined.

Paper	Inclination, (α)	Yaw, (β)	Attack in Plane, γ_{θ}
Hikami	45°	45°	37.2°
Flamand	25°	30°	14.5°
Gu	$30-35^{\circ}$	$25-40^{\circ}$	$15.8 - 27.3^{\circ}$
Zhan	30°	35°	20.4°

Table 5.7: Angles of inclination, yaw and angle of attack in plane at which previous experiments by Hikami [5], Flamand [39], Gu [43] and Zhan [74] determined RWIV. α , β and γ_{θ} of experimentally determined RWIV.

Furthermore in cases where an oscillating rivulet did form on the upper surface, while the mean maximum thickness of this rivulet \bar{h} did not change significantly for $\gamma_{\theta} < 25^{\circ}$, the mean angle $\bar{\theta}$ at which such rivulets originally formed did. As this varied in such a manner as to maintain an almost constant angle between where the rivulet originally formed and the stagnation point of the incident flow $\gamma_{\theta} + \bar{\theta}$, table 5.8, the indication is that the location of this upper rivulet is a direct consequence of the aerodynamic loading, figure 5.43. This table also highlights the independence of mean maximum thickness of this upper rivulet with angle of attack in plane and the relation γ_{θ} has to the location at which this upper rivulet originally formed $\bar{\theta}$. The latter of which is in good qualitative agreement with the experimental results of Bosdogianni [67] discussed in section 2.4.1 of the literature review.

Angle of attack	Mean rivulet	Mean location of original	Rivulet angle from
in plane, γ_{θ}	thickness, \bar{h}	rivulet formation, $\bar{\theta}$	stagnation, $\gamma_{\theta} + \bar{\theta}$
0°	$0.30\mathrm{mm}$	67°	67°
10°	$0.31\mathrm{mm}$	58°	68°
15°	$0.30\mathrm{mm}$	52°	67°
20°	$0.29\mathrm{mm}$	46°	66°
25°	$0.28\mathrm{mm}$	40°	65°

Table 5.8: Comparison of the mean maximum upper rivulet thickness \bar{h} , the mean location of formation $\bar{\theta}$ and the angle of formation from stagnation $\bar{\theta} + \gamma_{\alpha}$ with angle of attack in plane γ_{θ} .

Examination of the time averaged mean and fluctuating lift and drag coefficients (\bar{C}_L , \bar{C}_D and C'_L) rotated such that these were in-line with the incident flow, showed little variation with the angles of attack in plane investigated. This agrees well with the previous loading combinations as the maximum thickness of the rivulet formed, 0.004*D*, is below the limit at which the DVM solver can definitively detect the effect of an artificial rivulet.

Two results from the spectral analysis did stand out however; specifically those at $\gamma_{\theta} = 20^{\circ}$ and 25°. Illustrated in figure 5.46 for the former case, this indicates a suppression of Karman vortex shedding under these conditions as no



Figure 5.46: Spectral analysis of full loading conditions at various angles of attack in plane. Left: $\gamma_{\theta} = 10^{\circ}$, single peak at *St*. Right: $\gamma_{\theta} = 20^{\circ}$, no peak corresponding and reduced magnitude corresponding to Karman vortex shedding.

dominant peak could be determined. The magnitude of the PSD is also considerably reduced from the single peak at St detected at all other angles of attack in plane studied, here illustrated for the $\gamma_{\theta} = 10^{\circ}$ case. When RWIV occurs it does so at a much lower frequency than VIV and not in direct relation to the incident wind speed, section 2.3.1. Therefore that no dominant frequency could be determined in these cases is a major finding that agrees well with experimental results highlighted in section 2.2 of the literature review.

In addition to the determination of rivulets of self limiting thickness and the periodic formation and movement of an upper rivulet at $\frac{1}{3}f_s$ in locations that agree well with previous studies, the prediction of this characteristic feature of RWIV is excellent evidence of the capabilities of the present solver in tracking rivulet formation and evolution. It also highlights that this coupled solver has successfully taken the first steps towards numerically modelling RWIV and therefore that as a 'proof of concept' this thesis has greatly exceeded its original objectives. However without experimental evidence, determining whether this Karman vortex suppression is a real phenomenon and what role it might play in the governing mechanism of RWIV, cannot be determined at this stage. That the γ_{θ} at which this suppression occurs agrees so well with where RWIV was determined experimentally, table 5.7, only reinforces the significance of this finding.

5.5.3 Final Study

Within the two dimensional representation in which γ_{θ} is defined, figure 5.40, the incident wind speed U_{eff} , effective force of gravity g_{eff} and the angle of attack in plane are all interlinked. Therefore in the previous section while varying γ_{θ} independently from the other two allowed a range of inclination and yaw angles to be studied, it did not give an entirely representative solution. Both U_{eff} and g_{eff} should also be altered such that these act in the same plane. The former in accordance with equation (A.5), given here as $U_{eff} = U\sqrt{\cos^2\beta + \sin^2\alpha \sin^2\beta}$, and the latter by $g_{eff} = g \cos \alpha$. Therefore as a conclusion to this thesis one specific case was examined where all three parameters were modelled in accordance with the exact definitions. This was chosen to correspond to worst case of RWIV response determined by Gu [43] and subsequently by Zhan [74] of $\alpha = 30^{\circ}$ and $\beta = 35^{\circ}$; for which $\gamma_{\theta} = 20.4^{\circ}$, $U_{eff} = 8.27 \,\mathrm{m/s}$ and $g_{eff} = 8.946 \,\mathrm{m/s^2}$. That this worst case matches the value of γ_{θ} found by the present numerical solver to suppress Karman vortex shedding, figure 5.46, gives additional weight to this finding.



Figure 5.47: Numerical prediction of temporal evolution of film thickness in real time, under full loading conditions at $\gamma_{\theta} = 20^{\circ}$, with reduced incident velocity and effective gravity. Variation of h with θ and t, where θ is measured clockwise from the windward horizontal.

A comparison of the evolutionary surface of this 'inclined' case, figure 5.47,

with that for the previous 'flat' case where the gravitational loading and incident wind were unaltered, figure 5.41, highlights that these are very similar. Examination of the maximum thicknesses, locations and profiles of both the upper and lower rivulets and of the 'ripples' which form revealed that these were also largely unaltered. The only notable difference being that the 'inclined' case has a marginally longer initial start-up period. This confirms findings made previously in sections 4.6.2 and 4.6.3 where fixed distributions of C_P and C_F of the same profile but different magnitude were examined and the results were found to be 'scaled' images of one another. Thus with a smaller effective wind speed and thus smaller aerodynamic loading it therefore takes longer to reach a specific point in the evolution profile. Figure 5.48 illustrates this by comparing the maximum thickness and the corresponding location of the larger lower rivulets formed in each case over the two seconds of evolution studied. As can be seen these show very little variation after the initial start-up period which is marginally longer for the inclined case. That said, there may be slight instantaneous variations in the actual film profile.



Figure 5.48: Comparison of temporal evolution of maximum thickness and angular location of lower rivulet for full loading conditions at $\gamma_{\theta} = 20^{\circ}$ for the 'flat' case where only the angle of attack in plane was altered and the 'inclined' case where all three parameters were altered. Variation of θ and h/h_0 with t.

It can therefore be said that, for the limited cases and conditions presently

examined, the present solver predicts that the angle of attack in the plane has the greatest effect on temporal evolutionary profile, of the parameters used in this two dimensional representation. However before a broader statement can be made, larger parameter ranges and significantly more cases would need to be examined and verified with experimental evidence.

What can be said is that for the geometry of cable inclination and yaw found to cause the largest RWIV response by both Gu [43] and Zhan [74], the present solver predicts the formation of a lower rivulet whose maximum thickness is qualitatively in-line with that determined by Cosentino [19]. That a upper rivulet was found to periodically form in the locations determined by artifical rivulet studies to be dangerous for RWIV, section 3.5.2, and that Karman vortex shedding was suppressed. As such this is additional evidence that the present solver has successfully taken the first steps towards the numerical simulation of RWIV. However without experimental evidence to verify the present results, the demonstration of these characteristic features of RWIV is the best that can presently be achieved.

5.6 Summary

In conclusion it is worth reiterating several points addressed in this chapter and several of the results determined. Details of the coupling process, various refinements made and the rationale behind these were discussed and convergence studies performed on key parameters. Four combinations of loading were then investigated, examining the independent effects of shear and pressure, before combining these and finally adding gravity to ascertain full physical loading. This allowed effects particular to a given loading combination to be ascertained and the evolutionary surfaces of each loading combination to be compared with those obtained using fixed distributions of C_P and C_F previously determined in chapter 4. The location and growth rates of the rivulets formed agreeing well in each case. These were located marginally windward of the separation points of the dry cable and symmetric with respect to the incident flow when gravity was not considered. What was different, was that for the first time the thickness of these rivulets were found to be self limiting as aerodynamic loading varied due to rivulet growth. For the geometry and conditions typical of RWIV examined herein this value was found to be approximately 0.7×10^{-3} m. A value in excellent agreement with that determined experimentally by Cosentino [19].

When gravitational loading was considered, this symmetry was lost with a larger lower rivulet found to evolve with approximately the same growth rate and at approximately the same location as the fixed distribution case; the thickness of which was once again self-limiting. Unlike when fixed distributions of C_P and C_F were considered however, a rivulet was only found to form periodically on the upper surface, before moving to join the lower rivulet under the actions of combined loading. This prediction characteristically matches the unsteady moving upper rivulet previously determined to play a key role in the RWIV phenomenon, as outlined in section 2.4.1 of the literature review, which in the absence of experimental data, was seen to be significant in both verifying the results determined and in demonstrating the capability of the coupled solver to numerically simulate aspects of RWIV. Interestingly the period of formation of this upper rivulet was three times that of the Karman vortex shedding, matching that determined by Matsumoto [42, 53] to play an important role in HSV phenomenon and therefore indicated a tentative potential link between these.

Two case studies were then undertaken to investigate how the variation of specific parameters affected the overall evolutionary response. The first of these found that while the location and maximum self limiting thickness of the lower rivulet formed under full loading conditions was not affected by the initial thickness of the thin film, that the form of these rivulets was, with smaller initial film thicknesses producing narrower rivulets due to the condition for zero fluid flux imposed. The second study found that while the angle of attack in plane did not affect the thickness and growth rate of the lower rivulet, it did affect the mean location at which the lower rivulet formed. This rivulet was also found to oscillate about this mean in certain cases, $\gamma_{\theta} < 35^{\circ}$ and to cease when no upper rivulet could be detected, $\gamma_{\theta} \ge 35^{\circ}$. Therefore if the formation of this rivulet is taken as a required condition for a possible RWIV response, then this value is in excellent agreement with previous studies [5, 39, 43, 74] which likewise did not find RWIV at γ_{θ} significantly greater than 35°.

Time averaged aerodynamic parameters and the spectral response of the body were largely unaffected for most combinations of loading and parameter values considered, as the rivulets formed were below the thickness, 0.007D, for which the independent DVM solver was able to determine the effect of an artificial rivulet. That said under certain specific conditions, $20^{\circ} < \gamma_{\theta} < 25^{\circ}$, no dominant Strouhal peak could be determined within the PSD indicating that Karman vortex shedding had been suppressed. Together with the determination of periodic formation of the upper rivulet and the self-limiting thickness of the rivulets which formed, these were thought to represent significant evidence that the present solver has successfully taken the first steps towards the numerical simulation of RWIV and that as a 'proof of concept' this thesis has greatly exceeded its original objective.

As a final case, the cable geometry identified by Gu [43] and Zhan [74] as providing the largest RWIV response, $\alpha = 30^{\circ}$, $\beta = 35^{\circ}$, were fully represented in a two-dimensional sense through variation of effective in plane wind speed and gravity, which were considered fixed when investigating the effect of angle of attack in plane. Under these conditions Karman vortex shedding was once again found to be suppressed, while rivulets were found to form periodically on the upper surface in the regions previously identified as being important to RWIV.

Through the determination of these characteristic features the present solver is therefore seen to represent a significant first step toward the numerical simulation of RWIV and to have greatly exceeded its original objective as a 'proof of concept'. However without experimental data, detailed verification of the coupled solver could not be undertaken. It could however be partially achieved by qualitative comparison with specific results and through comparisons with studies previously undertaken in the development of the individual DVM and pseudospectral solvers. That said, as these two solvers were independently validated for problems related to RWIV before being combined and as the results determined were so consistent, the level of confidence in both the final coupled solver numerical model and the results it predicts is high. Suggestions for experimental data against which the final coupled solver could be verified, recommendations for future revisions and the next steps towards numerically modelling RWIV are given in the following chapter. Which by summarising the work undertaken and highlighting the advancements to knowledge made concludes the present thesis.

Chapter 6 Concluding Remarks

Within this final chapter, the development and application of the numerical solver created to track the formation and evolution of rivulets under an external aerodynamic field these influence, described in the previous chapters, is summarised. This is performed in two sections, the first provides a summary of the research presented in this thesis and outlines the main contributions to knowledge made. The second indicates areas in which future development could achieve improvement of the present code.

6.1 Summary

Given the uncertainty regarding the underlying mechanism of RWIV and the quantity, diversity and sometimes contradictory nature of the research published to date, an extensive review of previous literature was undertaken. This focused on; identifying characteristic features and parameter ranges for the occurrence and response of RWIV, how this can be distinguished from other aeroelastic instabilities and what can be done to design against or mitigate such a response. The requirement for the creation of a solver, such as that presented, to instigate numerical simulation of this phenomenon was also highlighted.

The DVM code developed by Lin [6] and generalised by Taylor [7] for analysing the aerodynamic field around bluff bodies, has been refined and improvements made to accommodate near circular geometries. Specific modifications implemented included methods to obtain wall shear stress and to allow limited temporal variation in body geometry. Extensive validation of a static plain circular cylinder at the sub-critical Reynolds number range of interest found excellent agreement with experimental and numerical data for; mean and fluctuating aerodynamic coefficients, mean and fluctuating pressure profiles, wake velocity profiles and spectral content of response. The level of quantitative and qualitative agreement demonstrating both the capability of the present solver for the bluff body flow of interest, and that the accuracy of the present solver was at least comparable, if not better, than other computational solvers. Similar validation of forced oscillation likewise showed good agreement with previous literature, should the body move. With reference to the original aims and objective as outlined in section 1.3 this validates the present solver in the case of a plain cylinder.

Validation of the solver with the addition of artificial rivulets was achieved by the successful identification of four distinct flow regimes with variation in rivulet location. The extents, boundaries and quantitative effect of these regimes on both fluctuating and time averaged aerodynamic parameters and on the spectral response, all agreed well with the experimental data of Matsumoto [81] and numerical data of Liu [80]. Examination of the pressure and velocity fields, which were also found to be in good agreement with these studies and that of Li [75], revealed the separation of the boundary layer at this rivulet and whether this subsequently re-attached, formed a 'saddle separation' or remained detached was the primary cause of these four regimes. This is reported explicitly for the first time and is but one example of the specified objective to clarify the effect that artificial rivulets have on the aerodynamic response. Another is the identification of a possible 'galloping' response in one of these regimes, $40^{\circ} < \theta < 60^{\circ}$, through a combination of Karman vortex suppression, generation of a net lift force and the fulfilment of the Glauert-Den Hartog criterion (2.6). That this likewise compared favourably with previous work [67, 74, 81] goes further toward the validation of the numerical solver.

Studies of different artificial rivulet configurations provided consistent result which inspired further confidence in the solver, as did independent numerical confirmation of a highly sensitive region when symmetric rivulets are located at 50° from the incident flow as previously identified by Matsumoto [81]. The latter of which was achieved by the detection of a \bar{C}_L of similar magnitude to experimental results when various methods of minor asymmetry were introduced at this location. Returning to the original aims and objectives, case studies examined numerically for the first time, highlighted that while variation of rivulet form or size can result in a change of the aerodynamic response, that this magnitude is significantly smaller than that which arises due to variation of rivulet location. These also showed that artificial rivulet width played a very minor role and that at central heights of less than 0.007D the presence of the artificial rivulet could no longer be detected on the time averaged aerodynamic response; a result which has serious implications during the growth of the rivulet. By virtue of such findings the solver thus exceeded its original objectives in determining the effect that the size, form and location of the rivulet have on the overall body response.

A pseudo-spectral solver was successfully created to solve the governing equation derived for the evolution of a thin film on the outer surface of a circular cylinder under the influence of an external aerodynamic field. Validation of this against existing and new analytical solutions to simplified problems, have shown this solver to be in both excellent temporal and spatial agreement, thus fulfilling the objective as stated at thesis outset. Comparisons with the existing numerical solver of Lemaitre [9] for a variety of loading combinations for which analytical solution do not exist showed excellent agreement in each case, further validating the present code and providing independent confirmation of these previous results. By determining and reporting full temporal evolution surfaces, the present solver also extended existing knowledge revealing that in each instance rivulets were found to form marginally windward of the separation points of a dry circular cylinder and that these continued to increase in thickness until the thin film approximation was violated. In the absence of gravitational loading these rivulets were found to be symmetric with respect to the incident flow, while the effects of shear and pressure were found to be of approximately equal importance, in direct confirmation of the results of Lemaitre [9].

To satisfy the aim of determining the effect that magnitude and profile of aerodynamic loading distribution (\bar{C}_P and \bar{C}_F) have on the evolutionary response, several cases were examined for the first time. The results showed that while varying the magnitude of the distribution caused a 'scaled' response of the same profile that it was the profile of the aerodynamic loading distribution which has the largest effect on the evolutionary profile and in particular the frequency content of this. By investigating different profiles of aerodynamic loading associated with different Reynolds number the effect that flow regime had on the formation and evolution of rivulets was also studied for the first time. This, like the other case studies examined, found that it was the profile and not the magnitude of aerodynamic loading which had the greatest effect on the form, location and size of the rivulets which evolved.

Having validated these solvers individually they were combined to form the first solver capable of numerically simulating the formation and evolution of rivulets on a circular cylinder under the effect of an external aerodynamic field these influence. The creation of which required several refinements to the independent DVM and pseudo-spectral solver. These included implementing a cubic-spline based method to interpolate between different numbers of nodes and smoothing of transient data obtained from the DVM solver, the latter being necessary to reduce noise and mitigate the very rapid variation which could cause problems for the pseudo-spectral method. As data with which to directly validate the solver was not available a series of convergence studies were undertaken to ensure the veracity of the solver and the results it produces.

A number of studies were performed on the final solver in line with stated aims and objectives. The first of these saw four combinations of loading investigated to allow the independent effects of shear and pressure to be examined, before a combined and a full loading case, which also included gravity were studied. In each instance the evolutionary surface particular to that loading combination was compared with that obtained from the independent pseudo-spectral solver using fixed distributions of C_P and C_F . This revealed that while the locations and the growth rates of the rivulets formed did not differ significantly, that rivulet thickness did. For the first time this thickness was found to be self limiting as a consequence of aerodynamic loading varying as the rivulet evolved. A determination given further credence, as for the geometry and conditions typical of RWIV examined herein, this value was found to be $\simeq 0.7 \times 10^{-3}$ m, in excellent agreement with that determined experimentally by Cosentino [19].

When gravitational loading was considered, the symmetry of rivulet evolution on the upper and lower surfaces found in its absence was lost and a larger lower rivulet found to evolve with approximately the same growth rate and at approximately the same location as that formed under fixed distributions of C_P and C_F ; the thickness of this though was again found to be self-limiting. Unlike when fixed aerodynamic loading was considered however, a rivulet was only found to form periodically on the upper surface, before moving to join the lower rivulet. The location and periodic nature of which characteristically matches that of the unsteady oscillating upper rivulet previously determined to play a key role in the RWIV phenomenon. This in turn allows conclusions to be drawn as to the importance of gravitational loading to the actual RWIV mechanism. Furthermore as several of the characteristic features of RWIV can be represented by this simplified representation it confirms the method chosen in constructing the coupled solver. Interestingly, the period of formation of this upper rivulet was found to be three times that of Karman vortex shedding. This ratio matches that determined by Matsumoto [42, 53] to play an important role in the HSV phenomenon, possibly indicating a tentative potential link between these. This however requires further study.

Finally to comply with the original aims stated in section 1.3 and to advance knowledge regarding the effect that coupling the aerodynamic field to rivulet formation and evolution has on the response, two case studies were undertaken. The first concluded that although the location and maximum thickness of the lower rivulet formed under full loading conditions was independent of the initial film thickness, that due to the condition for zero fluid flux, the form of these rivulets varied considerably, with initially thinner films developing 'narrower' rivulets. Secondly, for the geometry and loading considered, periodic formation of the upper rivulet was only discovered to occur at certain angles of attack in plane, specifically $\gamma_{\theta} < 35^{\circ}$. This agrees well with the literature [5, 39, 43, 74] if taken as a prerequisite for a possible RWIV response. As the rivulets formed under the conditions studied were below the thickness at which the independent DVM solver was able to determine the effects of an artificial rivulet, 0.007D, the aerodynamic parameters and the spectral response of the body were largely unaffected for most combinations of loading and parameter values considered. That said, at $20^{\circ} < \gamma_{\theta} < 25^{\circ}$ no dominant Strouhal peak could be determined within the PSD, signifying that Karman vortex shedding has been suppressed, a consequence of which is that the body is more susceptible to another instability. Given that these angles of attack in plane are in excellent agreement with those identified as causing the largest RWIV response [43, 74] it can be concluded that rivulet formation and oscillation does indeed play a major role in the governing mechanism which underlies this phenomenon. By predicting these characteristic features of RWIV for a simplified representation and validating the numerical method developed, this solver has taken a significant step in numerically simulating the overall instability and has therefore satisfied the underlying objective of this thesis.

6.2 Future Research

Although the low numerical diffusion inherent in a grid free technique and the high particle density presently used mean that small scale structures within the flow field could be captured; the existing DVM code does not incorporate a turbulence model. Implementing such a model should improve the capability of the present solver to predict the response in the region immediately leeward of separation. In conjuction with an improvement of the linear sub-layer model this should also obtain a more accurate prediction of the profile of wall shear stress in the same region.

The present DVM code allows small deformations of the free surface and ensures that neither the total nor the instantaneous change in circulation that results becomes significant. That said, true representation of the vorticity generation associated with the deformation of the gas/liquid interface requires reformulation and the addition of extra terms in the underlying governing equations and boundary conditions [115]. An attempt could be made to reduce the instantaneous peaks and noise determined in the pressure profile. However as stated by Sarpkaya [102] this is an inherent limitation in using a random vortex based DVM. As such undertaking either of these would require a far more fundamental study of the created solver before these could be re-examined within the present context.

When artificial rivulets were investigating in chapter 3, a brief study was undertaken to examine the effect that the oscillation of these has on overall response, section 3.5.6. However it quickly became apparent that this was a complex problem requiring the focus of individual study. Given that oscillation of artificial rivulets may represent an intermediate step towards a more complete understanding of the RWIV mechanism, numerically simulating this is a logical progression of the present work, particularly as experimental data with which to validate this is beginning to become available [36].

One requirement of any future work with the present code would be to obtain experimental data against which the coupled solver could be, at least partially, validated. Although wind-tunnel studies which allow the natural formation of rivulets have been undertaken [19, 39, 43, 71, 74], these have focused on obtaining data to determine the conditions under which RWIV occurs and not on the rivulets themselves. While ascertaining experimental evidence of some of the features determined herein may be difficult, specific results could be examined using techniques such as those implemented by Cosentino [19]. One pertinent example being the determination of the exact thickness and location of rivulets which form on a horizontal cylinder perpendicular to the free stream. The calculation of transient aerodynamic parameters would also be beneficial.

With regard to the governing evolution equation of the thin film, the present limitation of a uniformly coated surface does not best represent reality where rivulets are only found to form in certain specific locations [19]. Future revisions may remove this by incorporating some representation of contact lines, possibly through use of Tanner laws [136]. While a further logical extension of the present work would be to consider movement of the cylinder, for which Lemaitre [9] calculated the evolution equation before subsequently neglecting. This would enable studies to be undertaken to determine how cable oscillation influences both the formation and evolution of rivulets and aerodynamic response.

Finally while extending the present methodology into three dimensions is a long term goal, this is not the only technique available for numerical simulation of aspects of RWIV. Although not published at the outset of the present work, extending the DES model of Yeo and Jones [54] to incorporate an artificial rivulet would allow comparison of numerical predictions with experimental analysis. It would also enable examinations of how angles of inclination, yaw and rivulet location affect the aerodynamic response.

In Conclusion: the numerical solver presented is the most advanced computational tool for modelling RWIV and successfully predicts a number of the key features of this phenomenon. By determining characteristics such as periodic formation of an upper rivulet, the self limiting thickness of the lower rivulet and the suppression of Karman vortex shedding, the present solver has successfully taken the first steps towards the numerical simulation of RWIV by tracking rivulet formation and evolution. As a 'proof of concept' this thesis has therefore greatly exceeded its original objectives.

Appendix A Derivation of Modified Angles

In section 2.4 two additional angles were defined to help simplify the geometry of the three-dimensional yawed and inclined cable. These were the relative yaw angle β^* which defines the angle between the direction normal to wind and the cable axis, and the angle of attack in the plane normal to the cable axis γ_{θ} . This appendix provides derivations to these two equations, (2.15) and (2.16), illustrations of which can be seen in figure A.1.



Figure A.1: Definitions of angles α , β , β^* , U_{eff} and γ_{θ} , and numbering convention for vertices.

Derivation of Relative Yaw Angle

The incident wind speed U is parallel to sides 1-4, 2-3, 5-8 and 6-7. Assuming that this is horizontal, i.e. in plane 1-2-3-4 or 5-6-7-8, then we can derive the components of the wind parallel and perpendicular to line 5-7 in this horizontal plane as

$$U \parallel U_{5-7} = U \sin \beta \quad \text{and} \quad U \perp U_{5-7} = U \cos \beta.$$
 (A.1)

From these the components parallel and perpendicular to line 1-7 in the plane 1-5-7 can be derived as

$$U \parallel U_{1-7} = U \cos \alpha \sin \beta \quad \text{and} \quad U \perp U_{1-7} = U \sin \alpha \sin \beta.$$
 (A.2)

Therefore the angle between the direction normal to the wind and the cable axis is given by

$$\sin \beta^* = \frac{U \cos \alpha \sin \beta}{U},\tag{A.3}$$

which can be simplified as

$$\beta^* = \arcsin(\cos\alpha\sin\beta). \tag{A.4}$$

This is equivalent to the original definition given by Matsumoto [34], but in a slightly different notation which matches that given later given by Matsumoto [42] and which is more commonly used.

Derivation of Angle of Attack in Plane

Using the derivations of the previous section an effective wind speed in the plane normal to cable axis U_{eff} , can be determined from its perpendicular components, $U \perp U_{5-7} = U \cos \beta$ and $U \perp U_{1-7} = U \sin \alpha \sin \beta$, as derived in (A.1) and (A.2) respectively. The effective wind speed in the plane can therefore be given as
$$U_{eff} = \sqrt{(U \perp U_{5-7})^2 + (U \perp U_{1-7})^2} = U\sqrt{\cos^2\beta + \sin^2\alpha\sin^2\beta}.$$
 (A.5)

From this a derivation of the angle between the stagnation point and the horizontal axis in plane outlined in figure A.1 can be derived as

$$\sin \gamma_{\theta} = \frac{U \sin \alpha \sin \beta}{U \sqrt{\cos^2 \beta + \sin^2 \alpha \sin^2 \beta}}.$$
 (A.6)

Which can subsequently be simplified as

$$\gamma_{\theta} = \arcsin\left(\frac{\sin\alpha\sin\beta}{\sqrt{\cos^2\beta + \sin^2\alpha\sin^2\beta}}\right). \tag{A.7}$$

This agrees with the definition given in Wilde [10] and is equivalent to that originally developed by Matsumoto [34]. The present example being given in a slightly simplified manner.

Appendix B Particulars of DIVEX Implementation

Within section 3.1.2 several disadvantages of vortex methods were highlighted. This appendix provides additional details of the methods applied with the present DIVEX formulation to eliminate or alleviate these.

B.1 Cut-Off Functions

While point vortices provide solutions, as "real vortices are not concentrated singularities of infinite vorticity" [102], they are a potentially large source of error particularly when in close proximity to one another, as is the case at the body surface. Cut-off functions remove this singularity by representing a distribution of vorticity within some 'core radius' σ around the vortex centre. Although these are a mathematical artifice rather than a physical representation [102] several such schemes have been proposed with varying degrees of success. Here however a simple axisymmetric model from Spalart [104] with velocity distribution

$$\boldsymbol{U}(r) = \frac{\Gamma}{2\pi} \frac{r}{r^2 + \sigma^2},\tag{B.1}$$

and stream function

$$\Psi(r) = \frac{\Gamma}{4\pi} \ln\left(r^2 + \sigma^2\right),\tag{B.2}$$

is used, where Γ is the circulation. As can be seen from figure B.1, the velocities induced at large distances are both quantitatively and qualitatively correct but due to the de-singularisation the velocity is only qualitatively correct near the core.



Figure B.1: Vorticity and velocity distributions for various core functions, from Lin [6].

B.2 Operation Count

Using the Biot-Savart law to determine velocity requires $O(N^2)$ operations for a flow field of N vortex particles, this therefore becomes prohibitive at large N. In the present formulation two techniques are used to reduce this, vortex amalgamation and a zonal decomposition algorithm.

B.2.1 Vortex Merging

Vortex merging reduces the operation count by limiting the number of the particles within the flow. This is achieved by merging multiple vortex particles into a single particle should this procedure satisfy given criteria and tolerances. This is set to prevent the merging of vortex particles; with large circulation, that are close to the body surface or which are widely separated. The tolerance presently used being that that the error introduced in induced velocity, ΔU , by merging vortex particles be $< 1 \times 10^{-9} U_{\infty}$. Using this formulation [6] two vortex particles of position vector \mathbf{r}_1 and \mathbf{r}_2 , with circulation Γ_1 and Γ_2 are amalgamated into a singular vortex of strength Γ (B.3) and position \mathbf{r} (B.4) when the difference in induced velocity ΔU (B.5) is less than the specified tolerance.

$$\Gamma = \Gamma_1 + \Gamma_2 \tag{B.3}$$

$$\boldsymbol{r} = \frac{\Gamma_1 \boldsymbol{r}_1 + \Gamma_2 \boldsymbol{r}_2}{\Gamma_1 + \Gamma_2} \tag{B.4}$$

$$\Delta U(\boldsymbol{r}_0) = \frac{|\Gamma_1 \Gamma_2|}{\Gamma} \frac{|\boldsymbol{r}_1 - \boldsymbol{r}_2|^2}{|G_0 - (\boldsymbol{r}_1 - \boldsymbol{r}_0)|^{1.5} |G_0 - (\boldsymbol{r}_2 - \boldsymbol{r}_0)|^{1.5}}$$
(B.5)

Where r_0 is the position vector of the closest node on the body surface and G_0 is a vortex merging parameter which has a large influence close to the body. This is chosen as 0.05 in the present case.

B.2.2 Zonal Decomposition

This technique decomposes the flow field into a series of zones through a 'quadtree' method. In the present formulation [7] these are a hierarchical structure of square zones, each of which contains a number of vortex particles N_p , figure B.2. Provided that a given zone is sufficiently far from the point of interest, which is located at position vector \boldsymbol{r} , then the contribution of the particles within that zone to the velocity can be determined via series expansion rather than by direct

summation from the Biot Savart law, figure B.3. In the present formulation [7] the induced velocity of a given zone, U_z at this point is given as

$$U_z(\mathbf{r}) = \sum_{j=1}^{N_t} \frac{a_j}{(\mathbf{r} - \mathbf{r}_{zc})^j} + \epsilon_z, \qquad (B.6)$$

where the coefficients of each zone a_j are given by

$$a_{j} = \frac{1}{2\pi i} \sum_{k=1}^{N_{p}} \Gamma_{k} (\boldsymbol{r}_{k} - \boldsymbol{r}_{zc})^{j-1} \qquad j = 1, \dots, N_{t}.$$
(B.7)

In which \mathbf{r}_{zc} is the position vector of the zone centre, N_t is the number of terms in the series expansion, ϵ_z is the error due to the truncation expansion and the subscripts j and k are identifiers for the number of terms in the series expansion and the number of particles within a particular zone respectively. A fuller account of the derivation and implementation of (B.6) and (B.7) is given in Taylor [7], suffice to say the operation count is reduced to $O(N + N \log N)$, thus representing a significant saving.



Figure B.2: Sample flow field - Hierarchical zonal decomposition, from Taylor [7].



Figure B.3: Sample flow field - Series expansion, from Taylor [7].

B.3 Surface Vortex Distribution

As the vorticity within the flow originates at the body surface, two zones are defined within the fluid flow domain, the control zone and the wake. Once in the wake the vortex particles that originated in the control zone move according to the equations of convection (3.11) and diffusion (3.12), unless they pass back into the control zone, in which case they are re-absorbed. In the control zone however the body surface is represented by a series of nodes and interconnecting panels which define the surface geometry. Each of these panels is then subdivided further into a number of K_{sp} equal length sub-panels which form a one dimensional vortex sheet, figure 3.3. This varies linearly around the body with the strength of the circulation Γ at each sub-panel determined as the total vorticity within the control zone of that panel.

To calculate the vorticity within that region, requires the solution of a linear system of equations arising from the boundary condition ensuring zero mass flow though each panel. However without a source or sink within the body, for a given number of panels K_p there are only $K_p - 1$ independent equations. The system is thus under defined. Therefore to ensure the solution to this system of equations is unique, one further equation is required. This is based on Kelvin's theorem which states that the rate of change of circulation within a given system is zero in the absence of external vorticity within inviscid flows. For the present case however it can be shown that vorticity which enters the fluid through diffusion from the surface is offset by an equal and opposite loss of vorticity within the body. The resulting circulation condition can therefore be written as

$$\sum_{w} \Gamma_w + \sum_{jp=1}^{K_p} \sum_{mp=1}^{K_{mp}} (\Gamma_{jp})_{mp} + 2\Omega_i A_i = \Gamma_0.$$
(B.8)

Where jp is the panel identifier, mp is the subpanel identifier and Γ_0 is initial amount of circulation present. The resulting system of equations is therefore complete. Unique values of surface vorticity at the nodes and as such the subpanels can then be calculated at each timestep.

Appendix C Mean Curvature of Free Surface

Within section 4.3.1 an approximation of the mean curvature of the free surface κ was given to first order in equation (4.17). This appendix provides additional details as to the derivation of this asymptotic form of the curvature in the thin film limit.

C.1 Derivation

The outward unit normal vector to the free surface r = R + h, where $h = h(\theta, t)$, is given by

$$\mathbf{n} = \frac{\nabla (r - R - h)}{|\nabla (r - R - h)|} = \frac{1}{(r^2 + h_{\theta}^2)^{1/2}} \left(r \mathbf{e}_{(r)} - h_{\theta} \mathbf{e}_{(\theta)} \right),$$
(C.1)

so that the curvature κ of the free surface is given by

$$\kappa = \nabla \cdot \mathbf{n} = \frac{\partial}{\partial r} \left(\frac{r}{(r^2 + h_{\theta}^2)^{1/2}} \right) + \frac{1}{(r^2 + h_{\theta}^2)^{1/2}} - \frac{1}{r} \frac{\partial}{\partial \theta} \left(\frac{h_{\theta}}{(r^2 + h_{\theta}^2)^{1/2}} \right) \quad (C.2)$$

on r = R + h, leading to

$$\kappa = \frac{-(R+h)h_{\theta\theta} + 2h_{\theta}^2 + (R+h)^2}{[(R+h)^2 + h_{\theta}^2]^{3/2}}.$$
(C.3)

For a thin film we scale according to

$$h = \epsilon R h^*, \quad \kappa = \frac{\epsilon}{R} \kappa^*,$$
 (C.4)

where $\epsilon~(\ll 1)$ is the aspect ratio of the film. As such

$$\kappa^* = \frac{-(1+\epsilon h^*)\epsilon h_{\theta\theta}^* + 2\epsilon^2 h_{\theta}^{*2} + (1+\epsilon h^*)^2}{\epsilon[(1+\epsilon h^*)^2 + \epsilon^2 h_{\theta}^{*2}]^{3/2}},$$
(C.5)

which to $O(\epsilon)$, through use of binomial expansion gives

$$\kappa^* \sim \frac{1}{\epsilon} - (h^* + h^*_{\theta\theta}). \tag{C.6}$$

In terms of the original variables this is therefore given by

$$\kappa \sim \frac{1}{R} - \frac{1}{R^2} (h + h_{\theta\theta}). \tag{C.7}$$

Appendix D Analytical Solution for Constant Shear

Within section 4.3.1 an analytical formula for the evolution of an thin film subject to a constant shear was given. This appendix provides additional details as to the derivation of this equation.

D.1 Derivation

The evolution equation for the free surface profile $h(\theta, t)$ is given by

$$h_t + \left[-\frac{1}{3\hat{\mu}R} \left(\hat{\rho}g\cos\theta - \frac{\hat{\gamma}}{R^3} (h + h_{\theta\theta})_{\theta} + \frac{P_{\theta}}{R} \right) h^3 + \frac{Th^2}{2\hat{\mu}R} \right]_{\theta} = 0.$$
(D.1)

If surface tension, gravity and the external pressure gradient are neglected then this becomes

$$h_t + \left(\frac{Th^2}{2\hat{\mu}R}\right)_{\theta} = 0, \qquad (D.2)$$

which is a first order partial differential equation for $h(\theta, t)$. For simplicity we take T to be a constant; then (D.2) becomes

$$h_t + \frac{T}{\hat{\mu}R}hh_\theta = 0. \tag{D.3}$$

The characteristic equations associated with equation (D.3) are

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 0, \quad \frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{Th}{\hat{\mu}R},\tag{D.4}$$

whose solutions are

$$h = \text{constant}, \qquad \theta - \frac{Th}{\hat{\mu}R}t = \text{constant}.$$
 (D.5)

Therefore the general solution $h(\theta, t)$ of (D.3) satisfying the initial condition $h(\theta, 0) = h_0(\theta)$ (where $h_0(\cdot)$ is some prescribed non-negative function satisfying $h_0(\theta + 2\pi) = h_0(\theta)$) is given implicitly by

$$h = h_0 \left(\theta - \frac{Th}{\hat{\mu}R} t \right). \tag{D.6}$$

For example, if $h_0(\theta) = H(1 - a\cos(n\theta))$ (with |a| < 1, n = 1, 2, 3, ...) then (D.6) gives the implicit equation

$$\frac{h}{H} = 1 - a \cos\left[n\left(\theta - \frac{Th}{\hat{\mu}R}t\right)\right] \tag{D.7}$$

determining h as a function of θ and t.

We note that equation (D.6) predicts that h stays non-negative for all t and θ (as expected on physical grounds). Also if $h_0 = 0$ at some station $\theta = \theta_0$ then $h(\theta_0, t) = 0$ for all t, that is, any "three-phase contact line" does not move.

The solution (D.6) will become invalid if the wavelike profile of the free surface 'breaks' at some instant, that is, if $\partial h/\partial \theta$ becomes infinite. Differentiating (D.6) we obtain

$$\frac{\partial h}{\partial \theta} = \frac{h_0'(\xi)}{1 + h_0'(\xi) \frac{T}{\hat{\mu} R} t},\tag{D.8}$$

where

$$\xi = \theta - \frac{Th}{\hat{\mu}R}t; \tag{D.9}$$

Thus the free surface will 'break' at the instant when $1 + h'_0(\xi)Tt/\hat{\mu}R$ first takes the value zero, that is, at the instant $t = t_s$ defined by

$$t_{\rm s} = \frac{\hat{\mu}R}{T} \min_{\xi} \left(-\frac{1}{h_0'(\xi)} \right), \tag{D.10}$$

provided that t_s is positive. If $\xi = \xi_s$ is the value of ξ at which this minimum occurs then by (D.6) and (D.9) the shock occurs at height $h = h_0(\xi_s)$ (= h_s , say)

at position $\theta = \theta_s$ given by

$$\theta_{\rm s} = \xi_{\rm s} + \frac{Th_{\rm s}}{\hat{\mu}R} t_{\rm s} \;(\text{mod}\,2\pi). \tag{D.11}$$

D.2 Example

For example, if $h_0(\theta) = H(1 - a_s \cos(n_s \theta))$ (with $|a_s| < 1, n_s = 1, 2, 3, ...$) then $h'_0(\xi) = n_s H a_s \sin(n_s \xi)$, so that (D.10) gives

$$t_{\rm s} = \frac{\hat{\mu}R}{n_s H a_s T} \min_{\xi} \left(-\frac{1}{\sin(n_s \xi)} \right) \quad (t_{\rm s} > 0); \tag{D.12}$$

and as such

$$\xi_{\rm s} = \frac{3\pi}{2n_s}, \quad t_{\rm s} = \frac{\hat{\mu}R}{n_sHa_sT}, \quad h_{\rm s} = H, \quad \theta_{\rm s} = \frac{3\pi}{2n_s} + \frac{1}{n_sa_s} \; (\bmod \; 2\pi).$$
(D.13)

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